IR fixed point pattern of couplings in the standard model

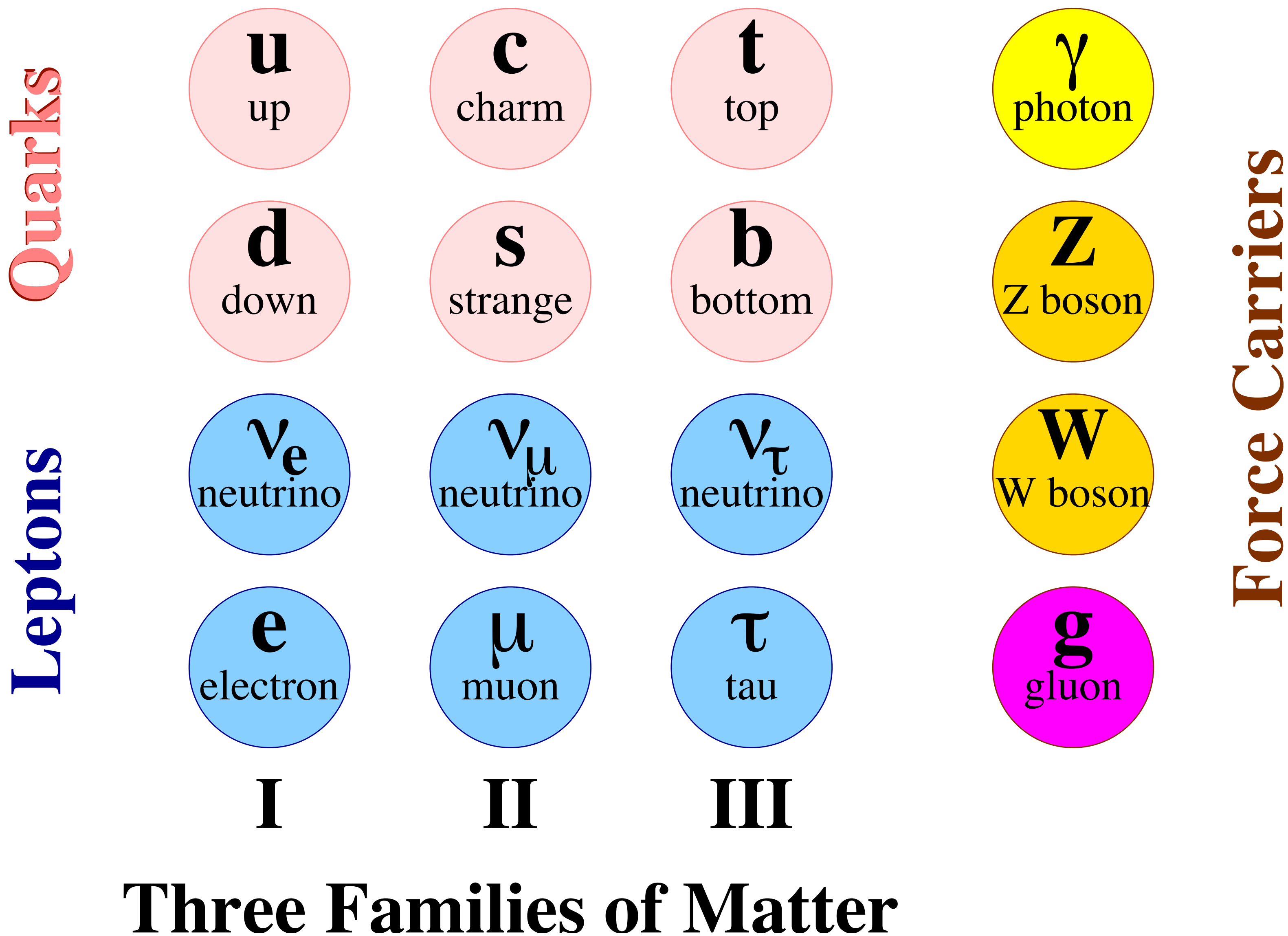
with N. McGinnis, arXiv:1812.05240
arXiv:1810.12474
arXiv:1712.03527
also several papers with E. Lunghi and S. Shin

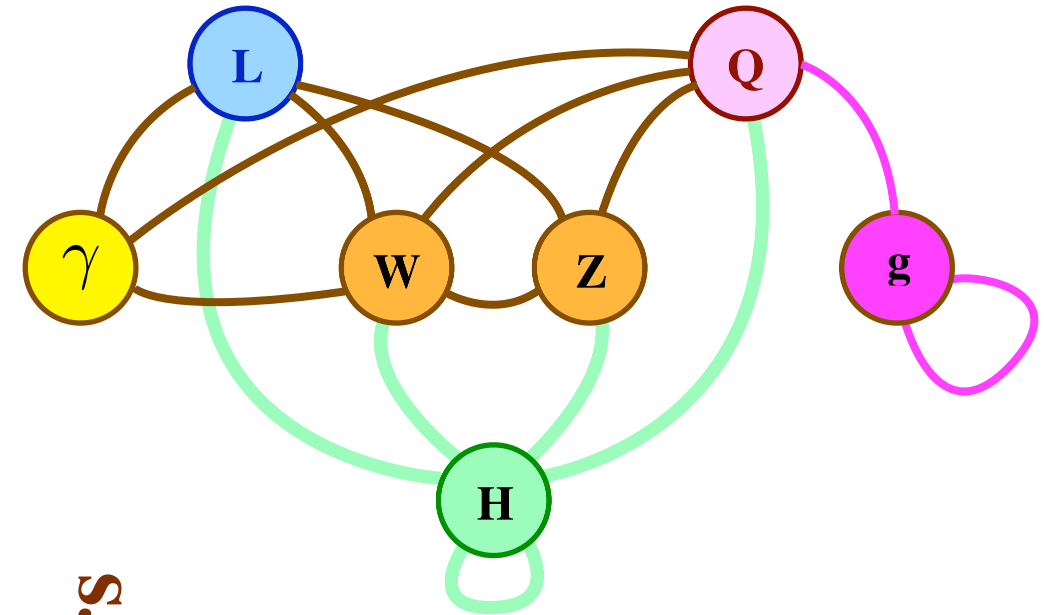
Radovan Dermisek

Indiana University, Bloomington

The Ohio State University, February 4, 2019

Standard model





17 dimensionless parameters

Standard model

Out of 17 dimensionless parameters:

$$\alpha_1, \ \alpha_2, \ \alpha_3, \ y_t, \ y_b, \ y_\tau, \ \lambda_h$$

only 7 couplings are sizable

all others = 0 (in the first approximation)

Can we understand any of them?

Seeking explanations

- **Numerology**

$$\alpha^{-1} = \frac{4\pi\epsilon_0\hbar c}{e^2} = 137.035\ 999\ 11(46)$$

Seeking explanations

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$$\alpha^{-1} = \frac{4\pi\epsilon_0\hbar c}{e^2} = 137.035\ 999\ 11(46)$$

must be an integer

Eddington (1930)

$$\alpha^{-1} = 2^4 3^3/\pi$$

Heisenberg

$$\alpha^{-1} = (8\pi^4/9)(2^4 5!/\pi^5)^{1/4}$$

Wyler (1969)

$$\alpha^{-1} = 108\pi(8/1843)^{1/6}$$

Aspden and Eagles (1972)

$$\alpha^{-1} = 2^{-19/4} 3^{10/3} 5^{17/4} \pi^{-2}$$

Robertson (1971)

$$\alpha^{-1} = (137^2 + \pi^2)^{1/2}$$

Burger (1978)

compiled by G. Giudice, arXiv:0801.2562 [hep-ph]

Seeking explanations

- **Numerology**

$$\frac{m_e + m_\mu + m_\tau}{(m_e + m_\mu + m_\tau)^2} = \frac{2}{3}$$

predicts tau mass 1.777 GeV

or the angle between
$(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$
and
(1,1,1)
is 45°

Koide (1981)

Seeking explanations

- **Numerology**

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

or the angle between $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ and (1,1,1) is 45°

Koide (1981)

predicts tau mass 1.777 GeV

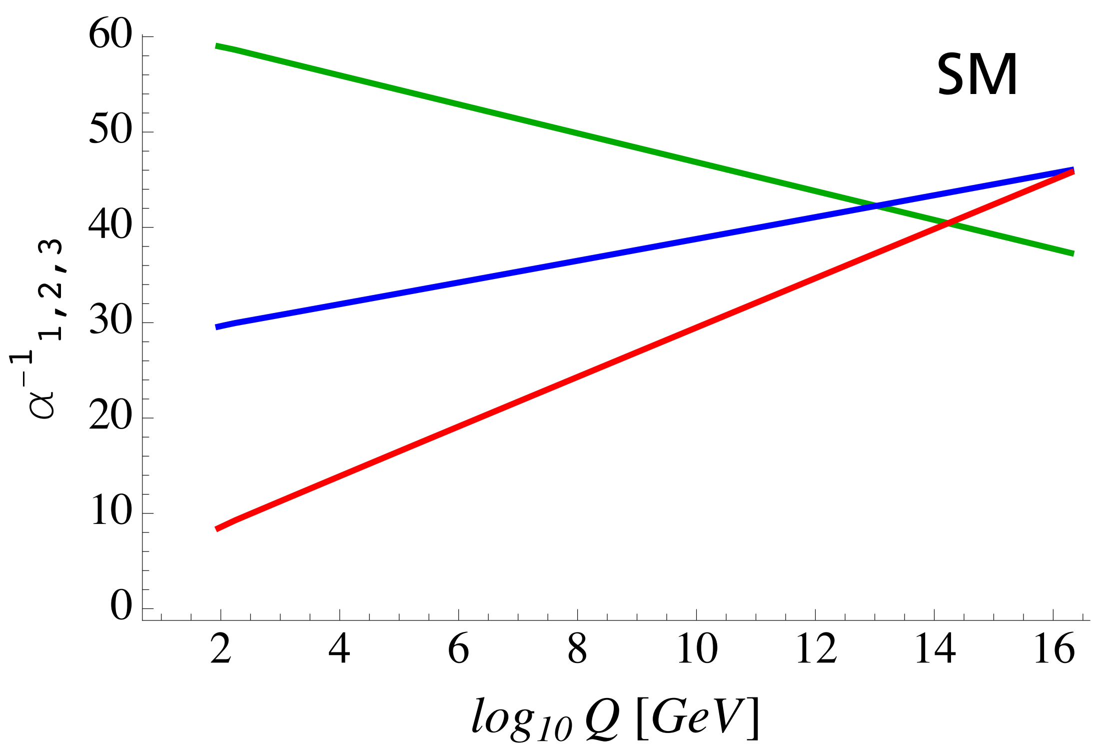
frequencies of the hydrogen spectral lines:

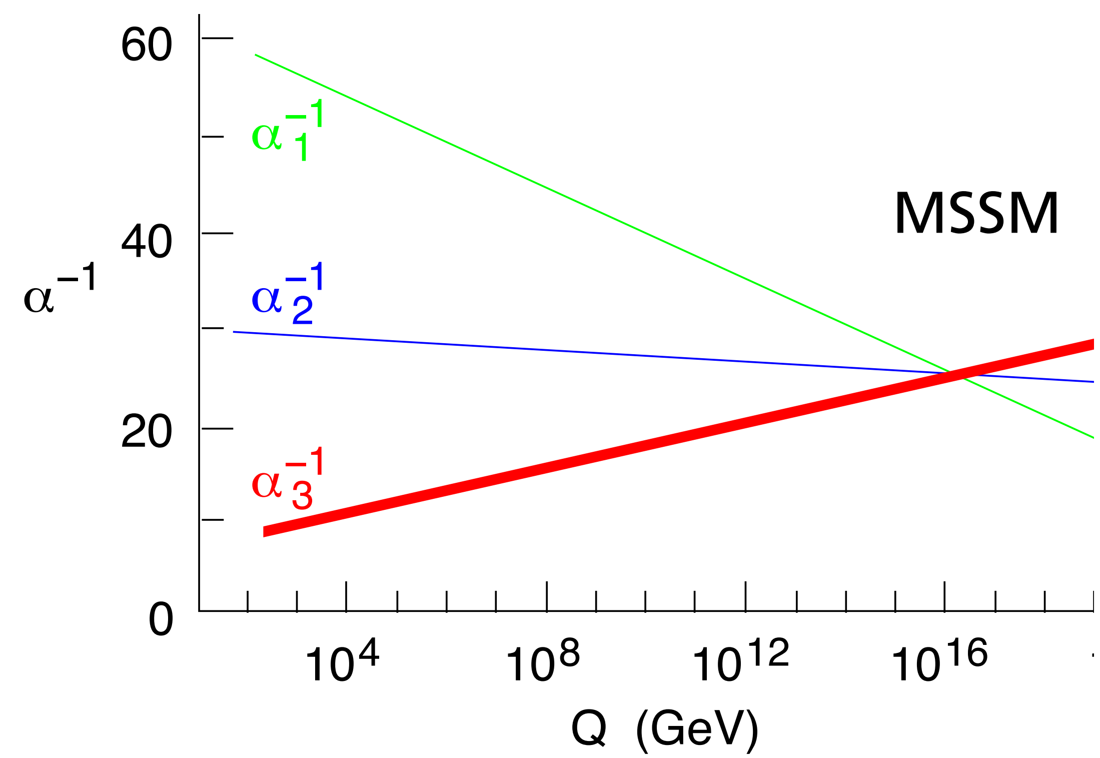
$$\nu = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Balmer (1885)

