Thermionic Emission

A. Goode Student and A. Finicky Professor

Abstract

Thermionic emission of electrons by a hot filament is observed in a space charge limited regime and temperature limited regime. From a study of the temperature dependent data, the work function of tungsten is found to be $5.4 \pm 0.8$ eV.

1 Introduction

In many everyday applications, the electron tube has now been replaced by transistors and integrated circuits. However, the physics behind the electron tube remains useful in such devices as cathode-ray tubes and magnetrons. For this reason, it is interesting to study the thermionic flux of electrons from a hot metal filament.

Metals exhibit excellent electrical conductivity. This property implies that the outer electrons in a metal are not bound to individual atoms but rather to the atomic lattice as a whole. In fact, they are held by some characteristic binding energy, which is the energy that must be given to an electron to remove it from the metal. Binding energy is represented by a metal's work function, $\Phi$.

There are several methods of giving electrons the needed energy to escape the lattice, such as by applying strong electric fields or bombarding the metal with high energy photons. In addition, the metal may be heated to a point at which electrons break free from the surface. When electrons are emitted from a surface due to heating, the process is known as thermionic emission, and this is the subject of interest in the experiment. If the emitted electrons do not escape the region around the emitter, a buildup of negative charge results; this effect is known as space charge, and it hinders the production of free electrons. However, when a sufficiently large potential difference exists between the emitter and some collection surface, electrons will be pulled away from the emitter and reduce this space charge effect, which results in a current flow between the emitter (cathode) and collector (anode).

Sir Owen Willans Richardson was awarded the Nobel Prize in 1928 for his work on thermionic emission, especially for the law that bears his name (today known as the Richardson-Dushman equation):

$$J = AT^2 e^{\frac{\Phi}{kT}}$$

which describes the current flow between the cathode and the anode [1]. Here $J$ is the emission $[A/cm^2]$ of the hot surface, $A$ is a constant for metals, $T$ is the temperature of the emitter [K], $\Phi$ is the work function in eV, and $k$ is Boltzmann's constant, $1.371x10^{-23} J/K$. An in-depth derivation and discussion of Richardson's law and thermionic emission in general may be found in [2].
2 Experimental Method

2.1 Principle of Measurement

The main goal of this experiment is to find the work function $\Phi$ of tungsten (W). Massaging the Richardson law given above, one obtains

$$\log\left(\frac{J}{T^2}\right) = \log(A) - \frac{\phi}{kT}. \tag{2}$$

Then $\phi$ may be taken out of the slope of $\log\left(\frac{J}{T^2}\right)$ vs. $1/T$.

Clearly the important parameters to measure are $J$ and $T$. The current density $J$ can be determined by measuring the current from a hot emitting cathode to a collector anode, combined with the geometry of the filament. The constant $A$, normally tricky to calculate correctly from theory, can be ignored as it does not affect the slope of the plot. Temperature $T$ can be calculated from resistance measurements [3].

2.2 Experimental Setup

The electron emitting element is a tungsten wire, $0.0127 \pm 0.0008$ cm in diameter and $10.5 \pm 0.2$ cm long. Surrounding this filament coaxially is a cylindrical copper anode, $6.4 \pm 0.1$ cm long and $1.3 \pm 0.1$ cm inner diameter. The filament is aligned as best as possible with the axis of the anode. Filament and anode together constitute a diode, and this diode is housed inside a vacuum system capable of pressures on the order of $10^{-5}$ torr.

Heating occurs when a current supply is connected to the filament; the potential difference between the cathode and anode is supplied by a voltage supply that also offsets the ground of the filament supply by up to 200V. See figure 1 for the applicable circuit diagram.

With the diode in place in the evacuated system and the circuit configured as shown in the diagram, one is ready to take data. For several values of filament current (and therefore several temperatures), $I$ (of the anode) vs. $V_p$ is recorded throughout the available voltage range. In addition, for each data point the value of filament current and filament voltage is recorded. These filament parameters should not change for a healthy filament at a given filament current, although some slight fluctuation is possible. It should be noted that the filament voltage will become unstable and eventually increase constantly when the filament becomes very deteriorated. Frequent, long, or particularly high-current operation will eventually burn though the filament. As this happens, it becomes thinner, which causes the resistance to increase at a given temperature, and since $V=IR$, the filament voltage will also increase.

3 Results

Five valid measurements were made of anode current vs. plate voltage. These correspond to five different filament currents and therefore temperatures. Figure 2 summarizes the experimental data.

