Indiana University
PHYSICS P221, Fall 2007
Final Exam
December 14, 2007

Do not turn the page until instructed to begin.

Instructions:

- Write your name and seat number on the front of the blue books.
- Write all your work including answers in the blue exam books provided. You may keep the exam.
- For long-answer problems, show your work, as credit will not be given for an answer only and partial credit may be given.
- It is best to plug numbers in to equations as a final step. Some quantities may cancel in equations and it is easier to grade (for partial credit!).
- Don’t forget units and answer with vector (not just a magnitude) when the problem requires. Use the correct number of significant figures.
- This is a closed-book examination. You may not refer to lecture notes, textbooks, or any other course materials.
- A list of (possibly) useful formulas is attached to this end of this exam.
- You may use a calculator, but solely for the purpose of arithmetic computation.
- There are 5 long-answer problems on this exam (with multiple parts). Point values are indicated with each problem.

Good Luck!
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1) (20pts) In March 1999, the Mars Global Surveyor (GS) entered its final orbit around Mars, sending data back to Earth. Assume a circular orbit around Mars with a period of $1.18 \times 10^2$ minutes and an orbital speed of $3.40 \times 10^3$ m/s. The mass of the GS is 930 kg and the radius of Mars is $3.43 \times 10^6$ m. Using this data...

a) (7pts) Calculate the radius of the GS orbit around Mars.

b) (7pts) Calculate the mass of Mars.

c) (6pts) In fact, the orbit of the GS is slightly elliptical with its closest approach to Mars at $3.71 \times 10^5$ m above the surface and the furthest distance at $4.36 \times 10^5$ m above the surface. If the speed of the GS at closest approach is $3.40 \times 10^3$ m/s, what is the speed at the furthest point?

2) (20pts) A fishing bobber is at rest and is attached to the bottom of a tank of water by a string as shown in the figure. The volume of the bobber is 75 cm$^3$ and its density is 0.15 times that of water. The density of water is 1.0 g/cm$^3$.

a) (10pts) What is the tension in the string?

b) (10pts) The top of the bobber is 1.5 m below the surface of the water. If the string is cut, how long does it take before the top of bobber reaches the surface? (Ignore the effects of drag from the water.)
3) (20pts) Two point sound sources, A and B, are separated by 1.0 m and are arranged as shown in figure. They emit sound of the same frequency and in phase (at the sources). Point P is 3.0 m from source B. The line BP is at a right angle to the line AB. The speed of sound in air is 343 m/s.

![Diagram of two sound sources and point P](image)

a) (10pts) If only source A is turned on (at first) and emits 0.15 W of sound power, what is the sound level (in dB) at point P?

b) (10pts) If sources A and B are turned on, what is the minimum frequency at which the two sources will interfere constructively at point P?

4) (20pts) A disk of mass \( M = 2.0 \) kg and radius \( R = 0.12 \) m with moment of inertia \( I = \frac{1}{2} MR^2 \) is released from rest at a height \( h = 0.44 \) m above the top of a table of height \( H = 1.3 \) m. It rolls without slipping down a ramp that is inclined at an angle \( \theta = 37^\circ \).

![Diagram of disk on inclined plane](image)

a) (10pts) What is the speed of the cylinder when it reaches the bottom of the ramp?

b) (10pts) It then continues to roll without slipping to the edge of table and then falls off. How far from edge of table \( (x) \) does it hit the floor?
5) (20pts) A block of mass $M = 4.5$ kg is at rest on a horizontal table and attached to a rigid support by a spring of constant $k = 5.0 \times 10^3$ N/m. A bullet of mass 7.5 g and speed $v$ hits the block in a completely inelastic collision (sticks in the block). The bullet, block, spring system then undergoes simple harmonic motion (SHM) with amplitude 0.054 m.

a) (10pts) What is the speed of the bullet just before hitting the block?

b) (10pts) What is the period of the resulting SHM?
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P221 Fall 2007 Formula Sheet

