

Indiana University
ACP PHYSICS P221
Final Exam

Spring 2004

This is a closed-book examination. You may *not* refer to lecture notes, textbooks, or any other course materials. You *may* use a calculator, but solely for the purpose of arithmetic computation. A list of potentially useful formulas, definitions, *etc* is given after the problems.

The exam consists of seven multiple-choice questions, each worth 3 points for a total of 21 points, and four problems to be worked out, together worth an additional 79 points. There is also an optional eighth multiple-choice question for extra credit. For the problems, you must show all work: clearly state and justify any arguments, assumptions, approximations *etc.*, as well as the use of any formulas. Unless otherwise specified, evaluate all integrals and derivatives, and perform any arithmetic calculations if a numerical answer is requested.

Multiple Choice Questions

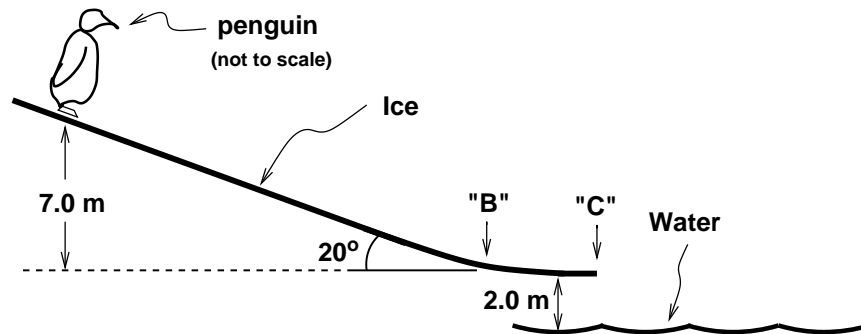
- [1] The net force on a moving object is suddenly reduced to zero, and remains zero thereafter. As a consequence, the object:
- A. stops abruptly.
 - B. stops after a short interval.
 - C. changes direction.
 - D. continues moving, with constant velocity.
 - E. changes velocity in an unknown manner.
- [2] Ben and Jen, each weighing 80 kg , are hiking up a mountain. Ben takes a long, gently sloping trail to get to the top, while Jen, starting from the same point as Ben, climbs a short, but very steep, trail. When they meet at the top,
- A. Jen's gravitational potential energy increase is larger than Ben's.
 - B. Ben's gravitational potential energy increase is larger than Jen's.
 - C. Ben and Jen experience the same increase in their gravitational potential energy.
 - D. To compare the potential energies, we must know the height of the mountain.
 - E. To compare the potential energies, we must know the length of the two trails.
- [3] Two blocks of mass M and $2M$ respectively are placed on a frictionless table, and are connected to each other by a spring. The blocks are pushed together by an external force so that the spring is compressed and the two blocks are at rest. This force is suddenly removed, giving rise to simple harmonic motion of the 2 blocks + spring system. The amplitude of oscillation of the heavier block is:
- A. the same as that of the lighter block.
 - B. 0.5 times that of the lighter block.
 - C. 2 times that of the lighter block.
 - D. 4 times that of the lighter block.
 - E. zero.
- [4] The upward acceleration of an elevator of mass 500 kg depends on its elevation according to $a_y = Cy^2$ where $C = 0.0020\text{ m}^{-1} \cdot \text{s}^{-2}$. The total work done (i.e., by all forces) on the elevator in lifting it from $y = 0.0\text{ m}$ to $y = 20\text{ m}$ is:
- A. $2.7 \times 10^3\text{ J}$
 - B. $4.0 \times 10^3\text{ J}$
 - C. $6.7 \times 10^3\text{ J}$
 - D. $8.0 \times 10^3\text{ J}$
 - E. $9.8 \times 10^3\text{ J}$

