In the figure, a non-conducting semi-circular rod of radius $R$ has a total charge $Q$ uniformly distributed along its length.

(a) What is the linear charge density, $\lambda$, of the rod?

(b) What is the direction of the electric field at the center of the circle $O$ due to two elements of charge, $dq$, labeled in the figure symmetrically located at angle $\theta$ with respect to the $y$-axis?

(c) What is the magnitude of the electric field at $O$, due to these two infinitesimal charge elements (in terms of $k$, $dq$, $R$ and $\theta$)?

(d) Express $dq$ in terms of $R$, $d\theta$ and the linear charge density $\lambda$. (Hint: What is the differential element of arc length, $ds$, subtended by $dq$ in terms of $R$ and $d\theta$?)

(e) Set up an integral for the magnitude of the total electric field at $O$ due to the semi-circular line of charge, and evaluate it. Express your final answer in terms of $k$, $Q$ and $R$. 

![Diagram of a semi-circular rod with labeled charges and angles $\theta$.]
(a) \[ \lambda = \frac{\text{linear charge density}}{\text{total charge}} = \frac{\text{total charge}}{\text{total length}} = \frac{Q}{\frac{1}{2}(2\pi R)} = \frac{Q}{\pi R} = \frac{Q}{\text{length of semicircle}} \]

(b) Direction of the electric field due to infinitesimal charge elements \( Q_1 \) and \( Q_2 \) is entirely \( \hat{z} \) (see figure):

(c) \[ dE_n + 0 = dE_x + dE_y = (dE_x \hat{i} - dE_y \hat{j}) + (dE_x \hat{i} - dE_y \hat{j}) \]

Now, \( |dE_i| = 1 \) and \( dE_x = dE_y \Rightarrow |dE| \sin \theta \)

\[ dE_y = dE_x \cos \theta \]

So,
\[ dE_n + 0 = 2|dE\| \cos \theta \hat{z} \]

\[ = 2 \frac{kQ}{R^2} \cos \theta \hat{z} \]

Note: Infinitesimal element \( d\theta \) change gives rise to infinitesimal contribution to E-field at \( O \).

(d) \[ dq = \lambda ds \]

\[ \leftarrow \frac{\text{change in charge}}{\text{length}} \]

\[ ds : \text{an length subtended by} \ dq = \frac{R \ d\theta}{\text{angle}} \]

\[ \Rightarrow dq = \lambda R \ d\theta \]
(c) \[ E_{a+0} = \int_{\text{semicircle}} dE_{a+0} \]

\[ = \int_{\text{semicircle}} \frac{2k}{R^2} d\theta \cos \theta \left( \vec{\jmath} \right) \]

\[ = \left( \frac{2k}{R^2} \right) \int_{0}^{\pi/2} \cos \theta \, d\theta \cdot \hat{\jmath} \]

Note: Integrate over \( \theta \in \left[ 0, \frac{\pi}{2} \right] \) since we have considered change pairs.

\[ = \frac{2k}{R} \left( \frac{\pi}{2} \right) \sin \theta \bigg|^{\pi/2}_0 = 1 \]

\[ = \frac{2k}{\pi R^2} \left( \vec{\jmath} \right) \]