3.1 Determination of Temperature
The first step in the analysis of the saturation (non space charge) regime is to determine the temperature that corresponds to each value of filament current. This process is somewhat complicated and has a large uncertainty. First, from each measured $V_f$ and $I_f$ and Ohm's law, the total resistance of the series combination of filament and leads is found. Then $R$ vs. $I_f$ is plotted and extrapolated to zero filament current, which would correspond to the room temperature value of $R$.

**Figure 1**: This circuit provides filament heating and the space-charge alleviating potential difference between filament and anode. Circles represent multimeters used to monitor $I_f$ (filament current), $V_f$ filament voltage, $I$ (anode current-thermionic!!), and $V_p$ plate (anode) voltage. Circuit taken from [3], p. 149.
Figure 2: Here the logarithm of the anode current is plotted versus the logarithm of plate voltage. The left side of the plot represents the space charge limited region of the voltage, while the right side, where one can distinguish between data sets, represents the temperature dependent region. Note that temperature uncertainty is 24%. In theory each approximately horizontal line in the saturated region could correspond to a higher temperature than the line below it. This theory holds when uncertainty is taken into account. NOTE: the legend on this figure appears to have a typo, the dots must represent T=2800 K and the triangles T=2735 K. (AFP).
Since only a limited range of current settings (~1.5 to ~2.0 A) was used for the actual experiment, it was necessary to measure separately $V_f$ vs $I_f$ for a wider range of filament currents in order to produce a more accurate plot. From the plot, the room temperature resistance $R$ is found to be 0.8 ± 0.1 Ω, where the uncertainty of $R$ is taken to be the error of the intercept of the fit line. See figure 3.

The room temperature resistance of the filament can be calculated from

$$R = \frac{\rho L}{A}, \quad (3)$$

where $\rho$ is the room temperature resistivity of tungsten, $L$ is the length of the filament, and $A$ is its cross-sectional area. The resulting value of $R_f$ for the filament is 0.5 ± 0.1 Ω. Error on $R_f$ is calculated through standard functional error propagation, where the error on the length is 0.2 cm and the area error is $3.0 \times 10^{-5}$ cm$^2$. Resistivity is taken as errorless, as no error was quoted in [3], where resistivity vs. temperature data are listed.

The filament resistance at room temperature is then subtracted from the room temperature total $R$ of filament and leads, which yields the resistance of just the leads, $R_l = 0.3 ± 0.1$ Ω. In theory, this lead resistance should not change with increasing filament current, so it is taken as constant. However, in reality there is probably some change in this resistance as the filament is heated. In addition, the filament has a temperature gradient; it is not at uniform temperature along its length. These effects merely add to the uncertainty in the temperature determination.

Now the lead resistance is subtracted from the total value of $R$ at each filament current in order to determine the filament resistance and corresponding resistivity. The resistivity of tungsten depends on its temperature. Therefore, it is helpful to plot resistivity vs. temperature of tungsten from the data given in [3] by Forsythe and Worthing (p.151). No error is stated in the Preston and Dietz data, so no error bars are plotted. Figure 4 shows the resultant plot of the relevant range of resistivities, fit to a line. The slope and intercept of the fit parameters allow temperature to be determined for resistivities in between the data points quoted in the reference. Thus by finding the resistivity of the filament for each filament current, one can deduce the temperature that corresponds to each current. Resulting temperatures are listed for each curve in figure 2, the data plot.

The error in the temperature ultimately comes from the error in the resistivity calculation, $\rho = \frac{RA}{L}$. For resistivity error, the standard propagation formula is used, with $\delta R = 0.1$ Ω, and errors in area and length remain as stated previously. Calculating $\delta \rho$, one finds a relative error of 24% on every $\rho$ value found. Since temperature and resistivity are directly related, one then takes the temperature uncertainty to be 24%. This is not strictly correct. The geometric errors are essentially constant, and one can greatly reduce their contribution to the uncertainty by measuring the resistance at very low currents (where the $T$ of the filament will remain at room temperature). If one then accurately accounts for the (constant) lead resistance, then the ratio of the measured resistance ($R(T1)/R(roomT)$) should be essentially the same as the ratio of the resistivities. The uncertainty in the $T$ determination is then primarily tied up in the accuracy of accounting for the lead resistance (which will have some $T$ dependence due to changes in contact resistance to the W wire), with the accuracy with which $T=roomT$ may be maintained in getting the resistance at low
current, and with the non-uniform T profile mentioned earlier. In figure 3 this amounts to roughly a 10% uncertainty in the middle of the temperature range.