- [5] A standing wave is excited on a stretched string of length $L = 2.0 \text{ m}$, mass per unit length $\mu = 0.10 \text{ kg/m}$ and tension $\tau = 2.5 \text{ N}$. Both ends of the string are fixed. A point on the string is observed to vibrate with a frequency of $f = 2.5 \text{ cycles/second}$. Not counting the ends of the string, the number of nodes is:
- A. 0
 - B. 1
 - C. 2
 - D. 3
 - E. 4
- [6] A wheel of radius R is rolling without slipping on a flat stationary surface. The instantaneous velocity of the point on the rim that is in contact with the surface is:
- A. zero.
 - B. equal to $R\omega$ in the direction of motion of the center of mass.
 - C. equal to $R\omega$ opposite the direction of motion of the center of mass.
 - D. equal to the velocity of the center of mass and in the same direction.
 - E. equal to the velocity of the center of mass but in the opposite direction.
- [7] A tennis racket strikes a stationary 0.058 kg tennis ball from below, delivering an average upward force of 1.30 N for 0.50 s . As a result, the ball flies upward. Including effects due to gravity, the velocity of the ball when it leaves the racket is:
- A. 2.0 m/s
 - B. 6.3 m/s
 - C. 9.8 m/s
 - D. 11.2 m/s
 - E. 22.4 m/s
- [8] **Extra Credit** (*Attempt only if you have time!*)
If the spring constant in problem [3] is equal to K , what is the angular frequency of the simple harmonic motion of the 2 block + spring system?
- A. $\sqrt{K/M}$
 - B. $\sqrt{2K/M}$
 - C. $\sqrt{K/(2M)}$
 - D. $\sqrt{K/(3M)}$
 - E. $\sqrt{3K/(2M)}$

Problems

[9] (26 points)

A 120 kg penguin is perched on a slab of ice that is slanted 20° with respect to the horizontal, such that it is located at an elevation of 7.0 m above the end of the slab. The coefficient of static friction μ_s between the ice and the penguin's feet is initially high enough that the penguin does **not** slide down.



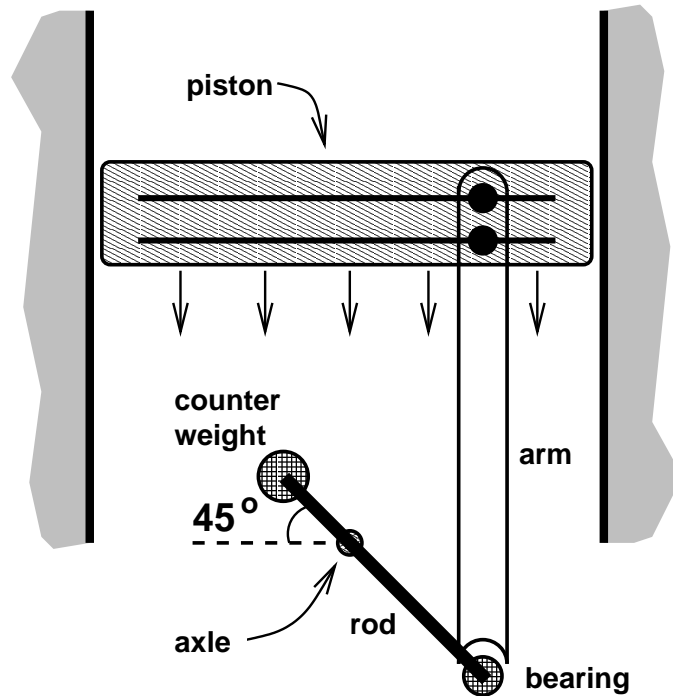
(a) [8 points] Draw a free-body diagram for the penguin, indicating all forces acting on it, and calculate the magnitudes for each of these forces.

(b) [4 points] The heat from the penguin's feet starts to melt a little bit of the ice, thereby gradually reducing the coefficient of static friction. At what value for μ_s will the penguin begin to slide down the ice?

(c) [6 points] Suppose that the coefficient of kinetic friction is $\mu_k = 0.075$. What will be the velocity of the penguin when it reaches point "B" at the end of the straight section of the ice ramp?

