One cylinder rolling on another: (with \( r \) and \( \theta_1 \) and \( \theta_2 \) as generalized coordinates)

(although there is just one degree of freedom, \( \theta_1 \), if cylinder is not slipping and remains in contact with the other one)

\[
L = \frac{1}{2}m[(r^2 + r^2\dot{\theta}_1^2) + \frac{1}{2}R_2^2\dot{\theta}_2^2] - mgr \cos \theta_1 - \text{const}
\]

kinetic energy of the center of mass

rotational kinetic energy

potential energy

\[
l_{\text{tot}} = \frac{1}{2}mR^2
\]

\[
l = l_{\text{com}} + l_{\text{rot}}
\]

constraints:

\[
r = R_1 + R_2
\]

\[
R_1\dot{\theta}_1 = R_2(\dot{\theta}_2 - \dot{\theta}_1)
\]

\[
\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_s} - \frac{\partial L}{\partial q_s} = \sum_{j=1}^{n} \lambda_j \frac{\partial f_j}{\partial q_s}, \quad s = 1, \ldots, n
\]

the forces of constraint are the normal force, and friction force:

\[
\delta W = Q_r \delta r + Q_{\theta_1} \delta \theta_1 + Q_{\theta_2} \delta \theta_2
\]

\[
\delta W = N \delta r \quad \delta \theta_1 = \delta \theta_2 = 0
\]

\[
Q_r = \lambda_1 = N
\]

5 eqns. for 5 unknown

---

One cylinder rolling on another: (with \( r \) and \( \theta_1 \) and \( \theta_2 \) as generalized coordinates)

5 eqns. for 5 unknown:

\[
m(r - r\dot{\theta}_1^2 + g \cos \theta_1) = \lambda_1 = Q_r
\]

(1)

\[
\frac{d}{dt}m\dot{r} - mgr \sin \theta_1 = (R_1 + R_2)\dot{\lambda}_2 = Q_1
\]

(2)

\[
\frac{1}{2}mR_2^2\dot{\theta}_2^2 = -R_2\dot{\lambda}_2 = Q_2
\]

(3)

constraints:

\[
r = R_1 + R_2
\]

(4)

\[
\dot{r} = \dot{r} = 0
\]

(5)

\[
R_1\dot{\theta}_1 = R_2(\dot{\theta}_2 - \dot{\theta}_1)
\]

\[
m(R_1 + R_2)\dot{\theta}_1 - mg(R_1 + R_2) \sin \theta_1 = (R_1 + R_2)\dot{\lambda}_2 = -\frac{1}{2}m(R_1 + R_2)^2\ddot{\theta}_1
\]

(2)

\[
\frac{1}{2}(R_1 + R_2)^2\ddot{\theta}_1 - g \sin \theta_1 = 0
\]

(5)

can be integrated:

\[
(R_1 + R_2)^2\ddot{\theta}_1 = \frac{3}{4}g(1 - \cos \theta_1)
\]

eq. of motion for the only independent coordinate

\[
N(\theta_1) = \frac{1}{2}mg(7 \cos \theta_1 - 4)
\]

cylinders stay in contact as far as \( N \geq 0 \)

angle of separation: \( 55.15^\circ \)

beyond this point we need all three variables, the motion is described by eqs. of motion with lagrange multipliers set to 0.

constant corresponds to cylinder starting at rest at the top