(a) $Q = CV \Rightarrow C = \frac{Q}{V} = \frac{7.0 \times 10^{-12} \, C}{20 \, V} = 3.5 \times 10^{-12} \, F$

(b) The capacitance of a system of conductors depends on their geometry (size and shape of conductors and their separation). Therefore, keeping the geometry fixed, the capacitance remains unchanged.

(c) $V = \frac{Q}{C} = \frac{200 \times 10^{-12} \, C}{3.5 \times 10^{-12} \, F} = 57.1 \, V$

---

HRW 25.11

Before that happens, what is the change on $C_1$? Find $C_{eq}$ first:

$C_{1, eq}$ in parallel: $C_{1, eq} = C_1 + C_2$

$C_{1, eq}$ in series: $C_{1, eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$

Change on $C_{eq}$: $\Delta V = \frac{V \cdot C_3 \cdot (C_4 + C_5)}{C_1 + C_2 + C_3}$

This is the same as the change on $C_{eq}$ and $C_2$. Hence, the potential difference across $C_{12}$ is:

$\Delta V = \frac{Q \cdot C_3 \cdot (C_4 + C_5)}{C_{12}} = \frac{V \cdot C_3 \cdot (C_4 + C_5)}{C_1 + C_2 + C_3}$

---

HRW 25.8

$c_{12}$ and $c_2$ are in series (because $V_0 = V_a \neq V_2 = V_b$, i.e., the potential difference across $C_1$ and $C_2$ are not necessarily equal; in fact $V_a = V_b = (V_0 - V_a) + (V_c - V_b)$)

$\Delta V$ across $C_1$ and $C_2 = \Delta V$ across $C_1 + \Delta V$ across $C_2$

which is fine for capacitors in series.

$\Rightarrow \frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C_{12}} = C_1 + C_2 \Rightarrow C_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2} = 10.5 \, \mu F$

$c_{12}$ and $c_2$ are in parallel (because the potential difference across $C_{12}$ is the same as that across $C_2$; points A and $A'$ and $C$ and $C'$ are equivalent, and hence at the same potential): $C_{eq} = C_0 + C_3 = 3.3 \, \mu F$

$\Rightarrow C_{eq} = C_0 + C_3 = 3.3 \, \mu F$
\[ AV_{\text{across } C_2} = AV_{\text{across } C_1} \text{ and } AV_{\text{across } C_3} \text{ which are in parallel. Therefore,} \]

\[ Q_1 = AV_{\text{across } C_1} \cdot C_1 = \frac{V \cdot C_2}{C_1 + C_2 + C_3} \]

\[ C_1 = \frac{V \cdot C_2}{C_1 + C_2 + C_3} \frac{C_1 + C_2 + C_3}{C_1 + C_2 + C_3} \]

When \( C_3 \) shunts out:

\[ \frac{100 \cdot 4}{10 + 5 + 4} = \frac{400}{19} = 21.1 \mu C \]

\[ AV'_{\text{across } C_1} = V \Rightarrow Q_1' = C_1 \cdot AV'_{\text{across } C_1} = C_1 \cdot V \]

\[ = 10 \cdot 100 = 1000 \mu C \]

Therefore, \( Q_1' = Q_1 = 1000 - 211 = 789 \mu C \).

Before \( C_3 \) shunts out:

\[ AV_{\text{across } C_1} = \frac{V \cdot C_2}{C_1 + C_2 + C_3} = \frac{100 \cdot 4}{10 + 5 + 4} = \frac{400}{19} = 21.1 \text{ V} \]

After \( C_3 \) shunts out:

\[ AV'_{\text{across } C_1} = V = 100 \text{ V} \]

Therefore, \( AV' - AV = 100 - 21.1 = 78.9 \text{ V} \)
Hence, $C_1$ and $C_2$ are two capacitors in series. The equivalent capacitance is

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

where $C_1 = K_1 \cdot \frac{\varepsilon_o A}{d_1} = 11 \cdot \left( \frac{8.85 \times 10^{-12} \text{ F}}{m} \cdot \frac{7.89 \times 10^{-4} \text{ m}^2}{2.31 \times 10^{-3} \text{ m}} \right) = 11 \cdot \left(3 \times 10^{-12} \text{ F} \right) = 33 \text{ pF}$

$$C_2 = K_2 \cdot \frac{\varepsilon_o A}{d_2} = 12 \cdot \left(3 \times 10^{-12} \text{ F} \right) = 36 \text{ pF}$$

$$\Rightarrow C_{eq} = \frac{K_1 K_2 C_1^2}{(K_1 + K_2) C} = \frac{K_1 K_2}{K_1 + K_2} \left( \frac{2 \times 10^{-12} \text{ F}}{11 + 12} \right) = 17.2 \text{ pF}$$

First, find energy stored in fully charged capacitor:

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \left( \frac{\varepsilon_o A}{d} \right) V_{bat}^2$$

When the capacitor is disconnected from the battery and pulled apart to a separation $d'$, the quantity that is recharged is the change on the plates ($Q' = Q = CV_{bat}$):

$$U' = \frac{1}{2} \frac{Q'}{C'} = \frac{1}{2} \frac{Q^2}{C'} = \frac{1}{2} \left( \frac{CV_{bat}^2}{C'} \right) = \frac{1}{2} V_{bat}^2 \left( \frac{\varepsilon_o A / d}{\varepsilon_o A / d'} \right) = \frac{1}{2} V_{bat}^2 \frac{\varepsilon_o A}{d} \frac{d'}{d} = U \cdot \frac{d'}{d} = U$$

