HW 26.8 a) Current density \( \vec{J} = n \times \vec{v} \cdot \frac{\text{charge}}{\text{area}} \)
\[ \vec{J} = (8.7 \times 10^6 \text{ protons}) \times (1.6 \times 10^{-19} \text{ C}) \times (4.7 \times 10^2 \text{ m/s}) \]
\[ = 6.5 \times 10^{-6} \text{ A/m}^2 \]

b) Integral over the half-sphere can be explicitly done in spherical polar coordinates to give the same result. Try it!

\[ I = \left( 6.5 \times 10^{-6} \text{ A} \right) \cdot \pi \cdot (2.4 \times 10^6 \text{ m})^2 = 8.3 \times 10^3 \text{ A} \]

Note that although the solar wind impacts half of the earth's surface area \((\frac{1}{2} \cdot 4\pi R_e^2)\), not all patches of area contribute equally to the number of protons captured per second:

\[ I = \int \vec{J} \cdot d\vec{A} \]

贴片 1: \( \vec{J} \cdot d\vec{A} = J dA \cos \theta \)

半球面

贴片 2: \( \vec{J} \cdot d\vec{A} = J dA \cos \theta \)

\[ = 0 \]

Therefore, \( I = \left( \frac{1}{2} \cdot 4\pi R_e^2 \right) \cdot |\vec{J}| \)

From conservation of charge, if the protons impacting the Earth's surface were to go through it, then the same number would have to leave through the shaded great circle which represents the projected area of the Earth's half-sphere. So,

\[ I = \int (\vec{J} \cdot (d\vec{A})) = \int J dA = \int \text{uniform} dA = \pi R_e^2 \]

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Due: 10/23/06
The total current density is the sum of the current densities due to 
the positive and negative ions:

\[ \mathbf{j} = \mathbf{j}_+ + \mathbf{j}_- = n_+ e \mathbf{v}_+ + n_- e \mathbf{v}_- \]

\[ \mathbf{v}_- = -n_- e \mathbf{V}_{\text{diff}} \mathbf{k} + v_- e \mathbf{V}_{\text{diff}} \mathbf{k} \]

\[ \mathbf{v}_+ = (n_+ + n_-) e \mathbf{V}_{\text{diff}} \mathbf{k} \]

Note that an upward current density due to negative charge carriers 
equal to \((n_- e \mathbf{V}_{\text{diff}})\) is the same as a downward current density due 
to positive charge carriers.

(a) We know \( \mathbf{J} \) and \( \mathbf{E} \) are related by:

\[ \mathbf{J} = \sigma \mathbf{E} \]

\( \sigma \) : conductivity of medium \( \) (in this case, air)

\[ |\mathbf{J}| = \sigma |\mathbf{E}| = \left(2.7 \times 10^{-14} \text{ A/m}^2\right) \left(120 \text{ V/m}\right) = 3.24 \times 10^{-12} \text{ A/m}^2 \]

Recall that \( \mathbf{F}_m = q \mathbf{E} \); therefore, \( \mathbf{F}_m^+ = (e) \mathbf{E} \) \( \) (downward),
and \( \mathbf{F}_m^- = (e) \mathbf{E} \) \( \) (upward). So the positive charge carriers 
are moving down and the negative charge carriers are moving up.
Since the amount of the charge on the carriers is the same \( \) (singly 
charged positive and negative ions) and the \( \mathbf{E} \)-field is uniform,
the drift speed for the positive and negative ions is the same. Therefore,

\[ \mathbf{v}^+_{\text{shift}} = \mathbf{v}_{\text{diff}} (-\mathbf{k}) \text{ and } \mathbf{v}^-_{\text{shift}} = \mathbf{v}_{\text{diff}} (+\mathbf{k}) \]

(b) Hence,

\[ |\mathbf{J}| = (n_+ + n_-) e \mathbf{V}_{\text{diff}} \Rightarrow \mathbf{V}_{\text{diff}} = \frac{|\mathbf{J}|}{(n_+ + n_-) e} = \frac{3.24 \times 10^{-12} \text{ A/m}^2}{[(6.2 + 5.5) \times 10^9 \text{ m}^2]/[1.6 \times 10^{-19} \text{ C}]} = 1.73 \times 10^2 \text{ m/s} \]

