

**Indiana University**  
**ACP PHYSICS P221**  
**Final Exam**  
*Spring 2005*

The exam consists of **8** problems, some with multiple parts, for a total of **100** points. The point values are listed for each part. Show all work as partial credit will be given. Use the back of the sheets if needed. Perform any calculations if a numerical answer is requested.

This is a closed-book examination. You may *not* refer to lecture notes, textbooks, or any other course materials. You *may* use a calculator, but solely for the purpose of arithmetic computation. A list of potentially useful formulas, definitions, *etc* is given on the last pages.

**Good Luck!**

Name: \_\_\_\_\_

(1) I am standing 25. m from a very tall building on level ground. I throw a water balloon with speed 18. m/s at an angle of  $55.^\circ$  (with respect to horizontal) toward the building. My hand is 2.0 m above the ground when I let it go.

a) (5 pts) How long does it take for the water balloon to hit the building?

b) (5 pts) How high from the ground does the balloon hit?

(2) You are carrying a 500. kg box in the back of a truck traveling at a constant speed of 28. m/s around a level, circular curve of radius 200. m.

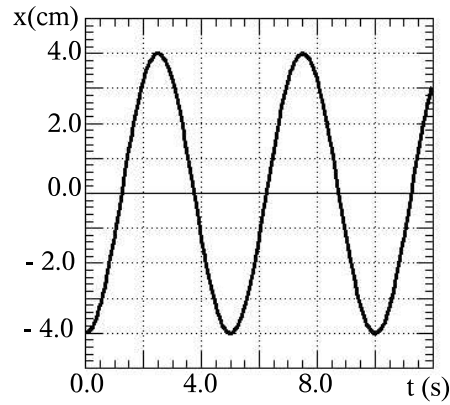
a) (10 pts) What is the minimum value for the coefficient of static friction,  $\mu_s$ , between the box and the truck bed if the box is not to slide while you make the curve?

(3) A small satellite ( $m = 200$  kg) is in orbit around the earth ( $M_{\text{earth}} = 6.0 \times 10^{24}$  kg) at a radius of  $2.0 \times 10^7$  m.

a) (5 pts) What is the period of its orbit?

b) (5 pts) What is its speed?

- (4) A mass of 2.2 kg is undergoing simple harmonic motion on the end of a horizontal spring. The motion of the mass may be described in the form,  $x(t) = A \cos(\omega t + \phi)$ , and is plotted below.



- a) (5 pts) Find the period of oscillation.
- b) (5 pts) Find the spring constant.
- c) (5 pts) Find the maximum speed of the mass

(5) Two disks are mounted on the same frictionless axle and can be brought together so that they turn together. The first disk, with a moment of inertia of  $3.3 \text{ kg m}^2$ , is spinning at  $45. \text{ rad/s}$ . The second disk, with a moment of inertia of  $6.6 \text{ kg m}^2$ , is spinning at  $90. \text{ rad/s}$  in the same direction as the first.

a) (5 pts) The disks are brought together so that they spin at the same angular speed. What is the angular speed of the system after they come together?

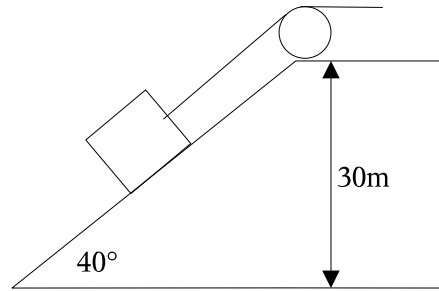
b) (5 pts) If instead the second disk is set spinning at  $90. \text{ rad/s}$  in the opposite direction as the first, what is the angular speed of the system after they come together?

(6) A king's crown is weighed in air with the result,  $m_{\text{true}} = 14.7$  kg. It is submerged in water and weighed again with the result,  $m_{\text{apparent}} = 13.4$  kg. The density of water is  $1.00 \times 10^3$  kg/m<sup>3</sup>.

a) (5 pts) What is the density of the material from which the crown is made?

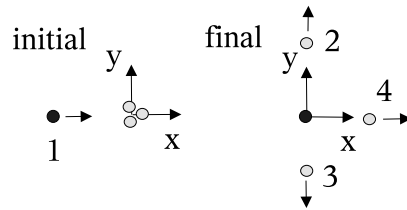
b) (10 pts) What would be the true and apparent masses if it was actually made of gold. The density of gold is  $1.93 \times 10^4$  kg/m<sup>3</sup>

- (7) A 1200 kg block of granite is pulled up a frictionless hill via a cable and winch at a constant speed of 1.5 m/s (parallel to the hill) as shown in the figure.



