Dyson Equations

\[ \Sigma(q) = \Sigma_0(q) + \Sigma_0(q) \Sigma(q) \Sigma_0(q) \]

The diagrammatic representation:

\[ \Sigma \] contains all graphs (converted to \( \Psi(x) \Psi^*(y) \))

Propagator self energy can be split into two pieces by cutting a single line (Hominiuk)

The proper self energy \( \Sigma^* \) = sum of proper diagrams
Thus:

\[ G(x, y) = G^0(x, y) + \int d^4z \int d^4z' G_0(x, z) \Sigma^x(z, z') G(z', y) \]

In momentum space (\( \Sigma^x(z, z') = \Sigma^x(t, t') \)

for nonrelativistic

\[ G(q) = G^0(q) + G^0(q) \Sigma(q) G(q) \]

This is still a matrix equation in spin.

But for isotropic system \( g_{\alpha\beta} = \delta_{\alpha\beta} G(q) \)
\[ \Sigma^* = \Sigma_{\text{pol}} \Sigma(g) \quad \text{and it becomes an algebraic equation.} \]

\[ \Sigma^*(q) = \frac{1}{[\Sigma_0(q)]^{-1} - i\Sigma^*(q)} \]

Recall

\[ \Sigma_0(q) = \frac{\Theta(1q_1-K_F)}{\omega - \omega_q + i\epsilon} + \frac{\Theta(K_F - 1q_1)}{\omega - \omega_q - i\epsilon} \]

\[ [\Sigma_0(q)]^{-1} = \omega - \omega_q \]

Then

\[ \text{such that } [\Sigma_0(q)]^{-1} \Sigma_0(q) = 1 = \Theta(1q_1-K_F) + \Theta(K_F - 1q_1) \]

Thus, we extract propagators beyond one-loop

\[ \Sigma(q) = \frac{1}{\omega - \omega_q - i\Sigma^*(q)} \]

and proper self-energy

\[ \Sigma(q) \text{ represents } \Sigma \text{ plus } \Sigma^*(q) \text{ gives } \]

renormalized energies of the system.

For \(\text{real } \omega \) then \( \text{Im}(\Sigma^*_{ \text{pol}}(q)) > 0 \) for \( \omega < \mu \)

- outside the Coulomb sphere

\( \text{Im}(\Sigma(q)) \leq 0 \) for \( \omega > \mu \) outside wedge

\[ \Rightarrow \text{this determines } \mu! \]
Example = sum of poles:

\[ \phi = 1 + \sum_{\omega_n} \frac{C}{\omega - \omega_n} + \ldots \]

\[ \Sigma^*(q) \approx \sum_{\omega_n}^* (q) = g V_0(q) \]

\[ G(q) \approx \frac{G_{\text{dis}}}{g V_0(q)} = \frac{1}{\omega - \omega_q - g V_0(q)} \]

This approximation at \( \omega = \omega_q + g V_0(q) \)

can shift in energy \( \Rightarrow \) effect of tunnel summation

is to include effect of mean field interaction of

particle with all the others \( \Rightarrow \) translational invariance

\( \Rightarrow \) \( x \)-independent \( \Rightarrow \) constant.
Polarization insertion.

Here are diagrams that look like

\[ \text{Diagram} \]

So we can also define a "dressed" interaction that includes modifications to \( M(\eta) \).

\[ U_0(\eta) = U_0(\eta) \rho_\eta \delta_\eta \]

For spin-dependent interactions:

\[ U_0(\eta, \rho_\eta, \delta_\eta) = U_0(\eta) U_0(\eta, \rho_\eta, \delta_\eta) \]

\[ M(\eta) = U_0(\eta) + U_0(\eta, \rho_\eta) + \ldots \]
As for the non-split π π υ u υ part

\[ U(q)_{\pi\pi\upsilon\upsilon} = U_0(q)_{\pi\pi} + U_1(q)_{\pi\pi} \]

\[\pi\pi\upsilon\upsilon = \pi\pi\upsilon + \upsilon\upsilon\]

approx 1 w spin independent

interference

\[ U(q) = U_0(q) + U_0(q) \pi^*(q) U(q) \]

\[ \Rightarrow U(q) = \frac{U_0(q)}{1 - U_0(q) \pi^* q} = \frac{U_0(q)}{K(q)} \]

generalized dipole transition

includes effects of screening

vacuum on the interaction U.
Vertex pulls

Proper vertex function

while could be absorbed by the

Which diagram was

work above on the
The idea is to write a system of coupled equations for $E$, $\Pi$, $\Gamma$ that when iterated countably to any order give the total contribution to $G$ to that order. Then these equations give an exact representation of the theory. Solving them is usually impossible, but they are a good starting point for nonsupractic approximation.

Start with skeleton drogous $=0$ (inadmissible)

$\Rightarrow$ remove all self-energy insertions

$\Rightarrow$ 11 polarization and vertex parts.

Example

\[ \begin{align*}
\{ & \rightarrow \\
\{ & \rightarrow \\
\{ & \rightarrow
\end{align*} \]
The theory says:

1) For any diagonal there exists a unique skeleton
2) all graphs can be reconstructed uniquely

The full 7, 11 and minor vertex in all skeletons (with some care to avoid double counting).

Easy to see that skeletons for $E$ and $\Pi^k$ are

\[ \scalebox{0.65}{\text{Diagram}} \]

and

\[ \text{Diagram} \]
So now we claim, that \( E^d \) and \( \Pi^d \) are given by

\[
\Sigma^d \rightarrow \quad + \quad \Pi^d
\]

Note that for hadune diagram, we used only "dress" 

\( G \) not a broad double counting.

The most complicated is the equation for \( \Pi \)

since it cannot be written in a compact form:

\[
\sqrt{\text{"the wall'"}}
\]

\[
\Pi = \frac{1}{4} NN + \text{other terms}
\]
Summary:

- three coupled equations for $Q, U, R$ ($u \in \mathbb{R}$) but had since eq. for $R$ cannot be given in a closed form.

- Dyson eq. give a compact summary of the perturbation theory

- big advantage is that they allow for manageable approximations (e.g. dynamical variational, breaking prints, ...)

- we use "derived" Dyson eq. using perturbative expansion for $Q$ but in fact they can be derived without reference to perturbation series (e.g. from path integral)