Seeking explanations

- **Numerology**
- **Symmetry: GUTs, SUSY, …**





extremely suggestive in the MSSM

Seeking explanations

- Numerology
- Symmetry: GUTs, SUSY, …
- Anthropic, multiverse, …

Seeking explanations

- **Numerology**
- **Symmetry: GUTs, SUSY, …**
- **Anthropic, multiverse, …**

- **Self-organization**

  particle content determines the strength of interactions (at least basis features)

  allows for predictions of SM parameters even if the fundamental symmetries or model parameters at the fundamental scale remain obscure; or even if the model has more parameters than observables

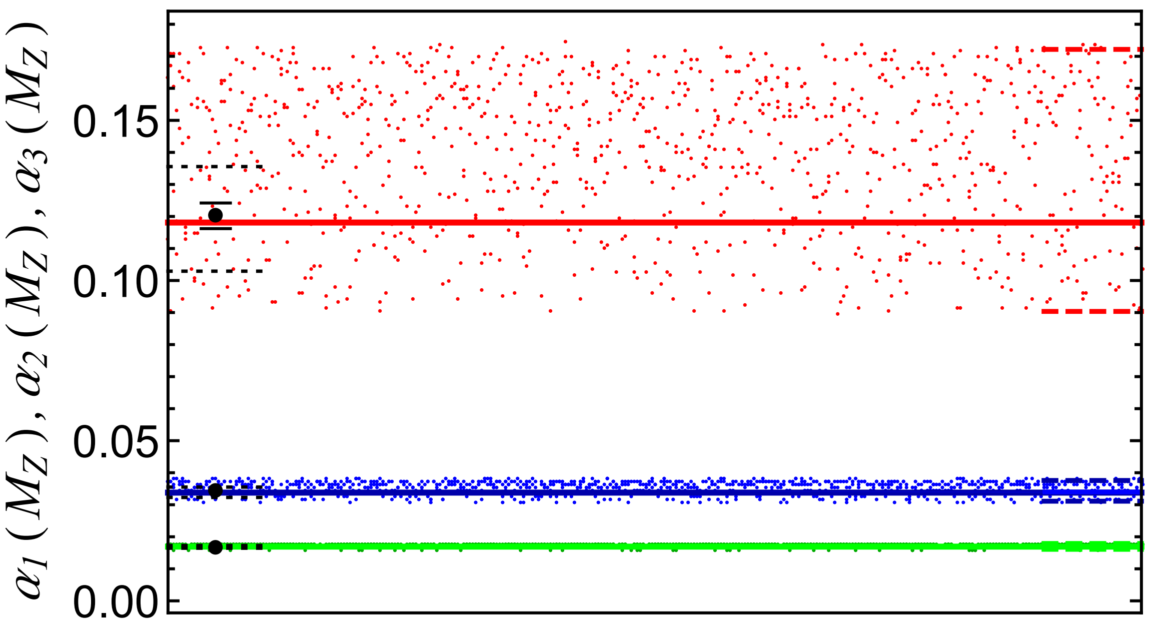
There is a model in which

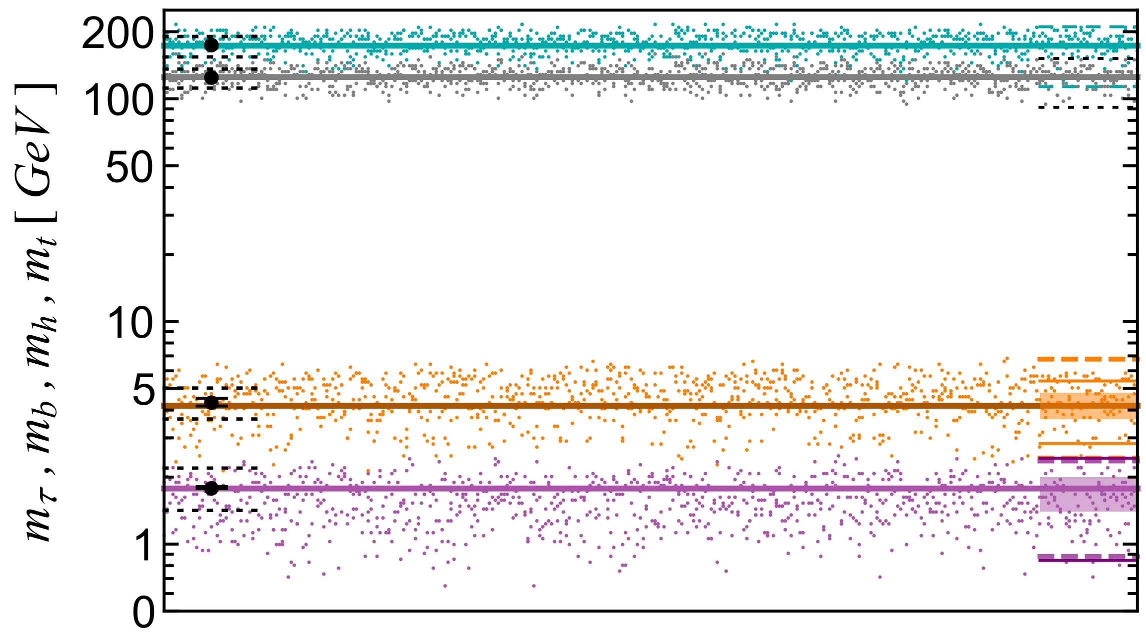
the pattern of seven largest couplings can be predicted:

for random (unrelated): $\alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1, 0.3]$, $y_t(M_G), y_b(M_G), y_\tau(M_G) \in [1,3]$

with fixed: $M_G = 3.5 \times 10^{16}$ GeV, $M = 7$ TeV and $\tan\beta = 40$

(larger values of couplings do not affect results significantly)





predicted couplings closely cluster around observed values

Model and assumptions

MSSM with a complete vectorlike family

We add to the MSSM:

$$Q,\ \bar{U},\ \bar{D},\ L,\ \bar{E} \quad + \quad \bar{Q},\ U,\ D,\ \bar{L},\ E$$

or $16 + \overline{16}$ **in SO(10) language**

We consider:

- unrelated or universal gauge couplings at the GUT scale
- unrelated or universal Yukawa couplings at the GUT scale:

$$y_t = y_b = y_\tau \equiv Y_0$$

motivated by SO(10)

- universal Yukawa c. of vectorlike fields at the GUT scale: Y_V
- common scale for superpartners: M_{SUSY} (and zero A-terms)
- common scale for vectorlike matter: M_V

often we identify the two scales: $M_{SUSY} = M_V \equiv M$

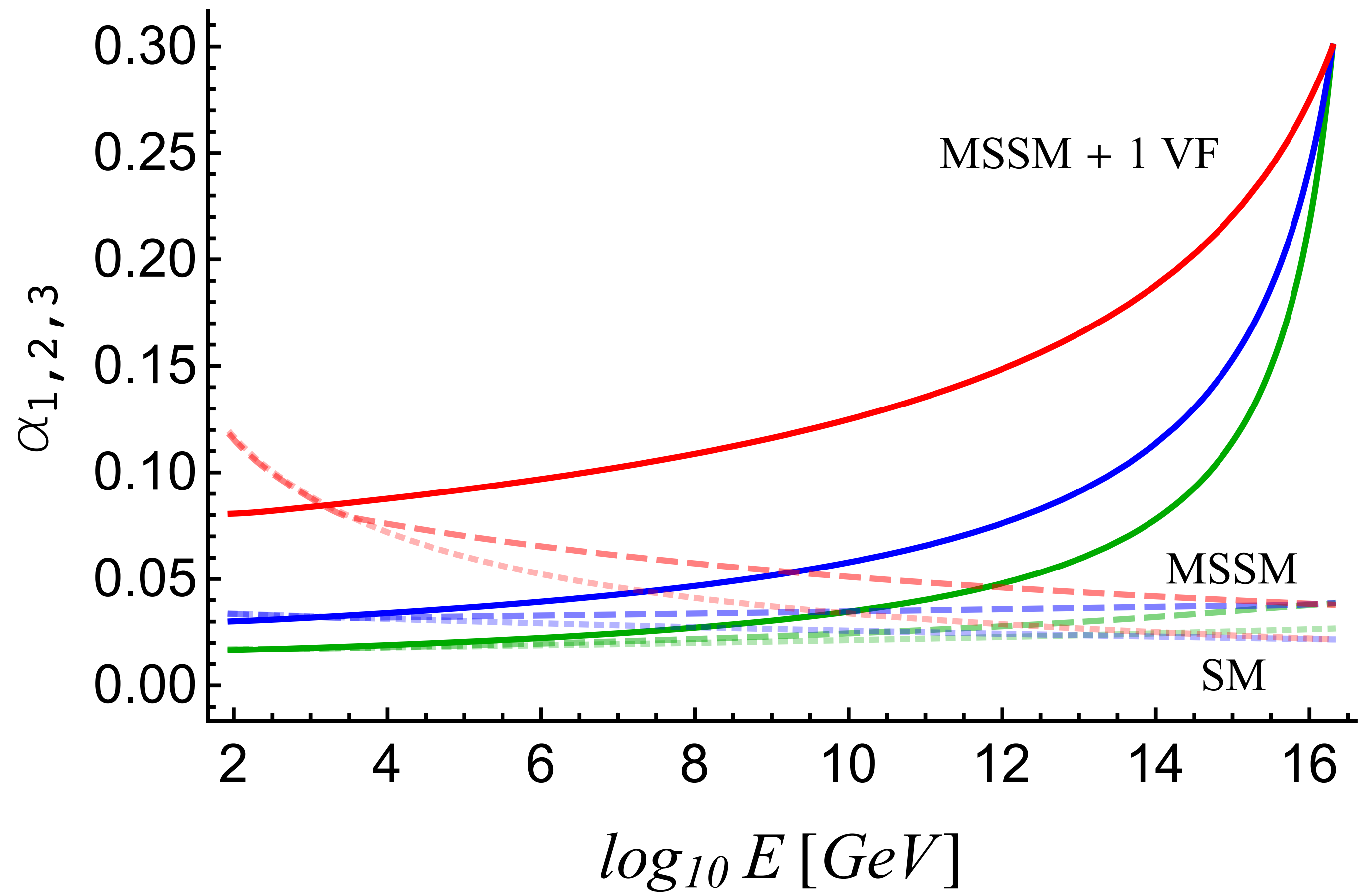
Gauge couplings

Gauge couplings in MSSM+1VF

RG equations:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$$b_i = (33/5, 1, -3) + n_5(1,1,1) + 3n_{10}(1,1,1)$$