(d) [8 points] Assume that the penguin's velocity at point "C" (after the short curved section) has the same magnitude as at point "B", but is now directed horizontally. What horizontal distance will the penguin travel after leaving the ice before splashing into the water 2.0 m below?

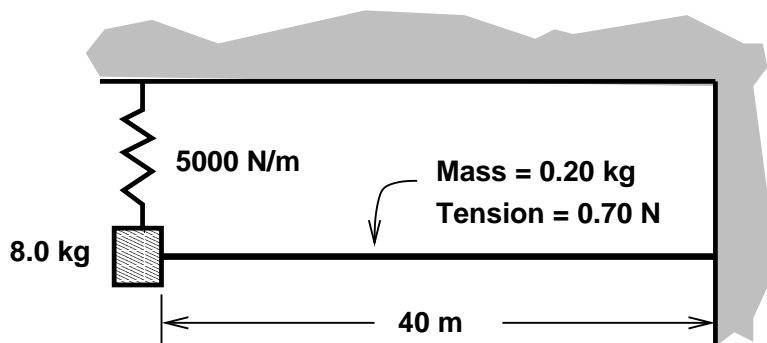
- [10] (16 points) A piston in an engine applies a constant downward force of 25 N on the crank structure shown in the diagram below. The crank structure consists of a light rod that is supported by and can rotate around a fixed axle oriented perpendicular to the page. The driving force is transmitted from the piston by a vertical arm which makes frictionless contact with a bearing of mass 2.0 kg mounted on the rod 0.18 m from the axle. At the other end of the rod 0.090 m from the axle is a counterweight of mass 4.0 kg .



*Note: Do not be misled by the complexity of the diagram.
This is not a difficult problem!*

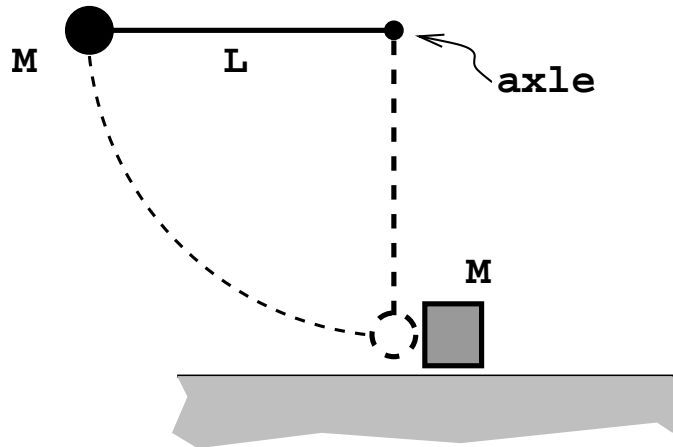
- (a) [8 points] What is the torque exerted on the crank by the piston at the instant shown?
- (b) [8 points] What is the angular acceleration of the crank about the axle at the instant shown?

- [11] (16 points) A wave-generating machine consists of a string of length $L = 40\text{ m}$ and mass $M_s = 0.20\text{ kg}$ under a tension of $\tau = 0.70\text{ N}$. One end of the string is connected to a point on a wall. The other end is attached to a $M_b = 8.0\text{ kg}$ block, which is suspended from the ceiling by a massless spring with spring constant $k_s = 5000\text{ N/m}$.



- (a) [4 points] The block is pulled downward by a small amount and then released. What is the frequency of its subsequent oscillatory motion in units of cycles per second? (Ignore the mass of the string and any forces exerted on the block by the string.)
- (b) [12 points] What is the wavelength of the transverse traveling wave generated in the string by the motion of the block?

- [12] (21 points) A pendulum consisting of a small ball of mass M connected to a fixed axle by a massless string of length L is shown in the diagram below. The ball is released from rest from the horizontal position shown. When it reaches the bottom point of its trajectory it undergoes an elastic collision with a stationary block, also of mass M , resting on a frictionless surface.



