Indiana University
Physics P521 Problem Set 4

1. A particle of mass $m$ moves in a three dimensional harmonic oscillator potential,

$$V(r) = \frac{1}{2}kr^2.$$ 

(a) Show that in general, the particle's orbit is an ellipse, centered at $r = 0$.
(b) Relate the parameters of the ellipse to the particle's angular momentum, $\ell$, and energy, $E$.

2. Consider the motion of a particle of mass $m$ in an attractive exponential potential

$$V(r) = -ke^{-r/a} \quad (k > 0, a > 0).$$

(a) Plot the effective potential for angular momentum $\ell$, and describe the different types of motion that can occur. Be sure to consider both small and large values of $\ell$.
(b) Find the condition which must be satisfied by $k, a, m$, and $\ell$ for there to be stable circular orbits in this potential.
(c) Find the frequency of small oscillations about a stable circular orbit of radius $R$, and compare it to the orbital frequency of the circular orbit. Can this perturbed orbit be closed?

3. A planet of mass $m$ is in a circular orbit around a massive star of mass $M$ ($m \ll M$). As a result of a supernova explosion, the star suddenly loses exactly 20% of its mass. The explosion is spherically symmetrical, and the ejected material does not directly affect the motion of the planet at the instant of explosion.

(a) Make a clearly labeled sketch showing the original orbit, the position of the planet at the moment of the explosion and the new orbit.
(b) Calculate the length of the semimajor axis of the new orbit as a multiple of the original orbit radius.