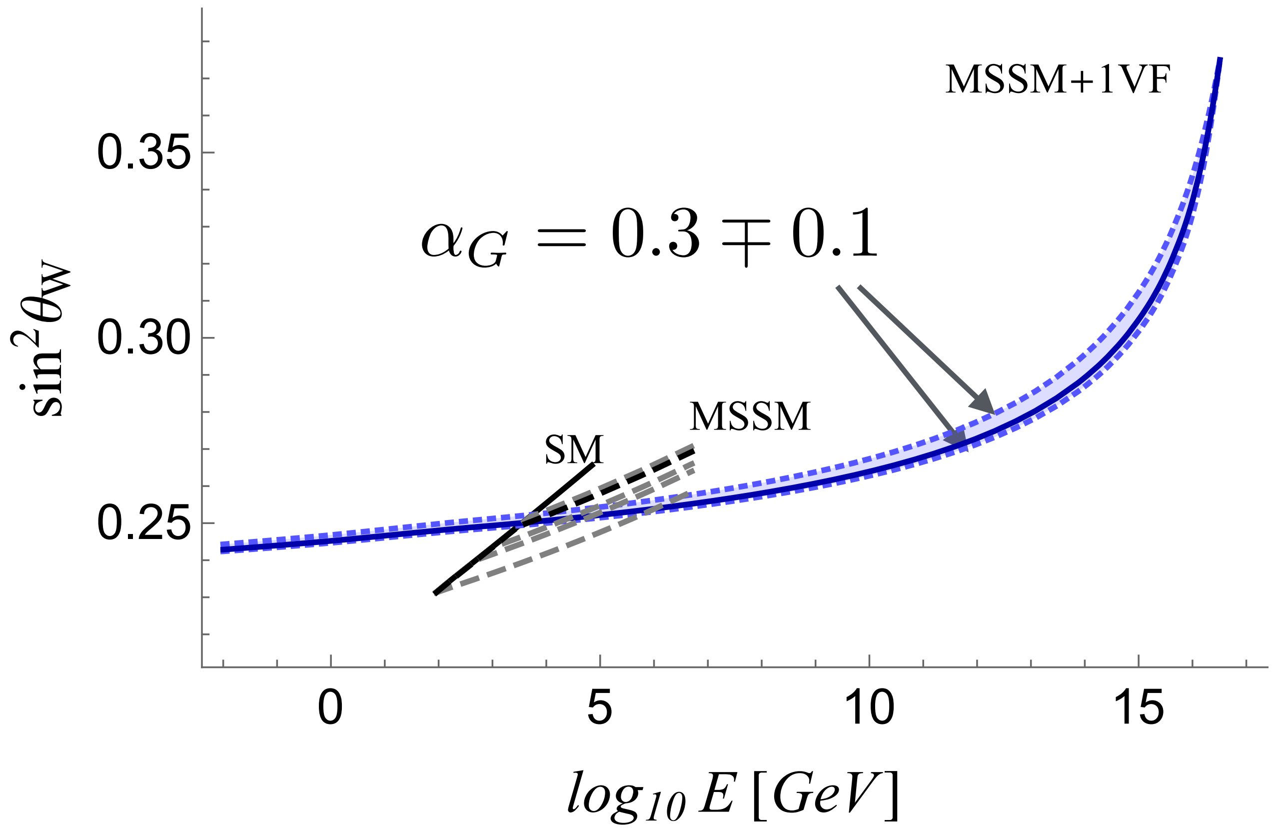


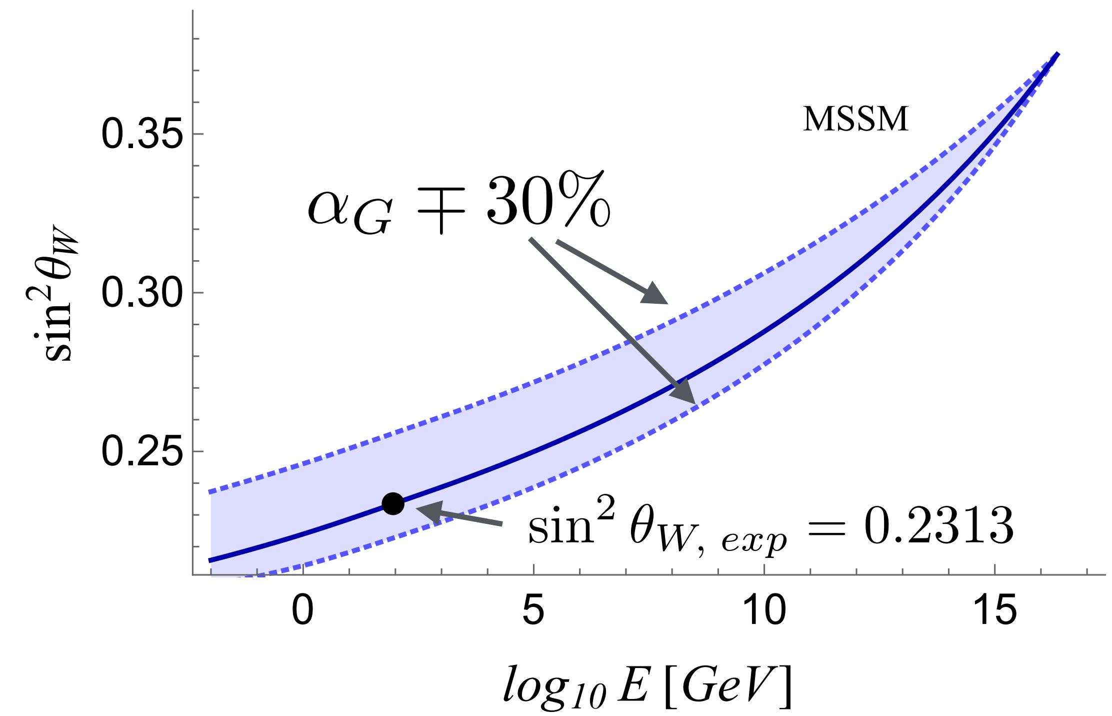
Solution:

$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha^{-1}(M_G)$$

Two parameter free predictions: $$\frac{\alpha_j(M_Z)}{\alpha_i(M_Z)} \simeq \frac{b_i}{b_j}$$

Weak mixing angle



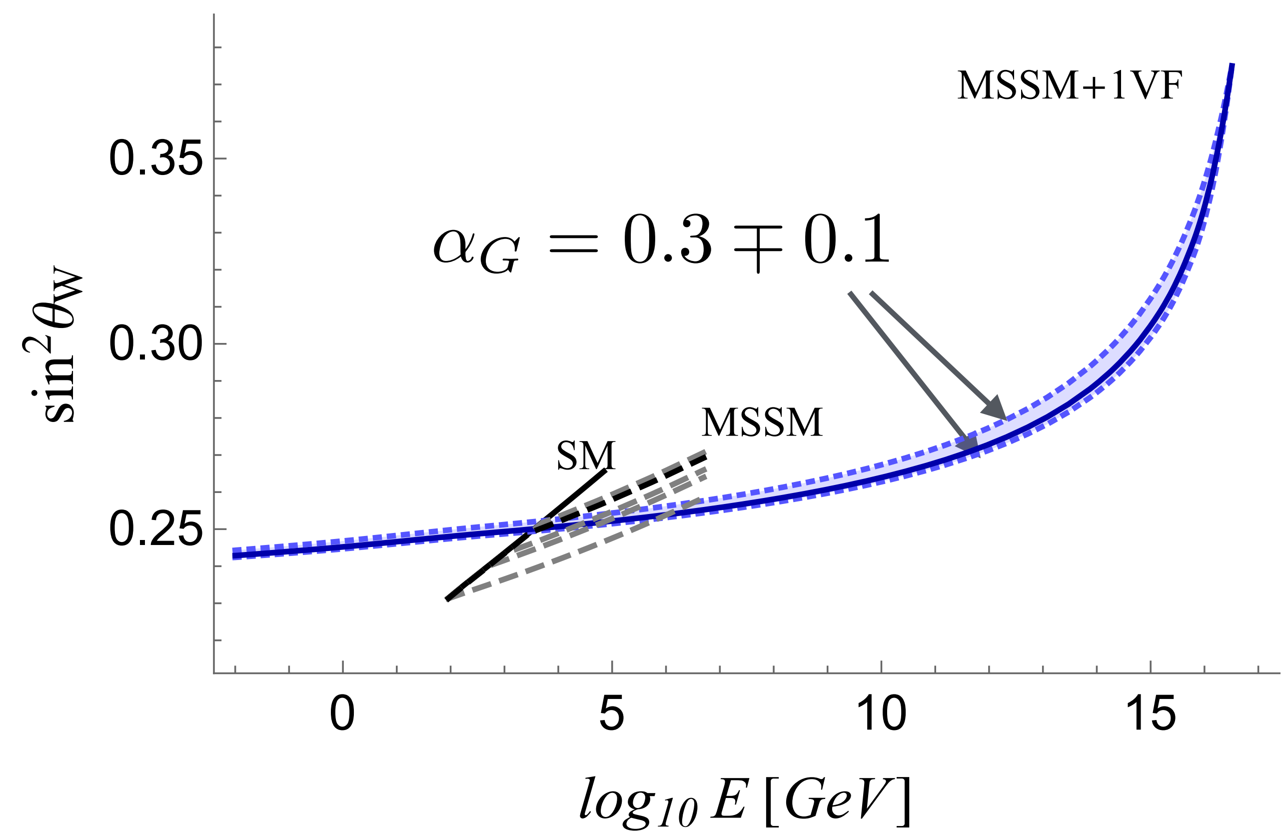


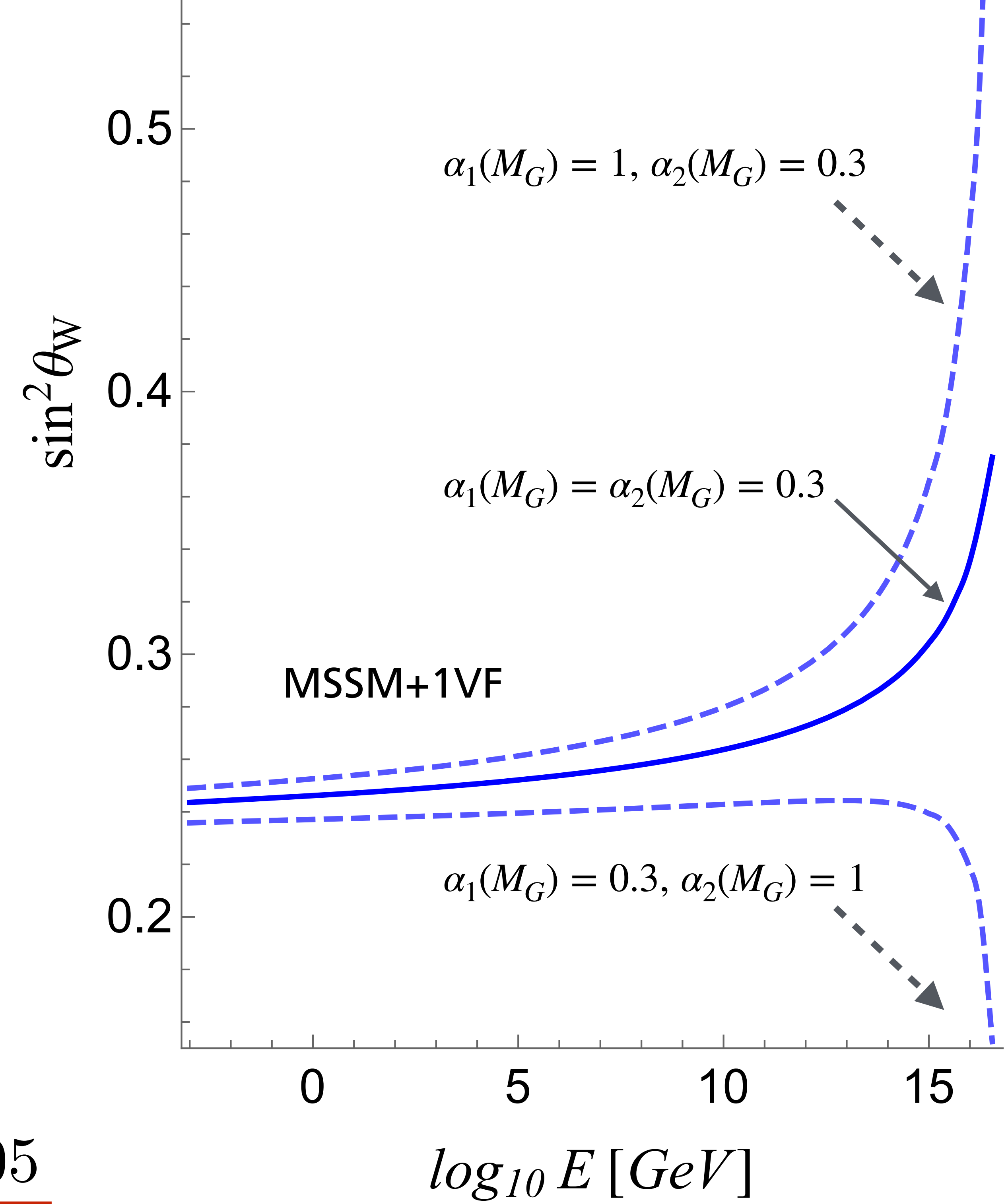
$$\sin^2\theta_W = \frac{\alpha'}{\alpha_2 + \alpha'} \simeq \frac{b_2}{b' + b_2} = \underline{0.2205}$$