Miscellaneous

Quadratic Formula:
\[ ax^2 + bx + c = 0; \quad x = \frac{1}{2a} \left( -b \pm \sqrt{b^2 - 4ac} \right) \]

Motion in 1 Dimension

Displacement: \( \Delta x = x_2 - x_1 \)
Average Velocity: \( v_{ave} = \frac{\Delta x}{\text{total distance}} \)
Average Speed: \( s_{avg} = \frac{\text{total distance}}{\text{time}} \)
Instantaneous Velocity: \( v = \frac{dx}{dt} \)
Average Acceleration: \( a_{ave} = \frac{\Delta v}{\text{time}} \)
Instantaneous Acceleration: \( a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \)
Constant Acceleration:
\[
\begin{align*}
v &= v_0 + at \\
x &= x_0 + v_0t + \frac{1}{2}at^2 \\
v^2 &= v_0^2 + 2a(x - x_0) \\
x &= x_0 + \frac{1}{2}(v_0 + v)t \\
x &= x_0 + vt - \frac{1}{2}gt^2
\end{align*}
\]
Free-fall acceleration: \( g = 9.8 \text{ m/s}^2 \)

Vectors

Components of a vector:
\[ a_x = a \cos \theta \quad \text{and} \quad a_y = a \sin \theta \]
\[ a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan \theta = \frac{a_y}{a_x} \]

Unit-Vector Notation: \( \vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \)

Scalar Product:
\[ \vec{a} \cdot \vec{b} = ab \cos \theta = a_x b_x + a_y b_y + a_z b_z \]

Vector Product:
\[ |\vec{a} \times \vec{b}| = ab \sin \phi \]
\[ \vec{a} \times \vec{b} = (a_yb_z - a_zb_y) \hat{i} + (a_zb_x - a_xb_z) \hat{j} + (a_xb_y - a_yb_x) \hat{k} \]

Motion in 2,3 Dimensions

Position Vector: \( \vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \)
Displacement: \( \Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k} \)
Average and Instantaneous Velocity:
\[
\begin{align*}
\vec{v}_{ave} &= \frac{\Delta \vec{r}}{\text{time}} \\
\vec{v} &= \frac{d\vec{r}}{dt} \\
\vec{v} &= v_x \hat{i} + v_y \hat{j} + v_z \hat{k}
\end{align*}
\]
Average and Instantaneous Acceleration:
\[
\begin{align*}
\vec{a}_{ave} &= \frac{\Delta \vec{v}}{\text{time}} \\
\vec{a} &= \frac{d\vec{v}}{dt} \\
\vec{a} &= a_x \hat{i} + a_y \hat{j} + a_z \hat{k}
\end{align*}
\]

Projectile Motion:
\[
\begin{align*}
x - x_0 &= (v_0 \cos \theta_0) t \\
y - y_0 &= (v_0 \sin \theta_0) t - \frac{1}{2}gt^2 \\
v_y &= v_0 \sin \theta_0 - gt \\
v_y^2 &= (v_0 \sin \theta_0)^2 - 2g(y - y_0) \\
R &= \frac{v_0^2}{2g} \sin 2\theta_0
\end{align*}
\]
Uniform Circular Motion:
\[
\begin{align*}
a &= \frac{v^2}{R}; T = \frac{2\pi R}{v}
\end{align*}
\]

Force and Motion

Newton’s Second Law: \( \vec{F}_{net} = m \vec{a} \)
Gravitational Force: \( F_g = mg \)
Weight: \( W = mg \)
Static Friction: \( f_{s,\text{max}} = \mu_s F_N \)
Kinetic Friction: \( f_k = \mu_k F_N \)
Drag Force: \( D = \frac{1}{2} \rho C D v^2 \)
Teminal Speed: \( v_t = \sqrt{\frac{2F_g}{C \rho A}} \)
**Kinetic Energy and Work**

