“If we take quantum theory seriously as a picture of what’s really going on, each measurement [of a system] does more than disturb; it profoundly reshapes the very fabric of reality.”

- - - Nick Herbert

1. De Generalization of De Generacy

In class, we argued that the first-order corrections to the energies of $d$ degenerate states are given by the eigenvalues of the matrix $H'_{ij}$, and the eigenvectors give us the “correct” set of states in the degenerate sub-space. These claims were based on working out the $d = 2$ case explicitly and then generalizing the results in an ‘obvious’ way.

For this problem, prove that these claims are true by considering a set of $d$ degenerate states, $\{\psi_j^0\}$, with $j = 1, 2, 3, \ldots, d$, that obey

$$H^0 \psi_j^0 = E^0 \psi_j^0 \quad \text{with} \quad \langle \psi_i^0 | \psi_j^0 \rangle = \delta_{ij}$$

In analogy with Eq. [6.17] in the text, consider the linear combinations

$$\psi^0 = \sum_{j=1}^{d} \alpha_j \psi_j^0$$

Now follow the steps in section 6.2.1 in the text to arrive at the generalized form of the eigenequation [6.22]:

$$\sum_{i=1}^{d} H'_{ji} \alpha_i = E^1 \alpha_j \quad \text{where} \quad H'_{ji} \equiv \langle \psi_j^0 | H' | \psi_i^0 \rangle$$
2. Welcome 2-D SHO!

Using non-degenerate perturbation theory, we worked a HW problem for the case of the one-dimensional harmonic oscillator. Moving to two dimensions, however, results in a system where most excited states are now degenerate in energy, and so more sophisticated techniques are needed. With this as a warning ... consider a 2-D isotropic SHO, which in Cartesian coordinates has the Hamiltonian

\[ \hat{H}^0 = -\frac{\hbar^2}{2\mu} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} \mu \omega^2 (x^2 + y^2) \]

where \( \omega = (\kappa/\mu)^{1/2} \), with \( \kappa \) being the spring constant and \( \mu \) the particle mass. The eigenvalues of the above Hamiltonian are

\[ E_n^{(0)} = \hbar \omega (n + 1) \]

with \( n = n_x + n_y \), and \( n_x, n_y = 0, 1, 2, \ldots \).

Now suppose we turn on an external potential of the form \( \hat{H}' = \varepsilon xy \). Treating this as a small quantity, calculate the first-order energy shift of the ground state and show that it vanishes.

3. I’m SHO Excited

Continuing the above: now solve for the first-order shift in energy of the first excited state(s).

4. Relatively Easy, Relativistically

When examining the first-order corrections to the hydrogen atomic energy levels due to relativistic effects, we found a term proportional to \( p^4 \), where \( p \) is the electron’s momentum. We attributed this to higher-order terms in the kinetic energy \( T \) when expanded in powers of \( p^2 \). What is the next term in this expansion, i.e., what is the third term in \( T(p) \) when \( pc/(mc^2) \) is a small quantity? Rewrite your answer in terms of \( T_0 \equiv p^2/2m \), and estimate its magnitude in eV.

5. Angular Anguish

Please do problem 6.16, found on page 276 in the text. Pay attention to the hint, or your ‘anguish’ will grow exponentially!