Indiana University
Physics P521 Problem Set 8

1. A two-dimensional square of uniform density has a smaller square removed, as shown. The total mass of the resulting object is \( M \). The \( \hat{x} \) and \( \hat{y} \) axes are shown, and \( \hat{z} \) points out of the page.

(a) Find the moments of inertia of this object for rotation about the \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \) axes.
(b) The system undergoes torque-free rotation about each axis. Describe whether the rotation is stable or unstable with respect to rotation about each of the axes.

2. You make a series of rotations through Euler angles with \( \phi = 45^\circ \), \( \theta = 30^\circ \) and \( \psi = 30^\circ \).

(a) Draw a picture of the location of the rotated axes \( \{ \hat{e}_1, \hat{e}_2, \hat{e}_3 \} \) following these rotations.
(b) Find the rotation matrix \( A \).
(c) Demonstrate that \( A \) is orthogonal.
3. A spherical bowling ball of mass $m$ and radius $r$ rolls without slipping inside a wedge which has a circular bowl of radius $R$. Gravity acts downward, and the bowling ball rolls without slipping on the surface of the circular bowl. The ball is released at an initial angle $\theta_0$ from rest.

(a) Derive the equations of motion for the ball. (Hint: If the angle of rotation of the ball through any axis through its center is $\phi$, what is the “rolling without slipping constraint”?)

(b) For small oscillations of the ball ($\theta \ll 1$), show that the ball undergoes simple harmonic motion, and find the frequency $\omega^2$ of the harmonic motion.

(c) The wedge which has mass $M$ is now allowed to move on the frictionless horizontal surface on which it rests. Derive the new equations of motion for the wedge-ball system.

(d) Find the frequency of harmonic motion for small oscillations of the wedge-ball system of part (c).

(e) For small oscillations, this system is harmonic and hence could be used to tell time like a clock. Does this “clock” speed up or slow down when $M$ is allowed to move?
4. (Extra Credit) The baseball player Johnny Damon at co-latitude \( \theta = 45^\circ \) hits the ball horizontally from a height \( h = 1000 \) m at \( v_0 = 20 \) m/sec due East. Assume the acceleration due to gravity is directed straight toward the center of the Earth with magnitude \( g = 9.8 \) m/sec\(^2\).

(a) Solve for the trajectory of the baseball to second order in the Earth’s angular speed of rotation about its axis, \( \omega_e \).

(b) Find the time that the ball would hit the ground if the Earth were not rotating. To lowest order in \( \omega_e \), find the time that the ball actually hits the Earth.

(c) How far East has the ball traveled when it hits the ground? Compare this with the distance if the Earth were not rotating.

(d) When the ball hits the ground, how large is its deflection in the North-South direction?