- **Kinetic Energy**: $K = \frac{1}{2}mv^2$
- Work Done by Constant Force: $\vec{F} \cdot \vec{d} = Fd\cos\phi$
- Work and Kinetic Energy: $\Delta K = K_f - K_i = W$
- Work Done by Gravity: $W_g = mgd\cos\phi$
- Spring Force: $\vec{F} = -kd$
- Work Done by Spring Force: $W_s = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$
- Work Done by Variable Force: $W = \int_{x_i}^{x_f} F(x)dx$
- Power: $P_{avg} = \frac{W}{\Delta t}$, $P = Fv\cos\phi = \vec{F} \cdot \vec{v}$

**P.E. and Conservation of Energy**

- Potential Energy: $\Delta U = -W = -\int_{x_i}^{x_f} F(x)dx$
- Gravitational Potential Energy: $\Delta U = mg\Delta y; U(y) = mgy$
- Elastic Potential Energy: $U(x) = \frac{1}{2}kx^2$
- Mechanical Energy: $E_{mec} = K + U$
- Cons. of Mech. Energy: $\Delta E_{mec} = 0$
- Potential Energy Curves: $F(x) = -\frac{dU}{dx}$
- Work Done on System by an Ext. Force:
  - no friction: $W = \Delta E_{mec}$
  - with friction: $W = \Delta E_{mec} + \Delta E_{th}$
- Conservation of Energy:
  $\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$

**COM and Linear Momentum**

- Center of Mass: $\vec{r}_{com} = \sum_{i=1}^{n} m_i \vec{r}_i$
- Linear Momentum of Particle: $\vec{p} = mv\vec{v}$
- Linear Momentum of System: $\vec{P} = M\vec{v}_{com}$
- Newton’s 2nd law for System: $\vec{F}_{net} = M\vec{a}_{com}; \vec{F}_{net} = \frac{d\vec{P}}{dt}$
- Cons. of Linear Momentum: $\Delta \vec{P} = 0$
- Impulse: $\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt = \vec{P}_{ave}\Delta t = \Delta \vec{p}$
- Inelastic Collision: $\Delta \vec{P} = 0$
- Elastic collision: $\Delta \vec{P} = 0; \Delta K = 0$

**Rotation**

- Angular Position: $\theta = s/r$
- 1 rev = $360^\circ = 2\pi$ radians
- Angular Displacement: $\Delta \theta = \theta_2 - \theta_1$
- Angular Velocity: $\omega_{ave} = \Delta \theta/\Delta t$, $\omega = \frac{d\theta}{dt}$
- Angular acceleration: $\alpha_{ave} = \Delta \omega/\Delta t$, $\alpha = \frac{d\omega}{dt}$
- Constant Angular Acceleration:
  - $\omega = \omega_0 + \alpha t$
  - $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
  - $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
  - $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
  - $\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$
- Relating Linear and Angular Variables:
  - $s = \theta r$, $v = \omega r$, $a = \alpha r$
  - $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$
- Rotational Kinetic Energy: $K = \frac{1}{2}I\omega^2$
- Moment of Intertia: $I = \Sigma m_i r_i^2$
- Parallel-Axis Theorem: $I = I_{com} + Mr^2$
- Torque: $\tau = rF_i = r_\perp F = rF\sin\phi$
- Newton's 2nd Law for Rotation: $\tau_{net} = I\alpha$

**Torque, and Angular Momentum**

- Rolling Bodies:
  - $v_{com} = \omega R$
  - $K = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}Mv_{com}^2$
  - $a_{com} = \alpha R$
  - $a_{com,x} = -g\sin\theta/(1 + I_{com}/MR^2)$
- Torque Vector: $\vec{\tau} = \vec{r} \times \vec{F}$
- Angular Momentum of Particle:
  - $\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) = mr_\perp \vec{v}$
- Newton’s 2nd Law of Rotation: $\dot{\vec{\tau}} = \frac{d\vec{l}}{dt}$
- Angular Momentum of Rigid Body: $\vec{L} = I\vec{\omega}$
- Conservation of Angular Momentum: $\vec{L} = const.$

