The figure shows a wire of total length $L$ that is bent in the middle. The wire carries a current $I_0$ in the direction shown. Note that the $z-$axis is oriented out of the page in the coordinate system shown. Recall that the magnetic force on a straight current-carrying wire of length $l$ is $\vec{F} = I l \times \vec{B}$.

(a) What is the magnitude and direction of the force on the wire if it is in a magnetic field given by $\vec{B} = B_0 \hat{k}$?

(b) What is the magnitude and direction of the force on the wire if it is in a magnetic field given by $\vec{B} = B_0 \hat{j}$?

Two ways to approach this problem:

1. Component-wise
2. Using R.H.R.

$\vec{F}_1 = \frac{B}{l} \left( \sin \frac{\theta}{2} \hat{\jmath} + \cos \frac{\theta}{2} \hat{k} \right)$

$\vec{F}_2 = \frac{B}{l} \left( \sin \frac{\theta}{2} \hat{\jmath} - \cos \frac{\theta}{2} \hat{k} \right)$

$\vec{F}_1 = I_1 I_2 \times \vec{B} = I_1 \frac{B}{l} \left( \sin \frac{\theta}{2} \hat{\jmath} + \cos \frac{\theta}{2} \hat{k} \right) \times B_0 \hat{k}$

$\vec{F}_2 = I_2 \frac{B}{l} \left( \sin \frac{\theta}{2} \hat{\jmath} - \cos \frac{\theta}{2} \hat{k} \right) \times B_0 \hat{k}$

(a) $\vec{F}_{1_{\text{hom}}} = \vec{F}_1 + \vec{F}_2 = -I_0 l B_0 \sin \frac{\theta}{2} \hat{j}$

(b) $\vec{F}_{1_{\text{hom}}} = \vec{F}_1 + \vec{F}_2 = I_0 l B_0 \cos \frac{\theta}{2} \hat{k}$
From R.H.R. \( \mathbf{F}_1 = I I_x x \mathbf{B} \) is \( \perp \) to leg \( \mathbf{3} \) of wire and to \( \mathbf{B} = \mathbf{B} \hat{k} \)

\[ \mathbf{F}_2 = I I_x x \mathbf{B} \]

as shown.

Since wire is in \( x-y \) plane \( \perp \) to \( \mathbf{B} \)

\[ \mathbf{F}_1 = I |I_x| |B| \cdot \sin \frac{\pi}{2} = I_x \frac{1}{2} - B_x \]

\[ \mathbf{F}_2 = I |I_x| |B| \cdot \sin \frac{\pi}{2} = I_x \frac{1}{2} - B_x \]

\[ \mathbf{F}_{\text{total}} = \mathbf{F}_1 + \mathbf{F}_2 = \left[ \mathbf{F}_1 \left( \sin \frac{\pi}{2} \hat{\mathbf{e}} - \cos \frac{\pi}{2} \hat{\mathbf{e}} \right) \right] + \mathbf{F}_2 \left( -\sin \frac{\pi}{2} \hat{\mathbf{e}} - \cos \frac{\pi}{2} \hat{\mathbf{e}} \right) \]

\[ = -I_x L B_x \sin \frac{\pi}{2} \hat{j} \]

From R.H.R. \( \mathbf{F}_1 = I I_x x \mathbf{B} \) is \( \perp \) to leg \( \mathbf{3} \) of wire and to \( \mathbf{B} = \mathbf{B} \hat{k} \)

\[ \mathbf{F}_1 = I I_x x \mathbf{B} \]

\[ = I_x \frac{1}{2} - B_x \sin \left( \frac{\pi}{2} \right) \hat{k} \]

\[ \mathbf{F}_2 = I I_x x \mathbf{B} \]

\[ = I_x \frac{1}{2} - B_x \sin \left( \frac{\pi}{2} \right) \hat{k} \]

\[ \mathbf{F}_{\text{total}} = \mathbf{F}_1 + \mathbf{F}_2 = 0 \]