This is a closed-book examination. You may not refer to lecture notes, textbooks, or any other course materials. You may use a calculator, but solely for the purpose of arithmetic computation. A list of potentially useful formulas, definitions and relations is given after the problems.

The exam consists of five multiple-choice questions, each worth 2 points for a total of 10 points, and two problems (questions 6 and 7) to be worked out, for a total of 20 points. For the two problems, you must show all work: clearly state and justify any arguments, assumptions, and approximations, as well as the use of any formulas. Unless otherwise specified, evaluate all integrals and derivatives, and perform any arithmetic calculations if a numerical answer is requested.
Multiple Choice Questions

(Choose only one answer and label your answer clearly.)

1. The figure shows a bar moving to the left on two stationary conducting rails connected by a wire. A constant external magnetic field in the region between the rails points out of the page. Which statement is true about the induced current in loop consisting of the bar, conducting rails and wire?

(a) The induced current is zero.
(b) The induced current is clockwise.
(c) The induced current is counterclockwise.
(d) The induced current is nonzero only if the magnetic field is changing with time.

2. The magnitude and direction of the torque on the circular current loop of area $A$ in the uniform external magnetic field shown is given by

(a) $IAB \sin 30^\circ$ into the page
(b) $IAB \sin 120^\circ$ into the page
(c) $IAB \sin 30^\circ$ out of the page
(d) $IAB \sin 120^\circ$ out of the page
3. A bent current-carrying wire that lies in the y-z plane shown above. A uniform magnetic field is directed in the x-direction (out of the page), $\mathbf{B} = B_0 \mathbf{i}$. Which of the following statements is TRUE?

(a) The net force is zero.
(b) The net force is $2IL_1B_0 \mathbf{j}$.
(c) The net force is $-2IL_1B_0 \mathbf{j}$.
(d) The net force is $IL_2B_0 \mathbf{j}$.
(e) The net force is $-IL_2B_0 \mathbf{j}$.

4. Which of the following statements is NOT true?

(a) The time constant $\tau = R/L$ determines the rate at which an inductor $L$ allows current $I(t)$ to change.
(b) At long times, when steady state currents are achieved in a circuit, an inductor behaves like a open switch.
(c) A back-emf is generated by an inductor when a switch is opened.
(d) When the current through an inductor is decreasing, the induced emf is in the same direction as the current flow.

5. Which of the following statements is NOT true?

(a) Both electric charges and time-varying magnetic fields can generate electric fields.
(b) Electric field lines that result from time-varying magnetic fields are closed, unlike electric field lines due to static charges which must start and end charges of opposite sign.
(c) Lenz's law states that the direction of the induced current is such that the resulting induced magnetic field opposes the change in flux that gives rise to it.
(d) The induced emf in Faraday's law depends on whether the magnetic flux varies due to motion of the source of the external magnetic field or as a result of a time-varying magnetic field.
Problems

(You must show all your work clearly.)

6. (6 points) A toroidal magnetic field is formed by winding $N$ turns of wire on a doughnut shaped form. By applying Ampere’s law around the path of integration shown (Amperian loop) find the $B$-field at the indicated radius $r$. (Note that by symmetry, inside the toroid, the $B$-field is tangent to the circular loop which is concentric with the toroid and zero outside the toroid.) What is the direction of the $B$-field, along or opposite the direction of integration around the loop?
7. A long solenoid of radius $a$, length $d$ and $N$ turns carries current $I$, as shown.

(a) (2 points) What are the magnitude and direction of the magnetic field of the solenoid?

(b) (4 points) Suppose the current in the solenoid varies with time, according to $I(t) = I_0 e^{-rt}$, where $I_0$ and $r$ are positive constants. What is the magnetic flux through a loop of radius $b < a$ concentric with the solenoid, as shown in the figure?

(c) (4 points) What is the induced electric field at radius $b$? Is the direction clockwise or counter clockwise (where the direction of the current $I$ in the solenoid is counter clockwise, as shown)?

(d) (2 points) If a copper ring of radius $b$ is placed inside the solenoid, what is the induced emf in the ring as a function of time?

(e) (2 point) If the copper ring has resistance $R$, what is the induced current in the ring (magnitude and direction)?
Possibly useful formulas, etc

- Lorentz force law: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$
- Force on current carrying wire segment: $d\vec{F} = I d\vec{s} \times \vec{B}$
- Magnetic dipole moment: $\vec{\mu} = IAN\hat{n}$
- Torque on magnetic dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$
- Potential energy of a magnetic dipole in an external magnetic field: $U = -\vec{\mu} \cdot \vec{B}$
- Biot-Savart law:
  \[
  d\vec{B} = \frac{\mu_0 I d\vec{s} \times \vec{r}}{4\pi r^2}
  \]
- Magnetic field of a long, straight wire: $|\vec{B}| = \frac{\mu_0 I}{2\pi r}$
- Magnetic field on the axis of a current loop of radius $R$ at height $h$:
  \[
  |\vec{B}| = \frac{\mu_0 I R^2}{2(R^2 + h^2)^{3/2}}
  \]
- Magnetic field at the center of an arc $\phi$ of radius $R$:
  \[
  |\vec{B}| = \frac{\mu_0 I \phi}{4\pi R}
  \]
- Magnetic field of a long solenoid: $|\vec{B}| = \mu_0 I n$
- Magnetic flux through closed surface:
  \[
  \Phi_B = \oint_S \vec{B} \cdot d\vec{A} = 0
  \]
- Magnetic flux through open surface:
  \[
  \Phi_B = \int_S \vec{B} \cdot d\vec{A}
  \]
  If $\vec{B}$ is uniform over surface $S$, then $\Phi_B = \vec{B} \cdot \vec{A}$.
- Ampere's law: $\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed by loop } C}$
- Faraday's law: $\mathcal{E}_{\text{induced}} = \oint_C \vec{E} \cdot d\vec{s} = -d\Phi_B/dt$
- Time constant of RL circuit: $\tau = L/R$
- Energy stored in inductor: $U_L = \frac{1}{2}LI^2$