- (a) [6 points] Obtain an expression for the velocity of the ball just before it collides with the block in terms of the quantities given and the gravitational acceleration constant.
- (b) [4 points] What is the tension in the string just before the ball collides with the block?
- (c) [4 points] What is the velocity of the block after the collision? Explain your answer in terms of conservation of energy and momentum.
- (d) [4 points] By how much, if at all, has the angular momentum of the ball + block system with respect to the axle changed between when the ball is released and the moment just before the collision? If it has changed explain what has caused this change; if not, explain why not.
- (e) [3 points] By how much, if at all, has the angular momentum of the ball + block system with respect to the axle changed between the moment just before the collision and a long time after the collision? If it has changed explain what has caused this change; if not, explain why not.

Possibly useful formulas, etc

Useful mathematical relations

Trig: $\sin \theta = \text{opposite/hypotenuse}$

Trig: $\cos \theta = \text{adjacent/hypotenuse}$

Trig: $\tan \theta = \text{opposite/adjacent}$
 $= \sin \theta / \cos \theta$

Pythagoras: $(\text{hyp.})^2 = \sqrt{(\text{adj.})^2 + (\text{opp.})^2}$

Vectors: $\vec{p} = p_x \hat{i} + p_y \hat{j} + p_z \hat{k}$
 $|\vec{p}| = p = \sqrt{p_x^2 + p_y^2 + p_z^2}$
 $\vec{a} \cdot \vec{b} = ab \cos \theta$
 $|\vec{a} \times \vec{b}| = ab \sin \theta$

Motion in one dimension

Displacement: $\Delta x = x(t_2) - x(t_1) = x_2 - x_1$

Average velocity: $v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$

Average speed: $s_{ave} = \frac{\text{total distance}}{\Delta t}$

Instantaneous velocity: $v(t) = dx/dt$

Acceleration: $a_{ave} = \frac{\Delta v}{\Delta t}$; $a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Acceleration due to gravity: $g = 9.8 \text{ m/s}^2$

Motion under constant acceleration:

$$v(t) = v_0 + at; \quad x(t) = x_0 + v_0 t + \frac{1}{2} at^2$$

$$v(t)^2 = v_0^2 + 2a \{x(t) - x_0\}$$

Force and motion

Newton's 2nd Law: $\vec{F} = m \vec{a}$

Gravitational Force: $F_g = mg$

Weight on earth: $W = mg$

Static friction: $f_{s,max} = \mu_s N$

Kinetic friction: $f_k = \mu_k N$

Hooke's Law for springs:

$$F_{spring} = -k(x - x_0)$$

Kinetic energy and work

Kinetic Energy (KE): $K = \frac{1}{2}mv^2$

Work (constant force): $W = \vec{F} \cdot \vec{d}$

Work (variable force, 1D): $W = \int_{x_i}^{x_f} F_x dx$

Work & KE: $W_{total} = \Delta K = K_f - K_i$

Work done by springs:

$$W_{spring} = -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

Average power: $P_{ave} = W/(\Delta t)$

Inst. power: $P(t) = dW/dt = \vec{F} \cdot \vec{v}$

Uniform circular motion

Centripetal acceleration: $a = v^2/r$

Potential energy; energy conserv'n

Potential Energy and work:

$$\Delta U = -W = \int_{x_i}^{x_f} F_x dx$$

Gravitational P.E.:

$$\Delta U_{grav} = mg \Delta y; \quad U(y) = mgy$$

Elastic P.E.: $U(x) = \frac{1}{2}k(x - x_0)^2$

Mechanical Energy Conservation:

$$K_i + U_i = K_f + U_f$$

P.E. & Force: $F_x = -dU/dx$

Work done by external agent:

$$W = \Delta E_{mech} + \Delta E_{thermal} + \Delta E_{internal}$$

Systems of particles; momentum

Center of mass: $\vec{r}_{com} = (1/M) \sum m_i \vec{r}_i$

Linear momentum: $\vec{p} = m \vec{v}$

Momentum of system of particles:

$$\vec{p}_{com} = M \vec{v}_{com}$$

Newton's 2nd Law for system of particles:

$$\vec{F}_{net} = M \vec{a}_{com} = d\vec{p}/dt$$

Collisions

Impulse: $\vec{J} = \Delta \vec{p} = \int \vec{F} dt = \vec{F}_{ave} \Delta t$

Inelastic collision, 1-D (momentum cons'n):

