One cylinder rolling on another: (with \( r \) and \( \theta_1 \) and \( \theta_2 \) as generalized coordinates)

(although there is just one degree of freedom, \( \theta_1 \), if cylinder is not slipping and remains in contact with the other one)

\[
L = \frac{1}{2}m[(\dot{r}^2 + r^2\dot{\theta}_1^2) + \frac{1}{2}R_2^2\dot{\theta}_2^2] -mgr \cos \theta_1 - \text{const}
\]

kinetic energy of the center of mass  
rotational kinetic energy \( I_{cyl} = \frac{1}{2}mR^2 \)  
potential energy

constraints:

\[
\begin{align*}
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} &= \sum_{j=1}^{n} \lambda_j \frac{\partial f_j}{\partial q_j} & \sigma = 1, \ldots, n \\
r &= R_1 + R_2 \\
R_1 \dot{\theta}_1 &= R_2(\dot{\theta}_2 - \dot{\theta}_1)
\end{align*}
\]

the forces of constraint are the normal force, and friction force:

\[
\delta W = Q_r \delta r + Q_1 \delta \theta_1 + Q_2 \delta \theta_2
\]

\[
\delta W = N \delta r \quad \delta \theta_1 = \delta \theta_2 = 0
\]

\[
Q_r = \lambda_1 = N
\]

5 eqns. for 5 unknown
One cylinder rolling on another: \( \text{(with } r \text{ and } \theta_1 \text{ and } \theta_2 \text{ as generalized coordinates)} \)

5 eqns. for 5 unknowns:

1. \[ m(r - r\dot{\theta}_1^2 + g \cos \theta_1) = \lambda_1 = Q_r \] \[(1)\]

2. \[ \frac{d}{dt} mr^2 \dot{r}_1 - mgr \sin \theta_1 = (R_1 + R_2)\lambda_2 = Q_1 \]

3. \[ \frac{1}{2}mR_2^2 \ddot{\theta}_2 = -R_2 \lambda_2 = Q_2 \]

4. \[ r = R_1 + R_2 \] \[(4)\]

5. \[ \dot{r} = \ddot{r} = 0 \]

\[ R_1 \dot{\theta}_1 = R_2(\dot{\theta}_2 - \dot{\theta}_1) \] \[(5)\]

Constraints:

\( -m(R_1 + R_2)\dot{\theta}_1^2 + mg \cos \theta_1 = \lambda_1 = N \) \[(1)\]

\[ -\frac{1}{2}mR_2^2 \ddot{\theta}_2 = -\frac{1}{2}m(R_1 + R_2)\ddot{\theta}_1 = \lambda_2 \] \[(3)\]

\[ m(R_1 + R_2)^2 \ddot{\theta}_1 - mg(R_1 + R_2) \sin \theta_1 = (R_1 + R_2)\lambda_2 = -\frac{1}{2}m(R_1 + R_2)^2 \dot{\theta}_1 \] \[(2)\]

\( \frac{1}{2}(R_1 + R_2)\dot{\theta}_1^2 - g \sin \theta_1 = 0 \) \[(5)\]

Eq. of motion for the only independent coordinate:

\( (R_1 + R_2)\dot{\theta}_1^2 = \frac{4}{3}g(1 - \cos \theta_1) \)

Cylinders stay in contact as far as \( N \geq 0 \)

Angle of separation: \( 55.15^\circ \)

Beyond this point we need all three variables, the motion is described by eqs. of motion with Lagrange multipliers set to 0.

Constant corresponds to cylinder starting at rest at the top.