**Equilibrium and Elasticity**

- Equilibrium: $\vec{F}_i = 0; \vec{\tau}_i = 0$
- Tension, Compression: $\frac{\vec{F}}{A} = E \frac{\Delta L}{L}$
- Shear: $\frac{\vec{F}}{A} = G \frac{\Delta \theta}{\theta}$
- Hydraulic Stress: $p = B \frac{\Delta V}{V}$
**Fluids**

Pressure: \( p = \frac{F}{A} \)

Pressure change with height: \( p = p_0 + \rho gh \)

Mass flow rate: \( R_M = \rho A v \) = constant

Bernoulli’s eqn: \( p + \frac{1}{2} \rho v^2 + \rho g y = \text{constant} \)

**Oscillations**

Period: \( T = \frac{1}{f} \)

Simple Harmonic Motion: \( x(t) = x_m \cos(\omega t + \phi) \)

Angular frequency: \( \omega = \frac{2\pi}{T} = 2\pi f \)

Linear oscillator: \( \omega = \sqrt{\frac{k}{m}}, \quad T = \frac{2\pi}{\sqrt{m/k}} \)

Energy of linear oscillator: \( E = \frac{1}{2} k x_m^2 \)

Physical Pendulum: \( T = \frac{2\pi}{\sqrt{I/mgh}} \)

**Waves**

Sinusoidal Waves:

\[
y(x,t) = y_m \sin(kx - \omega t),
\]

\( k = \frac{2\pi}{\lambda}, \quad \omega = f = \frac{2\pi}{T}, \quad v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \)

Wave speed on string: \( v = \sqrt{\frac{T}{\mu}} \)

Interference of waves: \( y(x,t) = 2y_m \times \cos\left(\frac{\lambda}{2} \Delta x - \frac{1}{2} \omega t - \phi/2\right) \sin(k_{ave} x - \omega_{ave} t + \phi/2) \)

Standing waves: \( y(x,t) = 2y_m \sin(kx) \cos(\omega t) \)

Speed of sound: \( v = \sqrt{\frac{T}{\mu}} \)

Sound Intensity at distance \( r \) from point source:

\( I = P_s / 4\pi r^2 \)

Sound Level: \( \beta = (10 \text{ dB}) \log \frac{I}{I_0}, \quad I_0 = 10^{-12} \text{ W/m}^2 \)

Interference:

\[
\phi = \frac{\Delta L}{2\pi} \]

fully constructive: \( \frac{\Delta L}{2\pi} = 0, 1, 2, 3, ... \)

fully destructive: \( \frac{\Delta L}{2\pi} = 0.5, 1.5, 2.5, 3.5, ... \)

**Gravitation**

Law of Gravitation: \( F = G \frac{m_1 m_2}{r^2} \)

Newton’s Constant: \( G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \)

Superposition: \( \vec{F}_{\text{net}} = \sum_{i=1}^{n} \vec{F}_i \)

Gravitational Acceleration: \( a_g = \frac{GM}{r^2} \)

Gravitation inside spherical shell: \( M_{\text{enc}} = \rho \frac{4\pi r^3}{3} \)

Gravitational Potential Energy: \( U = -\frac{GMm}{r} \)

Gravitational PE of a system:

\[
U = -\left( \frac{G m_1 m_2}{r_{12}} + \frac{G m_1 m_3}{r_{13}} + \frac{G m_2 m_3}{r_{23}} \right)
\]

Escape speed: \( v = \sqrt{\frac{2GM}{R}} \)

**Kepler’s Laws**

1. **law of orbits**: All planets move in elliptical orbits with the Sun at one focus

2. **law of areas**: A line joining any planet to the Sun sweeps out equal areas in equal time

3. **law of periods**: \( T^2 = \left( \frac{4\pi^2}{GM} \right) r^3 \)

Energy of orbital motion

(Circular orbit, radius \( r \)):

\[
U = -\frac{GMm}{r} \quad \text{and} \quad K = \frac{GMm}{2r}
\]

(Elliptical orbit, semimajor axis \( a \)):

\[
E = -\frac{GMm}{2a}
\]