= 173 cm/s
HRW 26.43) (a) Cost of running 100 W light bulb continuously for 31 days:

\[ \text{Cost} = \left(100 \, \text{W}\right) \left(31 \times 24 \, \text{h/d}\right) \left(\$0.06 \, \text{h/W}^{\text{th}}\right) \]

\[ = \$446 \]

(b) For a light bulb rated at 100 W for a standard 120 V outlet:

\[ P = \frac{IV}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120 \, \text{V})^2}{100 \, \text{W}} = 144 \, \Omega \]

(c) Current in the bulb when connected to 120 V outlet is:

\[ I = \frac{P}{V} = \frac{100 \, \text{W}}{120 \, \text{V}} = 0.83 \, \text{A} \]

HRW 26.47)

We are given:

- \( L_c = 1 \, \text{m} \)
- \( R_c = 2 \times 10^{-6} \, \text{m} \)
- \( R_D = 1 \times 10^{-6} \, \text{m} \)

Diameter of C = 1 mm = 1 \times 10^{-3} \, \text{m} \Rightarrow R_c = 5 \times 10^{-4} \, \text{m}

Diameter of D = 0.5 mm = 5 \times 10^{-4} \, \text{m} \Rightarrow R_D = 2.5 \times 10^{-4} \, \text{m}

Current J = 2 A flows through both wires. First, find resistances \( R_c \) and \( R_D \): 

\[ R_c = \frac{R_c L_c}{A_c} = \left(2 \times 10^{-4} \, \text{m}^{2}\right) \left(1 \, \text{m}\right) = 2.5 \, \Omega \]

\[ R_D = \frac{R_D L_D}{A_D} = \left(1 \times 10^{-4} \, \text{m}^{2}\right) \left(1 \, \text{m}\right) = 2 \, \Omega \]

HRW 26.50) Current, \( I \), and current density, \( \mathbf{J} \), are related according to:

\[ I = \int_S \mathbf{J} \cdot d\mathbf{A} \]

S: disc, radius \( R \)

In the circular wire given, assume \( \mathbf{J} \) is uniform (i.e., has constant magnitude and direction) and pointing out of the page (i.e., perpendicular to the cross-sectional area of the wire). Hence,

\[ \mathbf{J} \cdot d\mathbf{A} = |\mathbf{J}| |d\mathbf{A}| \cos \theta = \mathbf{J} \cdot d\mathbf{A} \]

\[ \theta = 0 \] since \( \mathbf{J} \) and \( d\mathbf{A} \) pointing out of page

If \( S \) is a disc of radius \( r \), then the current carried in that disc is
\[ J = \frac{\int \mathbf{J} \cdot d\mathbf{A}}{S} = \frac{\int \mathbf{J} \cdot dA}{\text{Area of } S} = J \cdot (\text{Area of } S) \]

\[ J = J \cdot \pi r^2 \]

Hence, \( J \) vs. \( r^2 \) is a straight line with slope = \( \pi J \).

(a) Yes, \( J \) is uniform, since \( J \) vs. \( r^2 \) is a straight line.

(b) \( \text{slope} = \pi J \Rightarrow J = \frac{\text{slope}}{\pi} = \frac{5 \times 10^{-3} \text{ A}}{(4 \times 10^{-3} \text{ m}^2)} = 1.25 \times 10^{-3} \text{ A} \)

\[ J = \frac{1}{\pi} \left(1250 \text{ A/m}^2\right) = 398 \text{ A/m}^2 \]

When the \( E \)-field in the gas discharge tube (\( \approx V_{\text{applied}} \)) exceeds the breakdown field for the gas in the tube, the gas ionizes into positive ions and negative electrons.

(a) Current is defined as the total positive charge flowing through a given cross-sectional area per unit time. Recall that electrons moving left (toward positive terminal) is equivalent to singly charged positive ions moving right (toward negative terminal). Hence,

\[ I = \frac{\Delta Q}{\Delta t} = \frac{\left(3.1 \times 10^{18} + 1.1 \times 10^{18}\right) \times 1.6 \times 10^{-19} \text{ C}}{1 \text{ sec}} = 0.67 \text{ A} \]

(b) The current is toward the negative terminal.