1. A planet with uniform mass density is a sphere of mass $M = 6 \times 10^{24}$ kg and radius $R$, with two mountains each of mass $m = 10^{12}$ kg located at diametrically opposite points on the planet’s surface. Denote this direction by $\hat{e}_3$. The planet rotates with an angular speed $\omega = 2\pi$ radians per 0.1 Earth day. The direction of $\vec{\omega}$ makes a 45° angle with $\hat{e}_3$.

(a) Find the principal moments of inertia of the planet. (The moment of inertia of a uniform sphere of mass $M$, radius $R$ is given by $2MR^2/5$.)

(b) If the external torques acting on the planet are very weak, such that its rotational motion can be considered torque-free, what is the rate $\Omega$ that the rotation axis precesses about $\hat{e}_3$?

2. A uniform washer of mass $M$ with outer radius $a$ and inner radius $a/2$ oscillates about a pivot point $P$. Find the frequency of small oscillations.

3. You perform rotations about a set of inertial axes, with the following Euler angles: $\phi = 45^\circ$, $\theta = -30^\circ$, $\psi = 60^\circ$.

(a) Sketch the location of the principal axes of the body-fixed system, relative to the inertial axes.

(b) Find the rotation matrix $A$. 
4. A particle is projected across a frictionless horizontal table with velocity \( \vec{v}_0 = v_0 \hat{e}_2 \). Determine to first order in the Earth’s angular frequency, \( \omega_e \):

(a) the displacement, \( \vec{r}(t) \), from the point of origin
(b) the normal force, \( N \), of the table on the particle

5. A uniform 2d disk of mass, \( M \), radius, \( a \), spins without slipping on a horizontal surface. At \( t = 0 \), the angular velocity of the disk is along the \( \hat{e}_2 \) axis, which is in the \( e_3^0 \) direction of the inertial lab frame (pointing vertically); hence, \( \theta(t = 0) = \pi/2 \). Suppose the motion of the disk is such that its center of mass falls slowly enough that we can take, \( d^2 \theta / dt^2 \approx 0 \), and choose the center of mass of the disk as the origin of the inertial lab frame.

(a) Set up the (approximate) Lagrangian for the disk in terms of Euler angles \( \phi, \theta \) and \( \psi \), neglecting the motion of the center of mass.
(b) Find the conserved generalized momenta for this system.
(c) Find \( \dot{\phi} \) and \( \dot{\psi} \) in terms of the angle \( \theta \); in particular, show that

\[
\dot{\phi} = \frac{c}{\sqrt{\sin \theta}},
\]

and find the constant \( c \).