“Now, my suspicion is that the universe is not only queerer than we suppose, but queerer than we can suppose . . . I suspect that there are more things in heaven and earth than are dreamed of, in any philosophy.”

- - - J.B.S. Haldane

1. Just a Couple of Angular Momenta

A spin-1 boson – a π meson, or ‘pion’ for example – has an orbital angular momentum $\ell = 2$. If these two angular momenta, $S$ and $L$, couple up to a state of total angular momentum $|jm\rangle = |31\rangle$, and now you measure $L_z$, what possible values might you get, and what is the probability of each? As always, check to make sure that these probabilities add up to 1.

2. You May Walk the Walk, But Do You Rock Dirac? (15 pts)

In class, I argued that the similarity in size between the first-order relativistic correction to the hydrogen atom and its spin-orbit interaction (the two contributions to fine structure) would have to be viewed as just a “coincidence” in the Schrödinger picture, but would emerge “naturally” in a fully relativistic treatment. To see this, here is Dirac’s exact version (no approximations) of the energy levels for the hydrogen atom:

$$E_{nj} = mc^2\left\{1 + \left(\frac{\alpha}{n - (j + 1/2) + \sqrt{(j + 1/2)^2 - \alpha^2}}\right)^2 \right\}^{-1/2} - 1$$

Taking $\alpha \approx 1/137$ to be a small number, expand this to order $\alpha^4$. If you do it right, your answer should look an awful lot like Eq. 6.67 in the text. Hint: you might want to start by treating the whole expression inside the large parentheses as a ‘small’ quantity, do a Taylor Series expansion, and then play around with the factors inside the parentheses.
3. A Devilish Derivation (15 pts)

Please do Problem 6.22, found on page 280 in the text. Most of this is just bookkeeping and rearranging constants, which is a boring but useful exercise every once in a while.

4. He Ain’t Heavy – He’s My Isotope

“Heavy hydrogen” is a common name for the hydrogen isotope deuterium, in which the proton in the nucleus is joined by a neutron. This has no effect on the Coulomb interaction that dominates atomic structure, but there are smaller, subtle effects, such as a different reduced mass. Another such effect is a large change in the size of the hyperfine splitting in the deuterium ground state. Following the steps from class, or section 6.5 in the text, calculate the wavelength (in cm) of the photon emitted during a hyperfine transition in $^2$H. You will need to know that the deuterium nucleus (the ‘deuteron’) has a spin of 1, a mass equal to twice the proton mass, and a $g$-factor of 1.71.