$b' = (5/3) b_1$

robust prediction away from the GUT scale

Weak mixing angle





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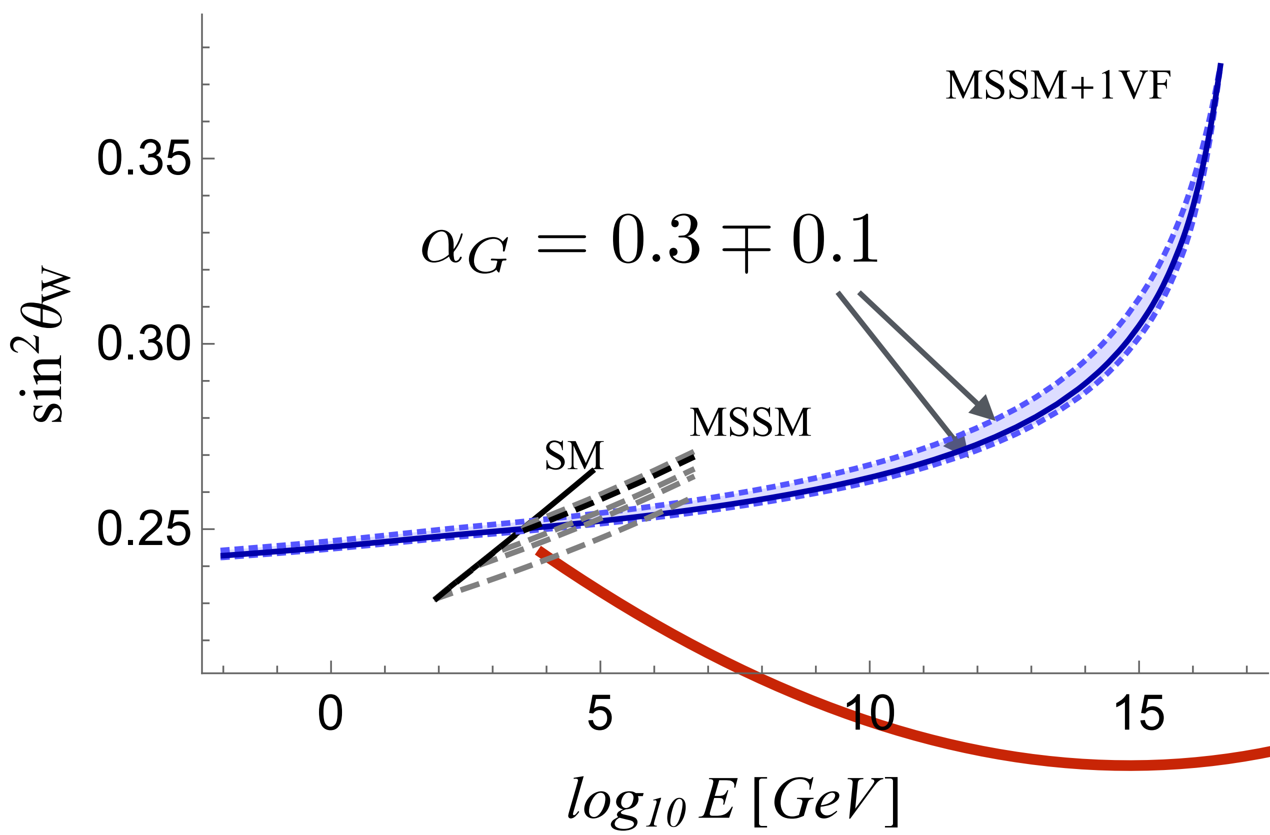
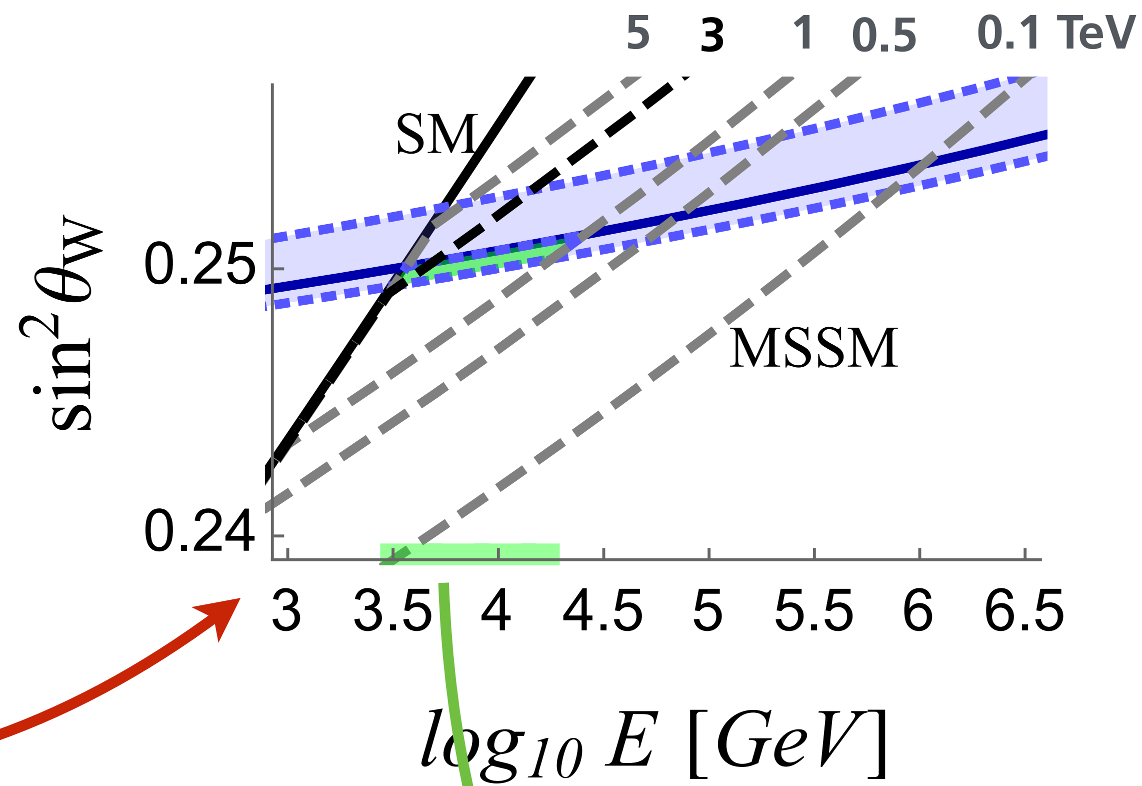
prediction highly insensitive to boundary conditions

Weak mixing angle





for any $\alpha_G > 0.3$ and superpartners above 1 TeV
vectorlike matter is expected below 20 TeV

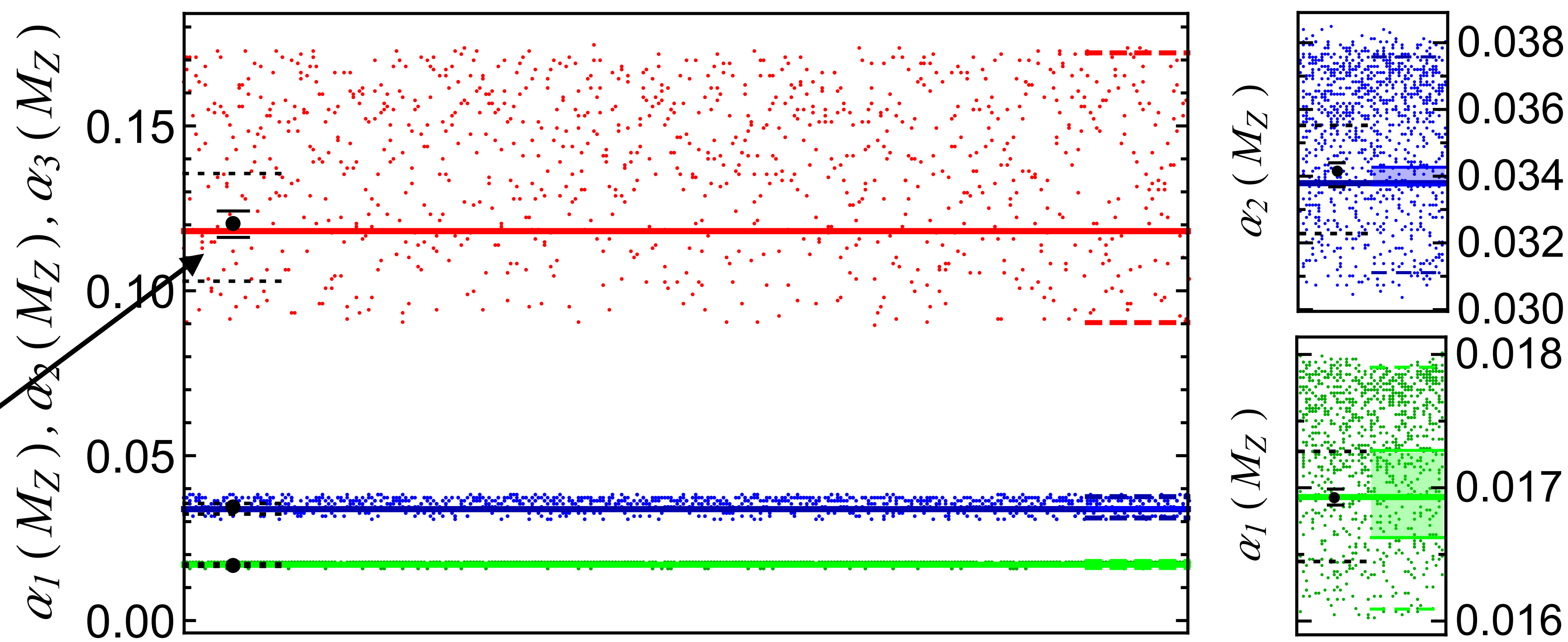
Predicted pattern of gauge couplings

In the MSSM+1VF:

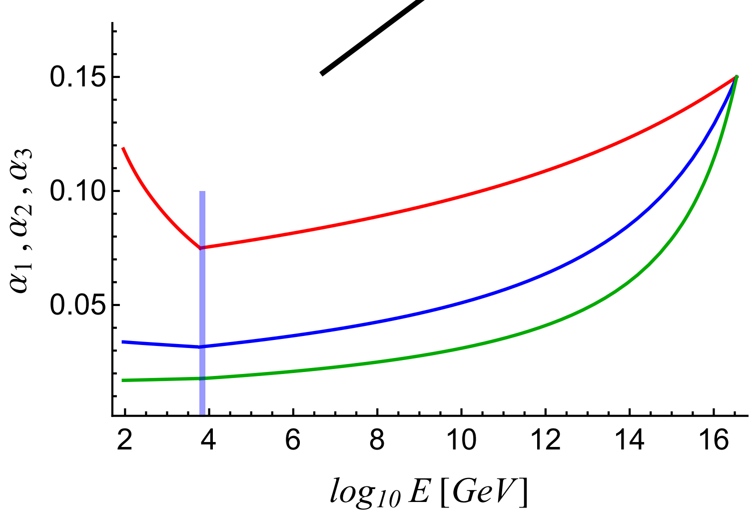
$\alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1, 0.3]$

