Cross section:

**incident flux**
(number of particles crossing unit transverse area per unit time)

**differential cross section**
(constant of proportionality)

**Number of events of type p per unit time**
\[ F \, d\sigma_p \]

**event p** = elastic scattering through a
deflection angle between \( \theta \) and \( \theta + d\theta \):

\[
F \, 2\pi b \, db = F \, d\sigma_{el}(\theta) = F \left( \frac{d\sigma}{d\Omega} \right)_{el} \, d\Omega
\]

**differential cross section**
(per unit solid angle)

\[
\left( \frac{d\sigma}{d\Omega} \right)_{el} = \frac{b}{\sin \theta} \left| \frac{db}{d\theta} \right|
\]

**total cross section**:
\[
\sigma_f = \int d\Omega \left( \frac{d\sigma}{d\Omega} \right)_{el}
\]

Rutherford scattering:

*(the scattering of a particles with charge \( ze \) and mass \( m \) from a
nucleus with charge \( Ze \) and mass \( M \))

Newton’s second law:
\[
\vec{\mu} = \vec{f}_{Coulomb}
\]

the same as for Kepler problem,
hyperbolic gravitational orbits with the replacement:

**Orbit solution**:
\[
r^{-1} = C\left( 1 - \epsilon \cos \phi \right)
\]

\[\epsilon = \left( 1 + \frac{2E_{f}}{\mu^2} \right)^{1/2} \quad C = \frac{\mu^2 \gamma}{l^2}
\]

for \( C<0 \) (repulsive potential):
\[
r^{-1} = |C| \left( \epsilon \cos \phi - 1 \right)
\]

is the right branch of hyperbola in polar
coordinates with the origin at the left focus.
Hyperbola:

constant difference in distance from two foci located at \( x=+f \) and \( x=-f \):

left branch

right branch

eccentricity: \( \epsilon = \frac{f}{a} > 1 \)

asymptotes of the two branches:

\[
\cos \alpha = \epsilon^{-1}
\]

right branch in polar coordinates with the origin at the right focus:

\[
r (1 - \epsilon \cos \phi) = a (\epsilon^2 - 1)
\]

follows from the law of cosines:

\[
d^2 = d^2 + 4f^2 - 4df \cos(\pi - \phi)
\]

right branch in polar coordinates with the origin at the left focus:

\[
r (\epsilon \cos \phi - 1) = a (\epsilon^2 - 1)
\]

follows from the law of cosines:

\[
d^2 = d'^2 + 4f^2 - 4d'f \cos \phi
\]

for positive \( C \) we found:

\[
\cot \frac{1}{2} \theta = \frac{v_e^2 b}{\gamma}
\]

\( b \) is identical for both signs of the potential

relation between \( b \) and the scattering angle:

\[
b = \left| \frac{Z e^2}{\mu v_e^2} \right| \cot \frac{1}{2} \theta
\]

Rutherford formula:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{ct} = \frac{1}{4E \sin^2 \frac{1}{2} \theta} \left( \frac{1}{2} \mu v_e^2 \right)^2
\]

even the total cross section is divergent for small scattering angles!

What does it mean?
Scattering by a hard sphere:

\[ \theta = 2\Phi_m - \pi \]

\[ b = a \sin \Phi_m \]

\[ b = a \cos \frac{1}{2}\theta \quad \text{for} \quad b < a \]

\[ \theta = 0 \quad \text{for} \quad b > a \]

**differential cross section:**

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{el}} = \frac{b}{\sin \theta} \left| \frac{db}{d\theta} \right|
\]

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{el}} = \frac{1}{4}a^2
\]

**isotropic**

**total cross section:**

\[
\sigma_T = \int d\Omega \left( \frac{d\sigma}{d\Omega} \right)_{\text{el}} = \pi a^2
\]

\[ \theta = 0 \quad \text{for} \quad b > a \]

Infinite for Rutherford-like scattering