- a) (5 pts) What is the work done by the winch on the block?
- b) (5 pts) What is the work done by the normal force on the block?
- c) (5 pts) What is the power output of the winch?

- (8) Consider a collision of 4 particles with masses,  $m_1 = 1.0$  kg,  $m_2 = m_3 = m_4 = 2.0$  kg, as shown in the figure. There are no external forces acting on the system of 4 particles. Initially: P1 is moving and P2, P3, P4 are all at rest:  $\vec{v}_1 = (3.0 \text{ m/s})\hat{i}$ ,  $\vec{v}_2 = \vec{v}_3 = \vec{v}_4 = 0$ . Finally: P1 is at rest and P2, P3, P4 are moving:  $\vec{v}_1 = 0$ ,  $\vec{v}_2 = (0.75 \text{ m/s})\hat{j}$ , P3 is moving at unknown speed in the  $-\hat{j}$  direction, and P4 is moving at unknown speed in the  $+\hat{i}$  direction.



- a) (5 pts) What is the final speed of particle 3?
- b) (5 pts) What is the final speed of particle 4?
- c) (5 pts) What is the change in kinetic energy for this collision?

# Formula Sheet

## Useful mathematical relations

Trig:  $\sin \theta = \text{opposite/hypotenuse}$

Trig:  $\cos \theta = \text{adjacent/hypotenuse}$

Trig:  $\tan \theta = \text{opposite/adjacent}$   
 $= \sin \theta / \cos \theta$

Pythagoras:  $(\text{hyp.})^2 = \sqrt{(\text{adj.})^2 + (\text{opp.})^2}$

Vectors:  $\vec{p} = p_x \hat{i} + p_y \hat{j} + p_z \hat{k}$   
 $|\vec{p}| = p = \sqrt{p_x^2 + p_y^2 + p_z^2}$   
 $\vec{a} \cdot \vec{b} = ab \cos \theta$   
 $|\vec{a} \times \vec{b}| = ab \sin \theta$

## Motion in one dimension

Displacement:  $\Delta x = x(t_2) - x(t_1) = x_2 - x_1$

Average velocity:  $v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$

Average speed:  $s_{ave} = \frac{\text{total distance}}{\Delta t}$

Instantaneous velocity:  $v(t) = dx/dt$

Acceleration:  $a_{ave} = \frac{\Delta v}{\Delta t}$ ;  $a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Acceleration due to gravity:  $g = 9.8 \text{ m/s}^2$

Motion under constant acceleration:

$$v(t) = v_0 + at; \quad x(t) = x_0 + v_0 t + \frac{1}{2} at^2$$

$$v(t)^2 = v_0^2 + 2a \{x(t) - x_0\}$$

## Force and motion

Newton's 2<sup>nd</sup> Law:  $\vec{F} = m \vec{a}$

Gravitational Force:  $F_g = mg$

Weight on earth:  $W = mg$

Static friction:  $f_{s,max} = \mu_s N$

Kinetic friction:  $f_k = \mu_k N$

Hooke's Law for springs:

$$F_{spring} = -k(x - x_0)$$

## Kinetic energy and work

Kinetic Energy (KE):  $K = \frac{1}{2}mv^2$

Work (constant force):  $W = \vec{F} \cdot \vec{d}$

Work (variable force, 1D):  $W = \int_{x_i}^{x_f} F_x dx$

Work & KE:  $W_{total} = \Delta K = K_f - K_i$

Work done by springs:

$$W_{spring} = -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

Average power:  $P_{ave} = W/(\Delta t)$

Inst. power:  $P(t) = dW/dt = \vec{F} \cdot \vec{v}$

## Uniform circular motion

Centripetal acceleration:  $a = v^2/r$

## Potential energy; energy conserv'n

Potential Energy and work:

$$\Delta U = -W = \int_{x_i}^{x_f} F_x dx$$

Gravitational P.E.:

$$\Delta U_{grav} = mg \Delta y; \quad U(y) = mgy$$

Elastic P.E.:  $U(x) = \frac{1}{2}k(x - x_0)^2$

Mechanical Energy Conservation:

$$K_i + U_i = K_f + U_f$$

P.E. & Force:  $F_x = -dU/dx$

Work done by external agent:

$$W = \Delta E_{mech} + \Delta E_{thermal} + \Delta E_{internal}$$

## Systems of particles; momentum

Center of mass:  $\vec{r}_{com} = (1/M) \sum m_i \vec{r}_i$

Linear momentum:  $\vec{p} = m \vec{v}$

Momentum of system of particles:

$$\vec{p}_{com} = M \vec{v}_{com}$$

Newton's 2<sup>nd</sup> Law for system of particles:

$$\vec{F}_{net} = M \vec{a}_{com} = d\vec{p}/dt$$

## Collisions

Impulse:  $\vec{J} = \Delta \vec{p} = \int \vec{F} dt = \vec{F}_{ave} \Delta t$

Inelastic collision, 1-D (momentum cons'n):