$M_G = 3.5 \times 10^{16}$ GeV, $M = 7$ TeV and $\tan\beta = 40$

– – – universal b.c.   M optimized for α_3



20% variations of model parameters



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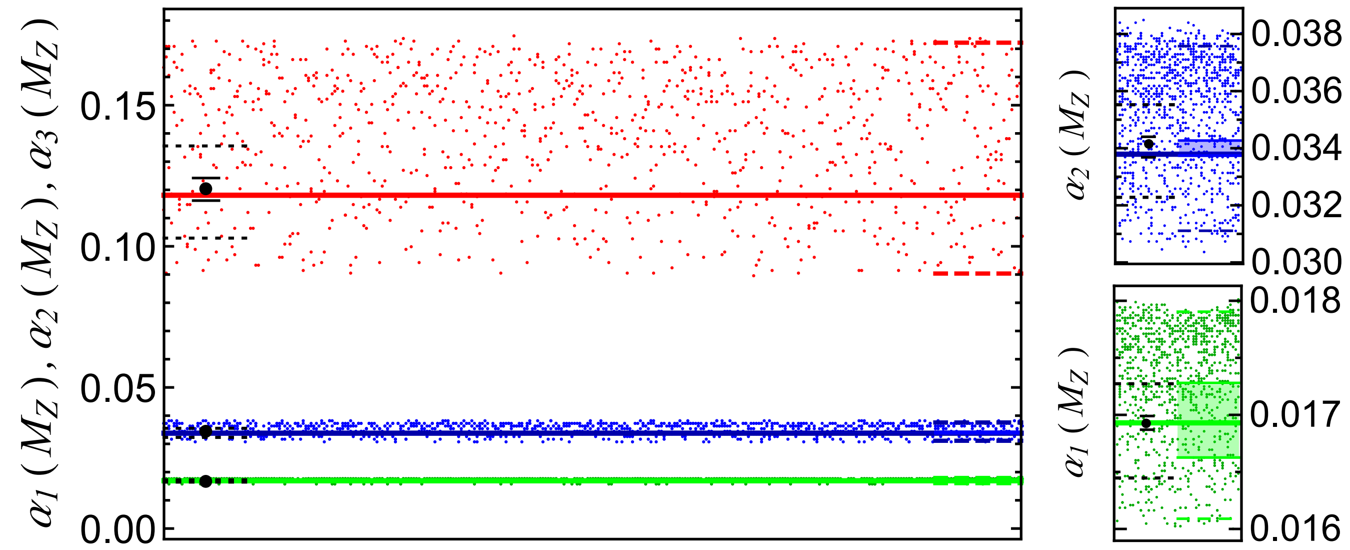
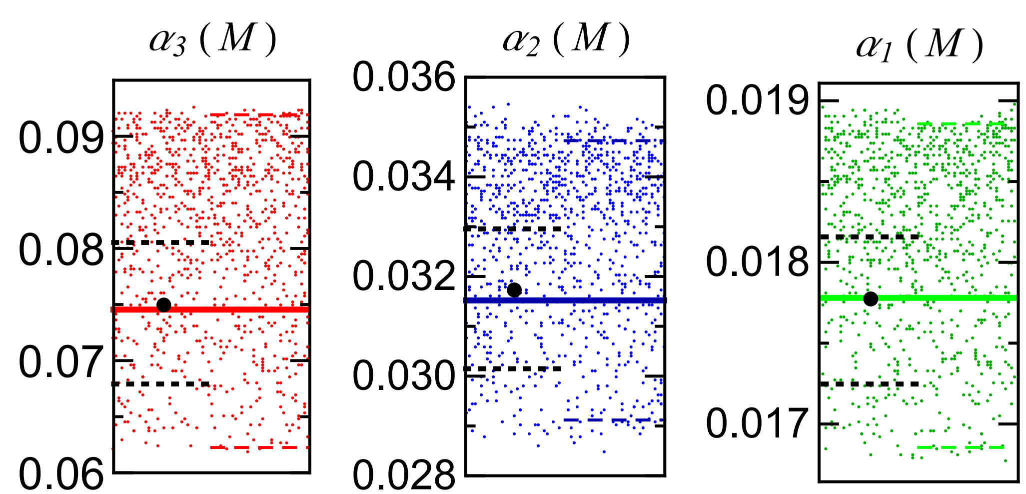


IR fixed point predictions should be compared with the measured values at M:



sharper predictions at the matching scale

Yukawa couplings

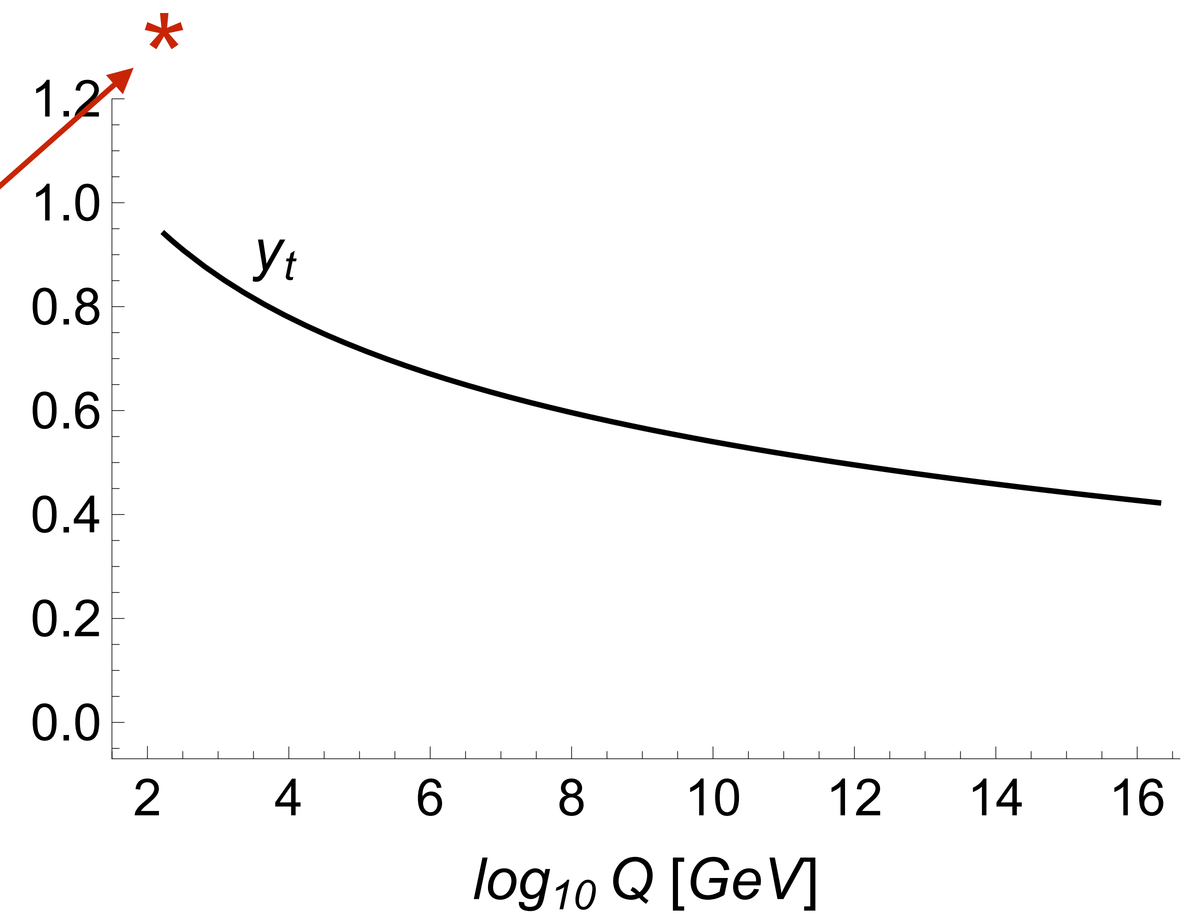
Top Yukawa coupling in the SM

The 1-loop RG equation for top Yukawa coupling:

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2\right)$$

top Yukawa coupling runs to a quasi-fixed point

B. Pendleton and G.G. Ross, 1981
C. Hill, 1981



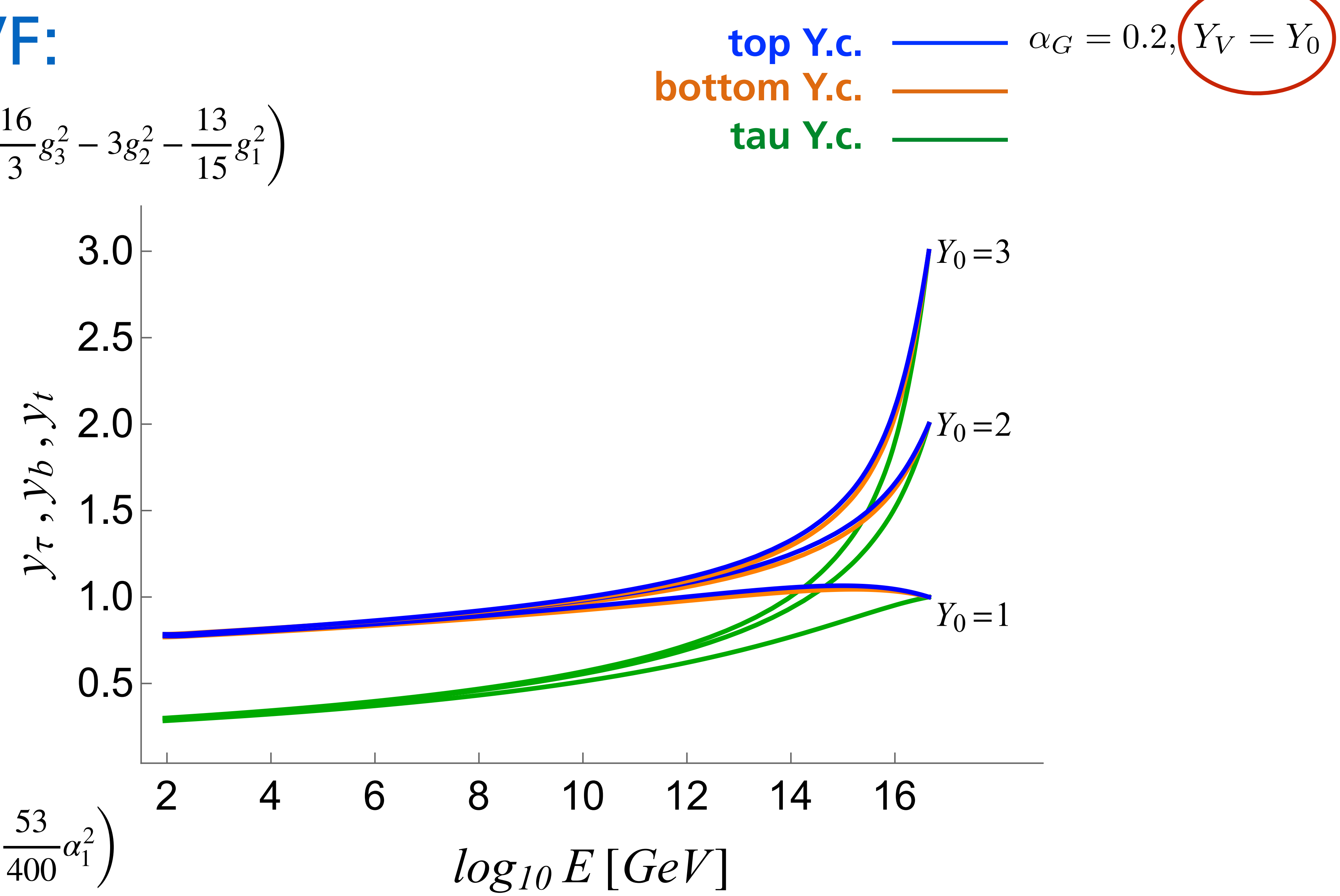
but approaches the fixed point very slowly

Evolution of top, bottom and tau Y.c.