The new potential difference between the plates is

$$V' = \frac{Q' / C'}{C} = \frac{Q}{C'} = \frac{C V_{bat}}{C'} = \frac{\varepsilon_o A}{d} \frac{V_{bat}}{d'} = V_{bat} \cdot \frac{d'}{d}$$

(a) $V' = V_{bat} \cdot \frac{d'}{d} = (6 V) \cdot \frac{3 \text{ mm}}{9 \text{ mm}} = 16 V$

(b) $U = \frac{1}{2} \left( \frac{\varepsilon_o A}{d} \right) V_{bat}^2 = \frac{1}{2} \left( \frac{8.85 \times 10^{-12} \text{ F}}{m} \frac{8.5 \times 10^{-4} \text{ m}^2}{3 \times 10^{-3} \text{ m}} \right) (6 V)^2 = 4.5 \times 10^{-11} \text{ J}$
(c) \( W' = U \cdot \frac{1}{d} = \left(4.5 \times 10^{-11} \text{ J}\right) \cdot \frac{3 \text{ mm}}{3 \text{ mm}} \)

\[ W' = 1.2 \times 10^{-10} \text{ J} \]

(d) \( W = U_f - U_i = U' - U = U \left(\frac{8}{3} - 1\right) = \frac{5}{3} U \)

\[ W = 3.5 \times 10^{-10} \text{ J} \]

H&N 25-67] The stationary plates are all at the same potential, and the movable plates are also all at the same potential. Hence, the variable air-gap capacitor constitutes a parallel capacitor, where

\[ C_{eq} = C_1 + C_2 + \ldots + C_n \]

The maximum capacitance is achieved when the movable plates are rotated such that the entire area \( A \) of each movable plate is adjacent to the corresponding area of the stationary plates:

\[ C_{eff} = \varepsilon_0 A \frac{1}{d} \]

\[ \Rightarrow C_{eq} = n \cdot C_1 = n \cdot \varepsilon_0 A \]

Given the geometry, 4 pairs of movable and stationary plates constitute 7 capacitors:

\[ C_{eq} = 7 \varepsilon_0 A = 7 \left(8.85 \times 10^{-12}\right)\left(1.25 \times 10^{-9}\right) \approx 2.3 \times 10^{-12} \text{ F} \]
Note: That for capacitors in series, the charge stored on each capacitor is the same, and equal to the charge stored on the equivalent capacitor:

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1C_2} \Rightarrow C_{eq} = \frac{C_1C_2}{C_1 + C_2}
\]

\[
Q = C_{eq}V = \frac{C_1C_2V}{C_1 + C_2} = 2.8 (800) = 480 \mu C
\]

The potential difference across the capacitor is the same:

\[
\frac{\Delta V'_{across C_1}}{\Delta V'_{across C_2}} = \frac{Q_1'}{Q_2'} = \frac{\frac{Q_1}{C_1}}{\frac{Q_2}{C_2}} \quad (2)
\]

Solving (1) and (2) for \(Q_1'\) and \(Q_2'\):

\[
\begin{align*}
(a) \quad \text{change on } C_1: & \quad Q_1 = Q = 480 \mu C \\
(b) \quad \Delta V_{across C_1} = \frac{Q_1}{C_1} = \frac{480 \mu C}{2 \mu F} = 240 V \\
(c) \quad \text{change on } C_2: & \quad Q_2 = Q = 480 \mu C \\
(d) \quad \Delta V_{across C_2} = \frac{Q_2}{C_2} = \frac{480 \mu C}{8 \mu F} = 60 V
\end{align*}
\]

The charged capacitors are disconnected from the battery and connected to each other with plates of the same sign wired together:

\[
\Delta V'_{across C_1} = \frac{Q_1}{C_1} = \frac{768 \mu C}{8 \mu F} = 96 V = \Delta V'_{across C_1}
\]

Once the switch is closed, charges \(Q_1' + Q_2' = Q + Q = 2Q\) redistribute among the top two plates, with \(Q_1'\) on \(C_1\) and \(Q_2'\) on \(C_2\). Similarly, the charge on the bottom plate redistribute, with \(-Q_1'\) on \(C_1\) and \(-Q_2'\) on \(C_2\). We have:

\[
Q_1' + Q_2' = 2Q \quad (1)
\]

The potential difference across the capacitors is the same:

\[
\frac{\Delta V'_{across C_1}}{\Delta V'_{across C_2}} = \frac{Q_1'}{Q_2'} \quad (2)
\]

Solving (1) and (2) for \(Q_1'\) and \(Q_2'\):

\[
\begin{align*}
(a) \quad Q_1' + Q_2' = 2Q \Rightarrow Q_1' = \frac{2Q}{1 + \frac{C_2}{C_1}} = \frac{2 \times 480 \mu C}{1 + \frac{8}{2}} = 192 \mu C \\
(b) \quad \Delta V'_{across C_1} = \frac{Q_1'}{C_1} = \frac{192 \mu C}{2 \mu F} = 96 V \\
(c) \quad Q_2' = 2Q - Q_1' = 2 \times 480 - 192 = 768 \mu C \\
(d) \quad \Delta V'_{across C_2} = \frac{Q_2'}{C_2} = \frac{768 \mu C}{8 \mu F} = 96 V = \Delta V'_{across C_2}
\end{align*}
\]

If the charged capacitors are instead disconnected from the battery and connected to each other with plates of the opposite sign wired together:
\[ Q_1 + (-Q_2) = Q + (-Q) = 0 \]

They will discharge: the charge on the plates will be zero, and the potential difference across the plates will be zero.

1) \( Q'_1 = 0 \)
2) \( \Delta V' \text{ across } C_1 = 0 \)
3) \( Q'_2 = 0 \)
4) \( \Delta V' \text{ across } C_2 = 0 \)