3.2 Temperature-Dependent Region

In the temperature dependent region, one can apply Richardson's law and determine the work function of tungsten. Recall that

$$\log \left( \frac{J}{T^2} \right) = \log (A) - \frac{\phi}{kT}$$

![Figure 3](image)

**Figure 3**: Total resistance versus filament current. The y-intercept P1 represents the zero-current or room temperature resistance of the total filament plus leads combination.
Figure 4: From a slope-intercept equation for the line, one can solve for the temperature corresponding to a resistivity within this region. Fit parameter $P_1$ is the intercept of the fit line, while $P_2$ is the slope. Uncertainties in the parameters are not accurate, as there are no error bars on the data points.

Table I
Approximate saturated current values based on data in figure 2 as extracted by AFP, assuming that the current at $\ln(V)=1.4$ is reasonably representative of the saturated current for all temperatures.

<table>
<thead>
<tr>
<th>T (K)</th>
<th>$I_{sat}$ (A)</th>
<th>$\ln(I_{sat}/T^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2592</td>
<td>0.00047863</td>
<td>-23.365</td>
</tr>
<tr>
<td>2637</td>
<td>0.00069183</td>
<td>-23.031</td>
</tr>
<tr>
<td>2735</td>
<td>0.001548817</td>
<td>-22.298</td>
</tr>
<tr>
<td>2785</td>
<td>0.003090295</td>
<td>-21.6435</td>
</tr>
<tr>
<td>2800</td>
<td>0.006309573</td>
<td>-20.9404</td>
</tr>
</tbody>
</table>
Figure 5: The work function of tungsten is equal to the magnitude of the slope of this line times the Boltzmann constant. The data points for this plot were taken from the raw data in figure 2 at a bias voltage of 25V. The data point at 2800K is suspect since we do not see the approach to saturation at this temperature as we do in the others (and hence we do not know that this point is not influenced by space charge effects). This point is effectively excluded from the fit by attaching to it a large uncertainty. These numbers were obtained naively by simply taking the current at a fixed voltage that is on the plateau for all temperatures considered. A somewhat better number may be obtained by fitting to expected dependence in the “saturated” region, but I suspect that this would not make a big difference and would be much more involved.

By plotting $\log(\log(J/T^2))$ vs. $1/T$, one may determine $\Phi$ by multiplying the slope of the line times Boltzmann's constant. See figure 5 for the plot. Error bars are calculated for the y-axis based on temperature uncertainty alone, as it should dominate. One then has the error of $\log(J/T^2)$ with regard to T. When evaluated, this error becomes $2\delta T/T$, or approximately $0.5$ in units of $\log(A/m^2/K^2)$. Recall that $\frac{\delta T}{T} = 24\%$.

From this fit one finds $\phi = 5.4$ eV. The fit error is $\pm 0.8$ eV, about $15\%$, and this is probably the dominant random uncertainty in the determination of the work function. Errors in the temperature determination introduce some additional uncertainty, but this is probably mainly systematic and should not have a large impact on the value of the work function.

3.3 A Note on Space Charge

The space charge region obeys a different law, Child's law (see [2]), which says that anode current should be proportional to the applied voltage raised to the 3/2 power. On the plot (figure 2), this
region is the section on the left, where the different filament currents should have points that combine to form a positive-slope line. Although rigorous analysis of the space charge region is not given here, one can easily see that the shape of the plot is correct, and the relation behaves qualitatively like $I \alpha V^\beta$.

4 Discussion/Conclusions

The final value of the work function of tungsten determined in this experiment is 5.4 +/- 0.8 eV. In the literature [2], this number is stated as 4.5 eV. The experimental result is in reasonable, but not perfect, agreement with the accepted value given the size of the uncertainty. This may reflect, in part, our decision to take a short-cut in determining the value of the saturated current at each temperature, or the difficulties mentioned above with determining a (unique) temperature for the filament at each value of the filament current. A potential source of systematic error, that is difficult to quantify, arises from the sensitivity of the work function to the surface chemistry of the particular sample under study. The accepted value quoted above is probably for pure W studied under much more stringent conditions of surface cleanliness than we can expect to achieve in the present apparatus (e.g. UHV conditions with in-situ ion cleaning of the material under study). It would not take much of an oxide on the surface to significantly perturb the work function of the wire.

5 References

   [Link](http://www.avseducation.org/pdffiles/vossenwinner01.pdf)


   [Link](http://www.astro.uwo.ca/~jlandstr/p359/writeup/Thermionic.pdf)