In the MSSM+1VF:

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(6y_t^* y_t + y_b^* y_b + T_{H_u} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right)$$

$$T_{H_u} \equiv 3Y_U^* Y_U + 3\bar{Y}_D^* \bar{Y}_D + \bar{Y}_E^* \bar{Y}_E$$



analytic solution for top Yukawa in terms of gauge couplings:

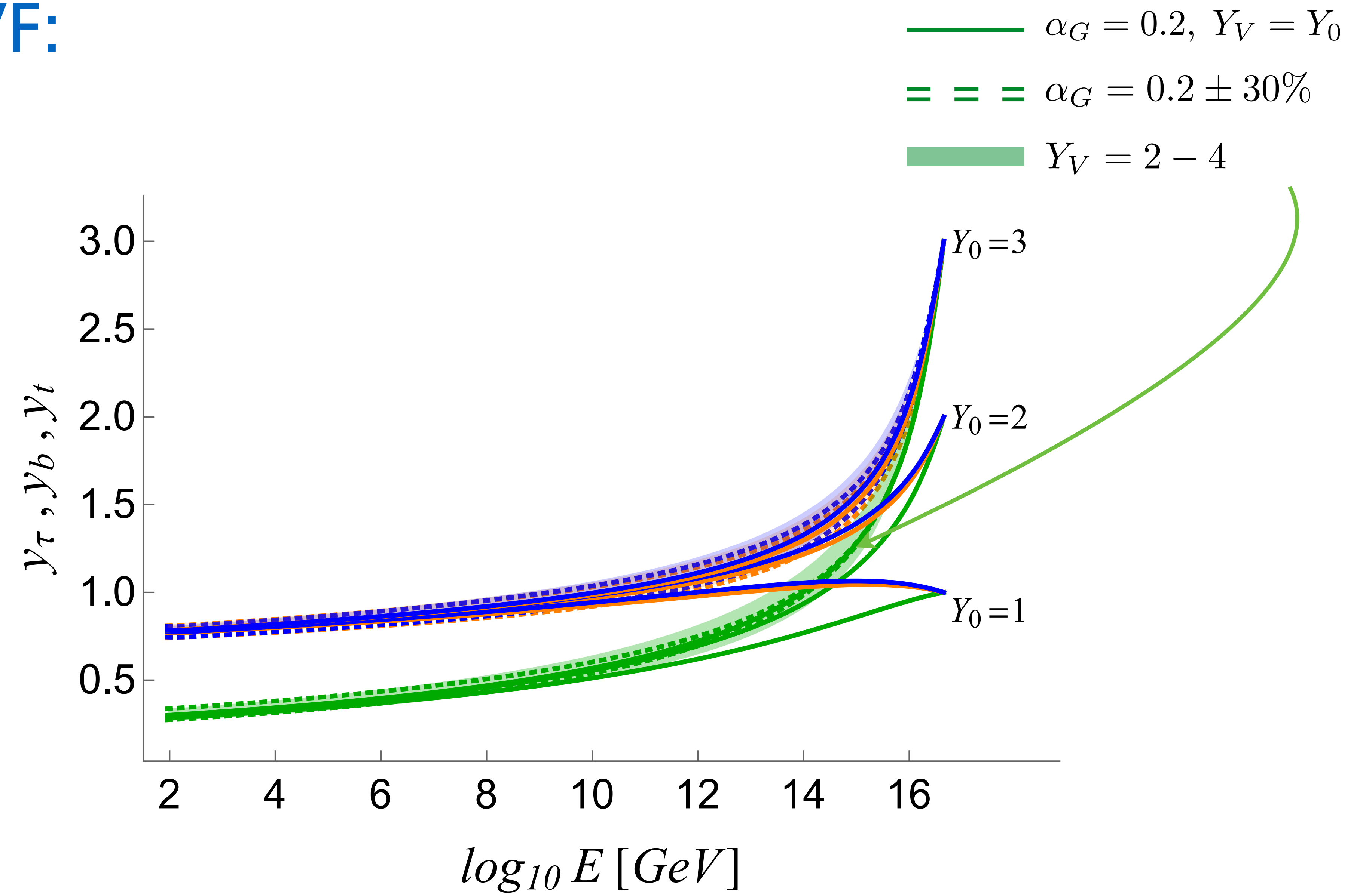
$$\frac{y_{t,IR}^2}{4\pi} = \frac{16}{39}\alpha + \frac{1}{\alpha}\left(\frac{1}{13}\alpha_3^2 + \frac{45}{208}\alpha_2^2 + \frac{53}{400}\alpha_1^2\right)$$

$$\alpha \equiv \alpha_3 + \frac{9}{16}\alpha_2 + \frac{13}{80}\alpha_1$$

can incorporate higher loop effects, agree at 0.1% level

Evolution of top, bottom and tau Y.c.

In the MSSM+1VF:



common IR fixed points remain good approximations for a large range of boundary conditions

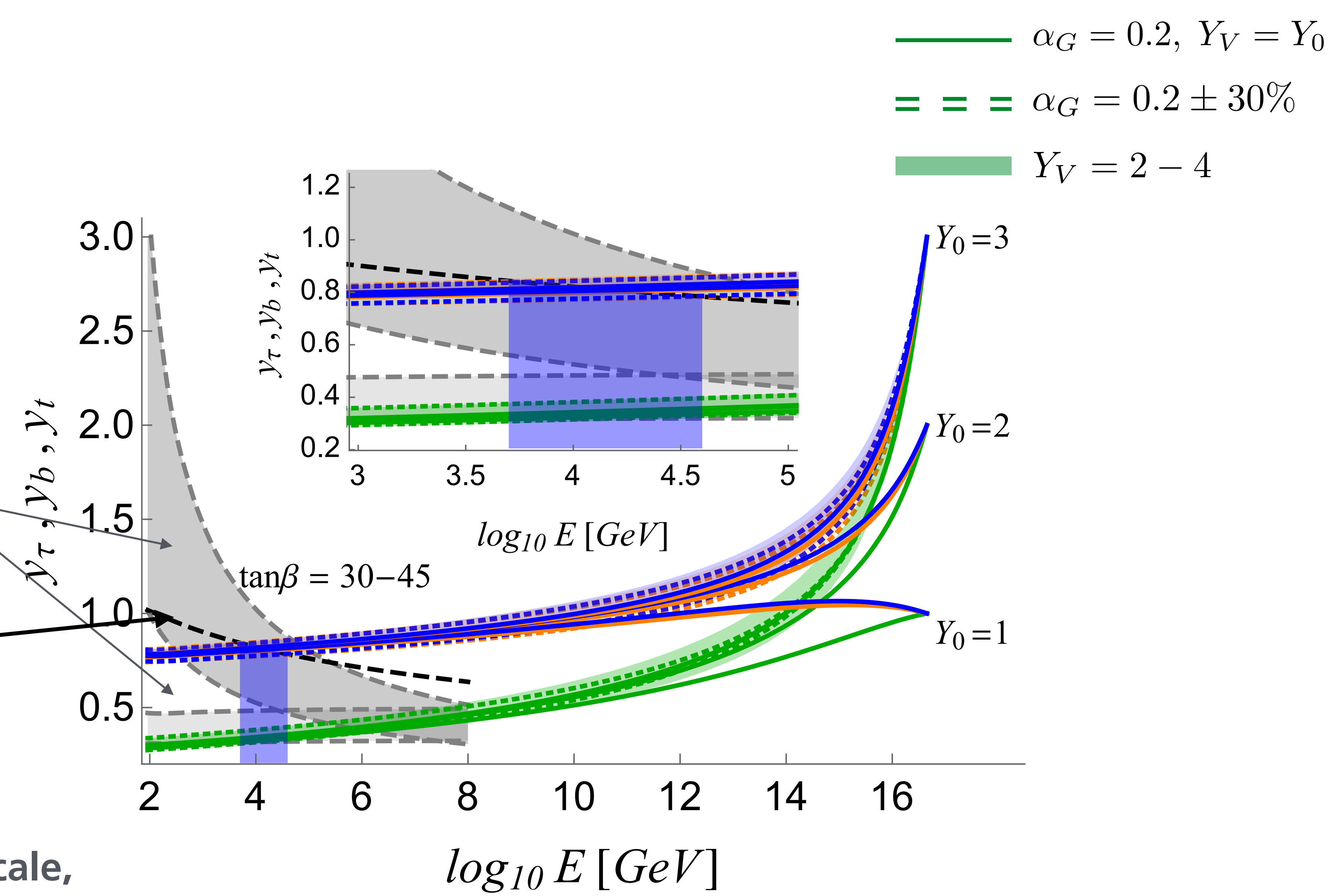
very effective IR fixed point behavior

Evolution of top, bottom and tau Y.c.

bottom, tau Y.c.:
$m_{b,\tau} = y_{b,\tau} v \cos\beta(1+\epsilon_{b,\tau})$

top Y.c.:
$m_t = y_t v \sin\beta(1+\epsilon_t)$

SUSY corrections assume all superpartners at the same scale, zero A-terms and $\mu = -\sqrt{2}M_{SUSY}$



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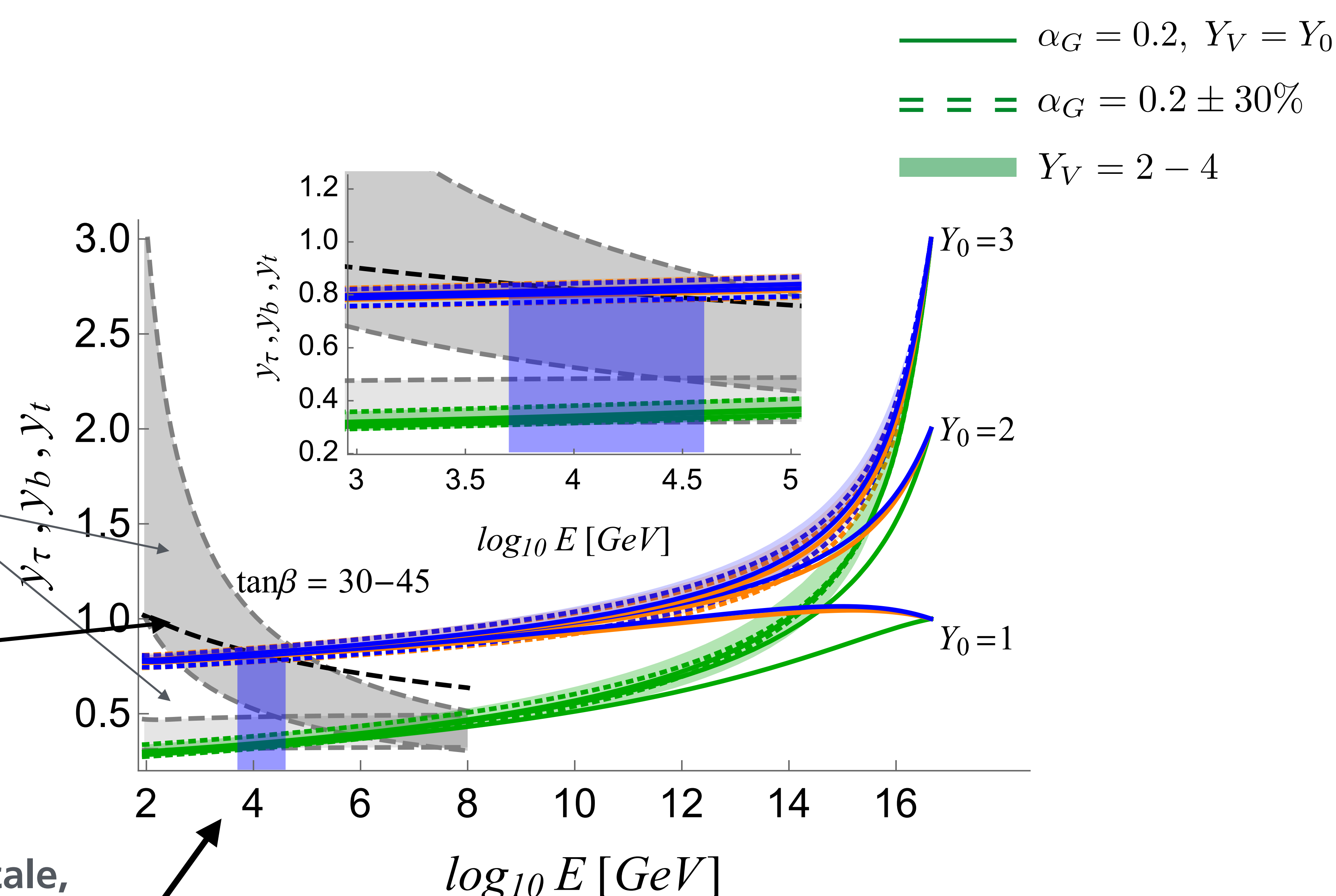
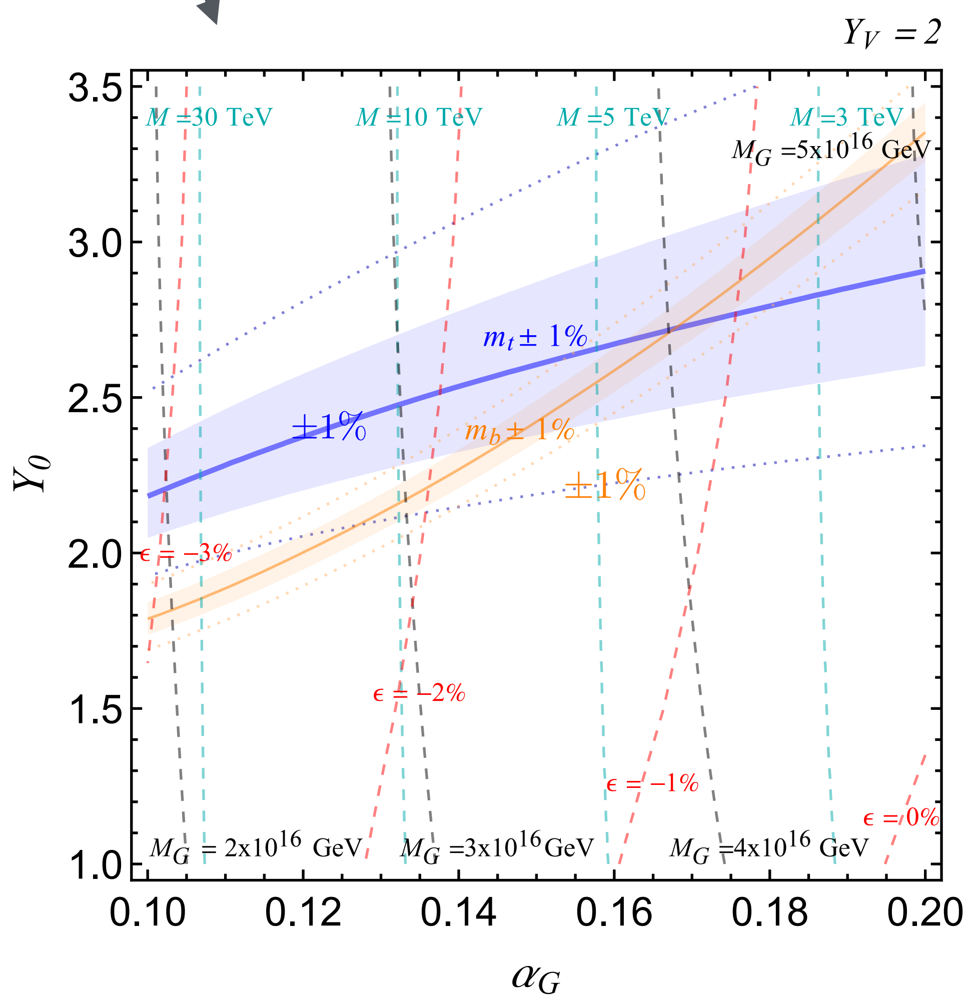