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Elastic coll'n, 1-D (mom. & K.E. cons'n):

case of m_2 initially at rest:

$$v_{1f} = v_{1i} \frac{m_1 - m_2}{m_1 + m_2}; \quad v_{2f} = v_{1i} \frac{2m_1}{m_1 + m_2}$$

case of m_2 initially **not** at rest:

$$v_{1f} = v_{1i} \frac{m_1 - m_2}{m_1 + m_2} + v_{2i} \frac{2m_2}{m_1 + m_2}; \quad \& \text{ v.v. for } v_{2f}$$

Rotation

Angular position: $\theta = s/r$

1 Revolution = $360^\circ = 2\pi \text{ rad}$

Angular Displacement: $\Delta \theta = \theta_2 - \theta_1$

Angular speed: $\omega_{ave} = \Delta \theta / \Delta t$; $\omega = d\theta/dt$

Angular accel.: $\alpha_{ave} = \Delta \omega / \Delta t$; $\alpha = d\omega/dt$

Motion under constant angular accel'n:

$$\omega(t) = \omega_0 + \alpha t; \quad \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

Relating linear and angular variables:

$$s = r\theta, \quad v = r\omega, \quad a_t = r\alpha$$

Centripetal acceleration: $a_c = v^2/r = \omega^2 r$

Period of rotation: $\omega T = 2\pi$

Possibly useful formulas, cont'd

Rotation, cont'd

Rotational kinetic energy: $K = \frac{1}{2}I\omega^2$

Moment of inertia: $I = \Sigma_i(m_i r_i^2)$

Parallel-axis theorem: $I = I_{com} + Mh^2$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$, $\tau = rF \sin \phi$

Newton 2nd Law: $\vec{\tau}_{net} = I\vec{\alpha}$

Rolling, angular momentum, etc.

Condition for rolling (no slip): $v_{com} = R\omega$

Kinetic energy: $K = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}Mv_{com}^2$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$

Ang. momentum rigid body: $\vec{L} = I\vec{\omega}$

Newton 2nd Law (again):

$$\vec{\tau} = I\vec{\alpha} = d\vec{L}/dt$$

Gravitation

Newton's Law of gravitation:

$$\vec{F}_{on\ 1} = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21}$$

Newton's constant:

$$G = 6.67 \times 10^{-11} \text{ m}^3 / (\text{kg} \cdot \text{s}^2)$$

Gravitational potential energy:

$$U(r) = -G \frac{m_1 m_2}{r}$$

Escape velocity: $v_{esc} = \sqrt{\frac{2GM}{R}}$

Kepler's laws:

(1) *Elliptical orbits*: All planets move in elliptical orbits with the sun at one focus.

(2) *Law of areas*: A line joining a planet to the sun sweeps out equal areas in equal time.

(3) *Law of periods*: $T^2 = 4\pi^2 r^3 / (GM)$.

Simple Harmonic motion:

General solution: $x(t) = x_m \cos(\omega t + \phi)$

Angular frequency: $\omega = 2\pi/T = 2\pi f$

Mass+spring: $F = -kx$

$\rightarrow d^2x(t)/dt^2 = -\omega^2 x(t)$, $\omega = \sqrt{k/m}$

Waves:

Sinusoidal traveling waves:

$$y(x, t) = y_m \sin(kx - \omega t + \phi)$$

Frequency: $f = \omega/(2\pi) = 1/T$

Wavelength: $\lambda = 2\pi/k$

Velocity: $v = \omega/k = f\lambda$

Stretched string: $v = \sqrt{\tau/\mu}$

Standing waves:

$$y(x, t) = y_m \sin(kx) \cos(\omega t)$$