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Elastic coll'n, 1-D (mom. & K.E. cons'n):

case of  $m_2$  initially at rest:

$$v_{1f} = v_{1i} \frac{m_1 - m_2}{m_1 + m_2}; \quad v_{2f} = v_{1i} \frac{2m_1}{m_1 + m_2}$$

case of  $m_2$  initially **not** at rest:

$$v_{1f} = v_{1i} \frac{m_1 - m_2}{m_1 + m_2} + v_{2i} \frac{2m_2}{m_1 + m_2}; \quad \& \text{ v.v. for } v_{2f}$$

## Rotation

Angular position:  $\theta = s/r$

1 Revolution =  $360^\circ = 2\pi \text{ rad}$

Angular Displacement:  $\Delta \theta = \theta_2 - \theta_1$

Angular speed:  $\omega_{ave} = \Delta \theta / \Delta t$ ;  $\omega = d\theta/dt$

Angular accel.:  $\alpha_{ave} = \Delta \omega / \Delta t$ ;  $\alpha = d\omega/dt$

Motion under constant angular accel'n:

$$\omega(t) = \omega_0 + \alpha t; \quad \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

Relating linear and angular variables:

$$s = r\theta, \quad v = r\omega, \quad a_t = r\alpha$$

Centripetal acceleration:  $a_c = v^2/r = \omega^2 r$

Period of rotation:  $\omega T = 2\pi$

## Formula Sheet (cont.)

### Rotation, cont'd

Rotational kinetic energy:  $K = \frac{1}{2}I\omega^2$

Moment of inertia:  $I = \sum_i(m_i r_i^2)$

Parallel-axis theorem:  $I = I_{com} + Mh^2$

Torque:  $\vec{\tau} = \vec{r} \times \vec{F}$ ,  $\tau = rF \sin \phi$

Newton 2<sup>nd</sup> Law:  $\vec{\tau}_{net} = I\vec{\alpha}$

### Rolling, angular momentum, etc.

Condition for rolling (no slip):  $v_{com} = R\omega$

Kinetic energy:  $K = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}Mv_{com}^2$

Angular momentum:  $\vec{L} = \vec{r} \times \vec{p}$

Ang. momentum rigid body:  $\vec{L} = I\vec{\omega}$

Newton 2<sup>nd</sup> Law (again):

$$\vec{\tau} = I\vec{\alpha} = d\vec{L}/dt$$

### Gravitation

Newton's Law of gravitation:

$$\vec{F}_{on\ 1} = -G\frac{m_1 m_2}{r_{21}^2} \hat{r}_{21}$$

Newton's constant:

$$G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)$$

Gravitational potential energy:

$$U(r) = -G\frac{m_1 m_2}{r}$$

Escape velocity:  $v_{esc} = \sqrt{\frac{2GM}{R}}$

Kepler's laws:

(1) *Elliptical orbits*: All planets move in elliptical orbits with the sun at one focus.

(2) *Law of areas*: A line joining a planet to the sun sweeps out equal areas in equal time.

(3) *Law of periods*:  $T^2 = 4\pi^2 r^3/(GM)$ .

### Simple Harmonic motion:

General solution:  $x(t) = x_m \cos(\omega t + \phi)$

Angular frequency:  $\omega = 2\pi/T = 2\pi f$

Mass+spring:  $F = -kx$

$\rightarrow d^2x(t)/dt^2 = -\omega^2 x(t)$ ,  $\omega = \sqrt{k/m}$

### Waves:

Sinusoidal traveling waves:

$$y(x, t) = y_m \sin(kx - \omega t + \phi)$$

Frequency:  $f = \omega/(2\pi) = 1/T$

Wavelength:  $\lambda = 2\pi/k$

Velocity:  $v = \omega/k = f\lambda$

Stretched string:  $v = \sqrt{\tau/\mu}$

Standing waves:

$$y(x, t) = y_m \sin(kx) \cos(\omega t)$$