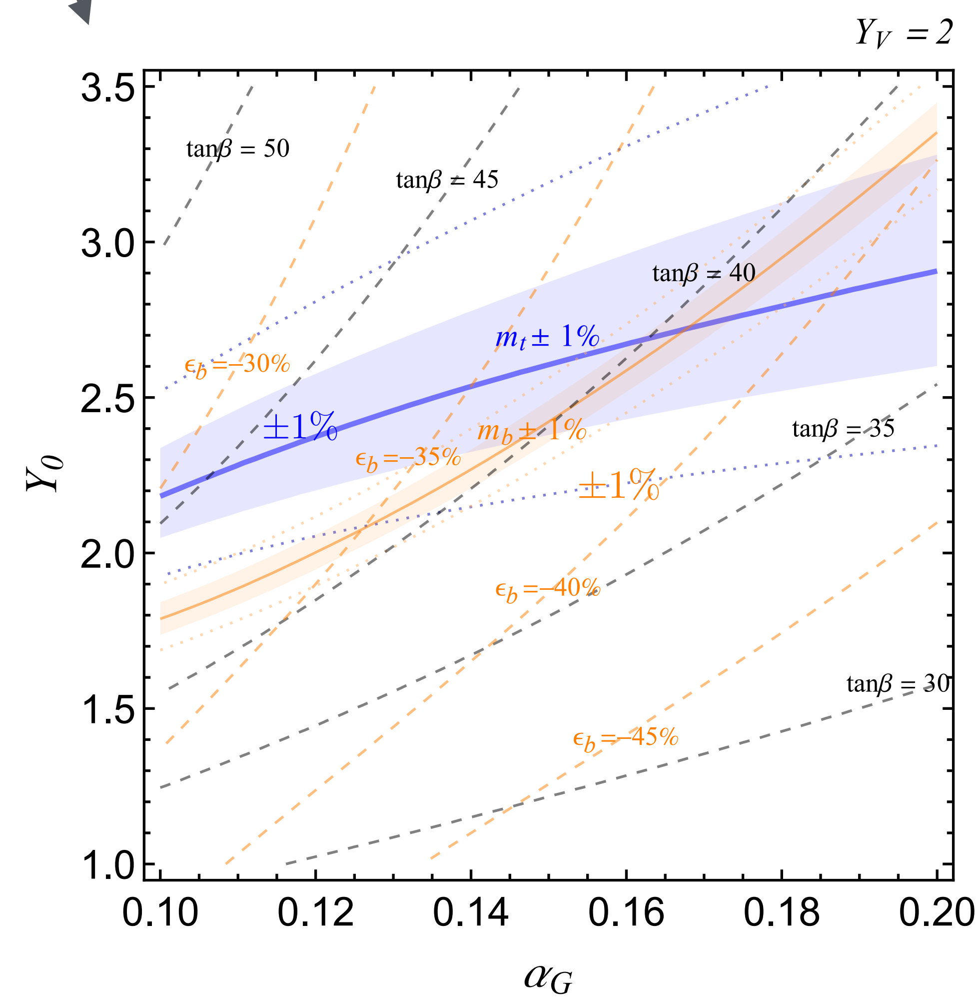
very sharp IR fixed point predictions again point to a multi-TeV scale for VF and SUSY

Exploring universal boundary conditions

Gauge couplings and tau mass are fit to central values:

M_G, ϵ, M $\tan\beta$

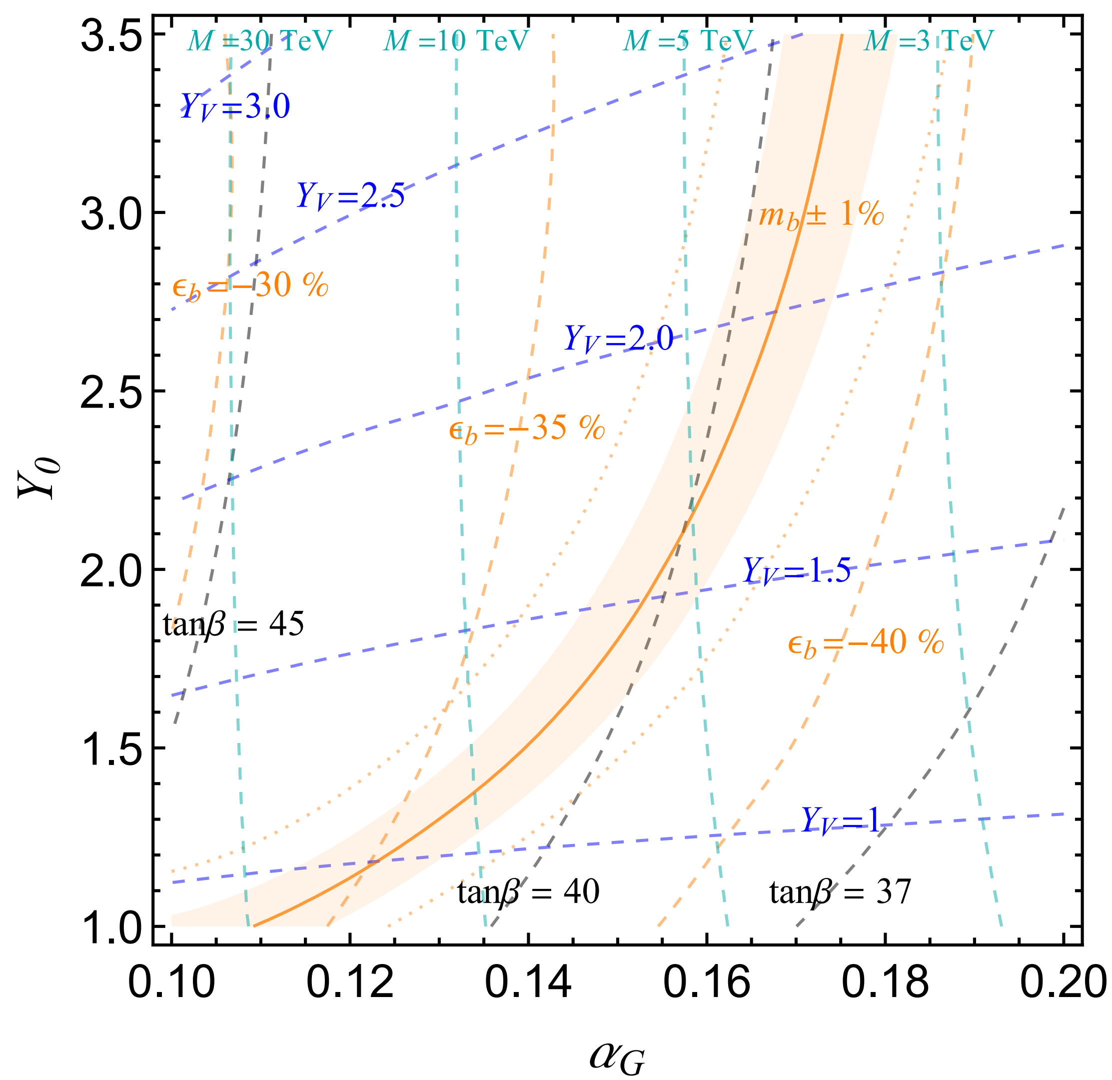




exact Yukawa coupling unification possible

Exploring universal boundary conditions

Everything fit to central values except for bottom mass:



fitting bottom mass in the whole plane requires SUSY corrections of a typical size

fitting everything suggests M = 3 - 30 TeV!

