Applications of Green's function methods.

Hartree-Fock approximation

Start with $\Phi_0 = \sum_{\ell} \alpha_{\ell} \phi_{\ell}$ (suppress spin).

and $|\Phi_0\rangle = \text{ground state}.$

Each particle is in or in a single-particle orbital independent from others. Let's try to do a similar approach for an interacting system to describe individual particles or many in an average potential due to other particles.

Overall self-energy in first order:

$$\Sigma^{\text{sc}} = \Sigma^{\text{sc}}(0) + \frac{i}{\hbar} \frac{\partial}{\partial \mu} \Sigma^{\text{sc}}(\mu)$$

gives interaction that is averaged over all occupied shells. ($\Sigma^{\text{sc}}$ gives the response of single-particle density)

If the interaction is strong we cannot use a single term in perturbative expansion.
In terms of propagators:

\[ \begin{align*}
\Gamma &= \Gamma_0 + \frac{i}{2} \langle D_{\text{HF}} \rangle \\
\Sigma &= \Sigma_0 + \Sigma_{\text{HF}} + \Sigma_{\text{HF}}
\end{align*} \]

This helps \[ \Sigma \equiv \Sigma_0 \] to first order.

But it is taken to all orders.

In conclusion, however, since the added particle experiences the interaction from all other particles taken to be free, \[ \Sigma_0 \] appears in loops we can would be self-consistent by writing:

\[ \begin{align*}
\Sigma_{\text{HF}} &= \Sigma + \Sigma_{\text{HF}} \\
\Sigma_{\text{HF}} &= \Sigma_{\text{HF}} + \Sigma_{\text{HF}}
\end{align*} \]
HF equations $\Rightarrow$ self-consistent summation of the first-order contribution to self-energy $\Sigma^\times$. Self-consistent means that HF is not only determined by $\Sigma^\times$ but also some terms $\Sigma^\times$ determine $\Sigma$. This results in a highly-nonlinear equation. Look at the diagrams:

![Diagrams](image)

We know that we can derive HF equations the "standard way" by using first quantization. So why bother with Green's functions? We can see how to improve the example:

$\Sigma^\times \Rightarrow \Sigma_{(1)}^\times + \Sigma_{(2)}^\times + \ldots$