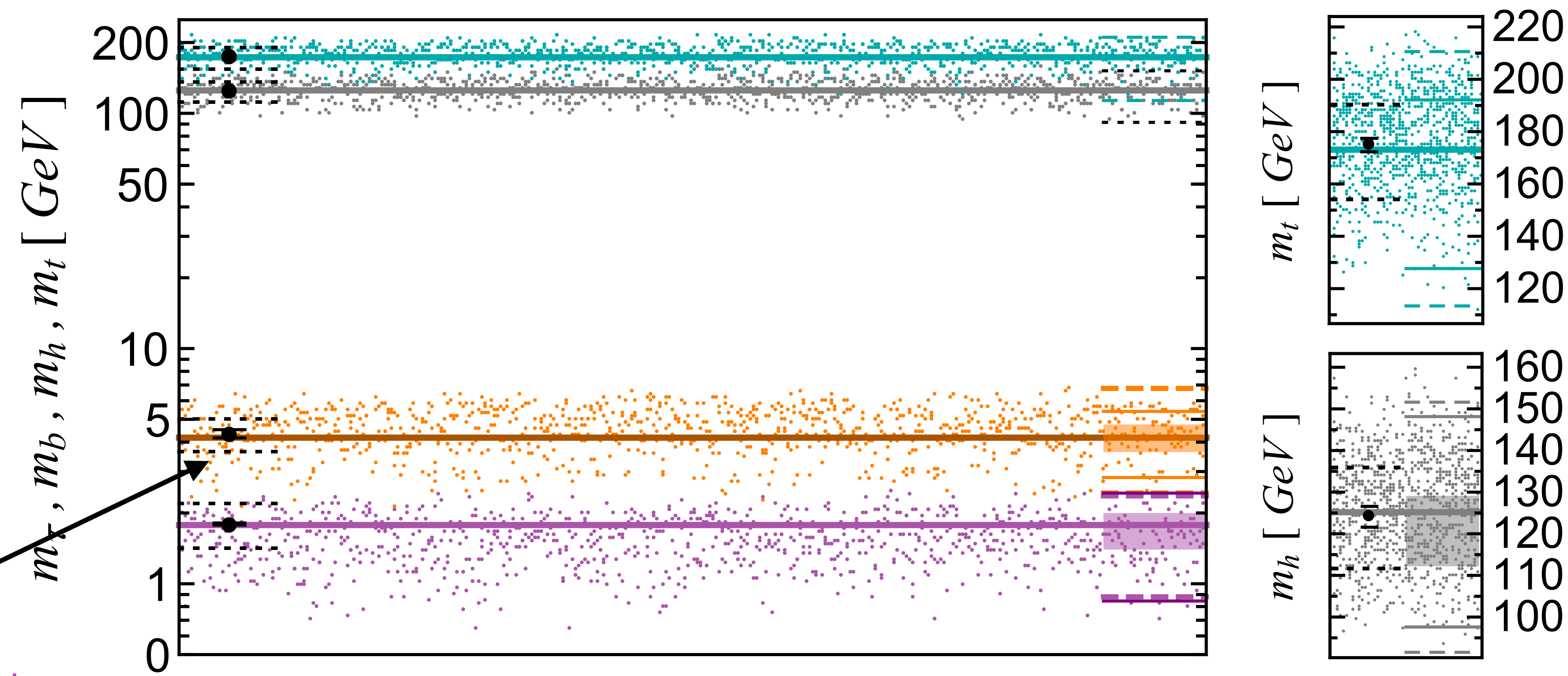
Predicted pattern of fermion masses

In the MSSM+1VF:

$\alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1, 0.3] \quad y_t(M_G), y_b(M_G), y_\tau(M_G), Y_V(M_G) \in [1,3]$

$M_G = 3.5 \times 10^{16}$ GeV, $M = 7$ TeV and $\tan\beta = 40$

– – – universal b.c. ▬ Y_V optimized for m_t



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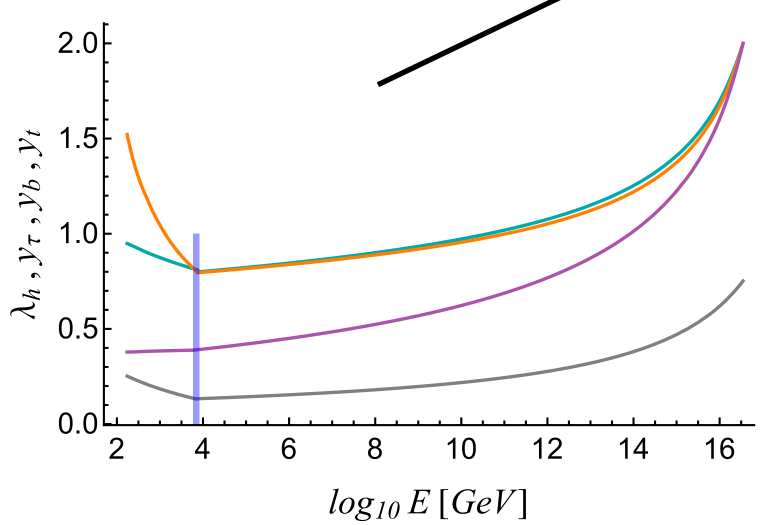

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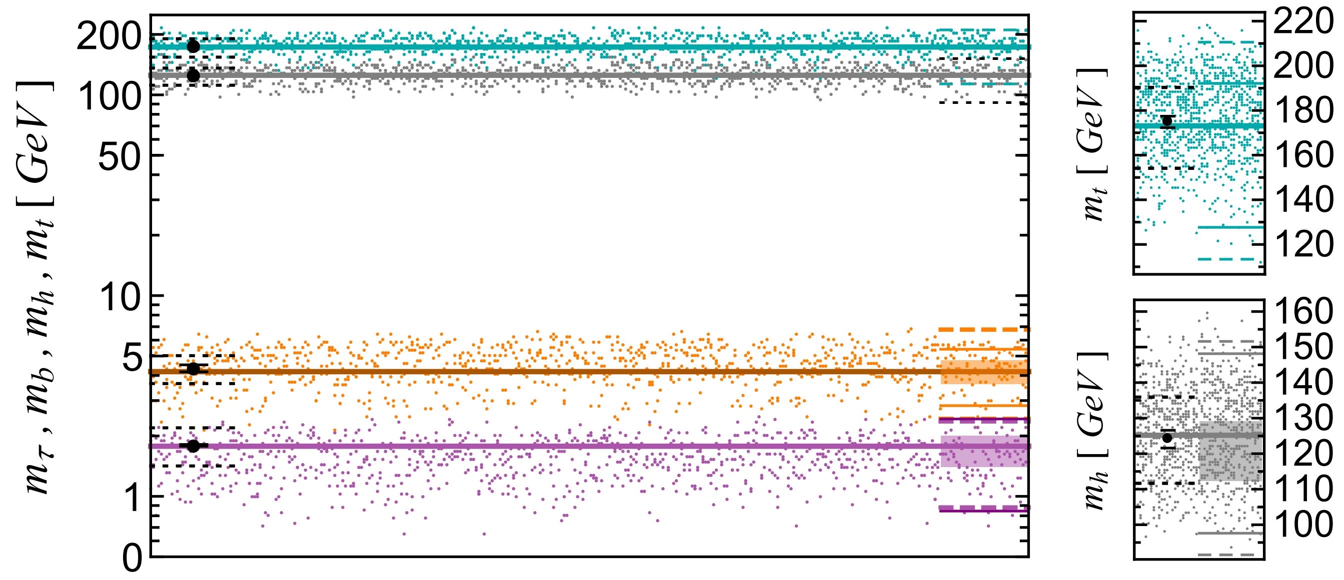

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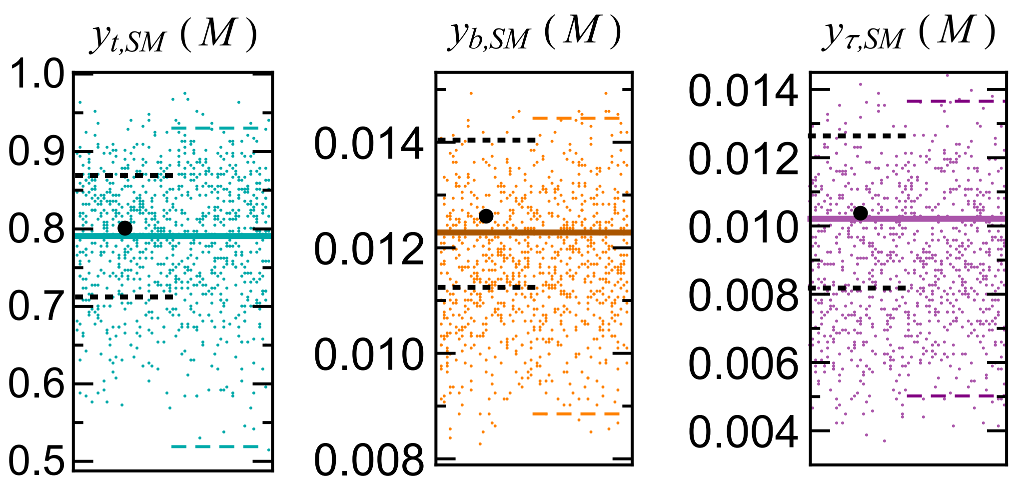

sharper predictions at the matching scale

Higgs quartic coupling

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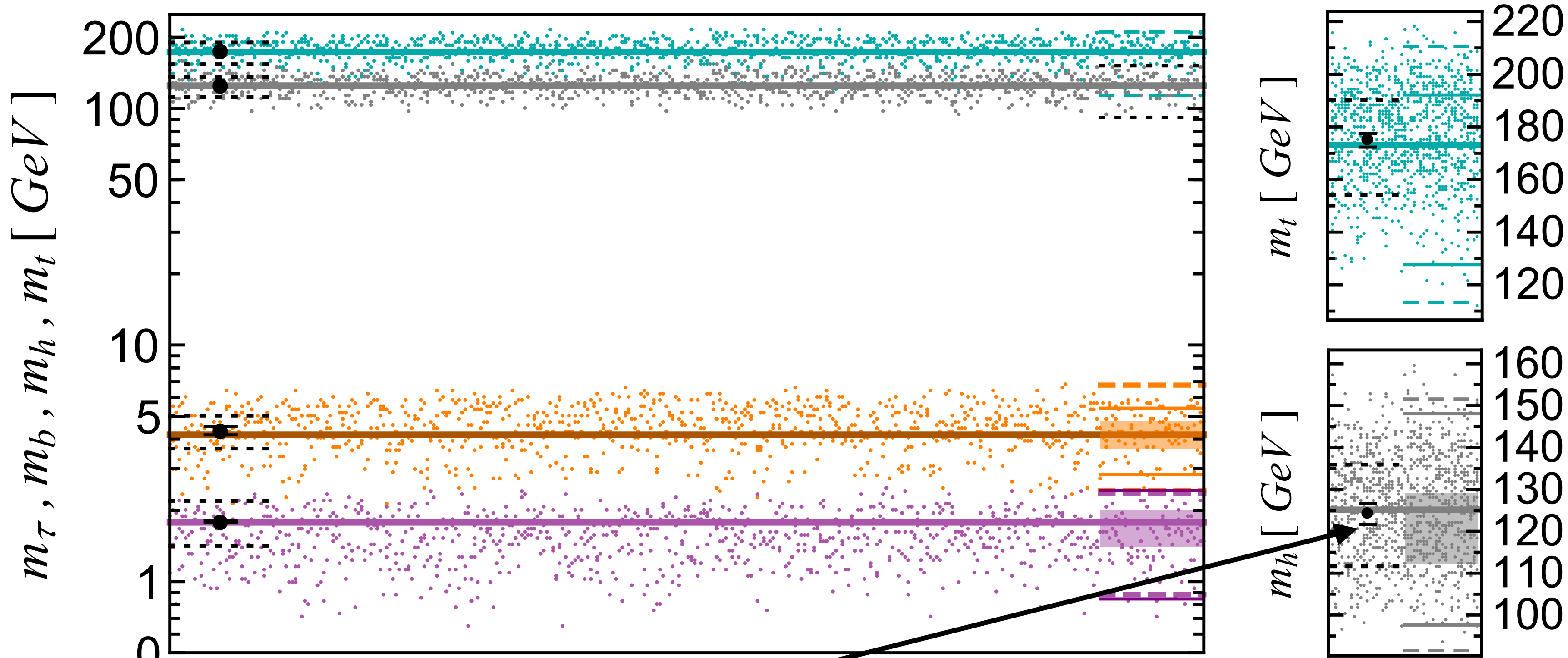

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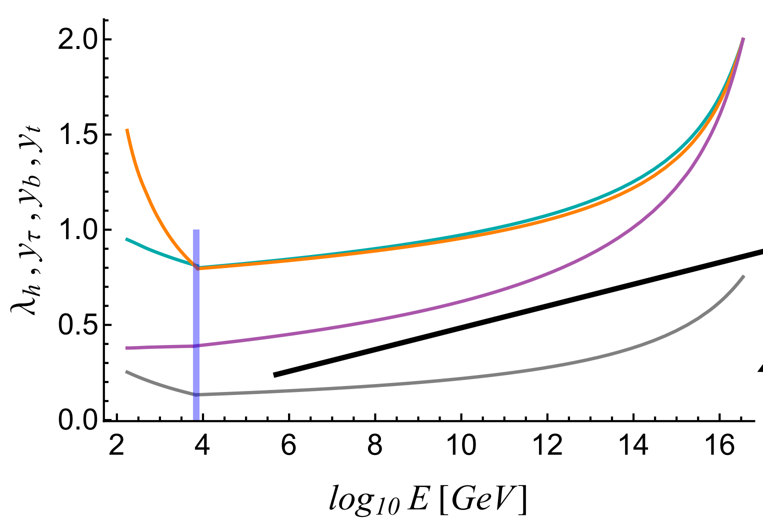
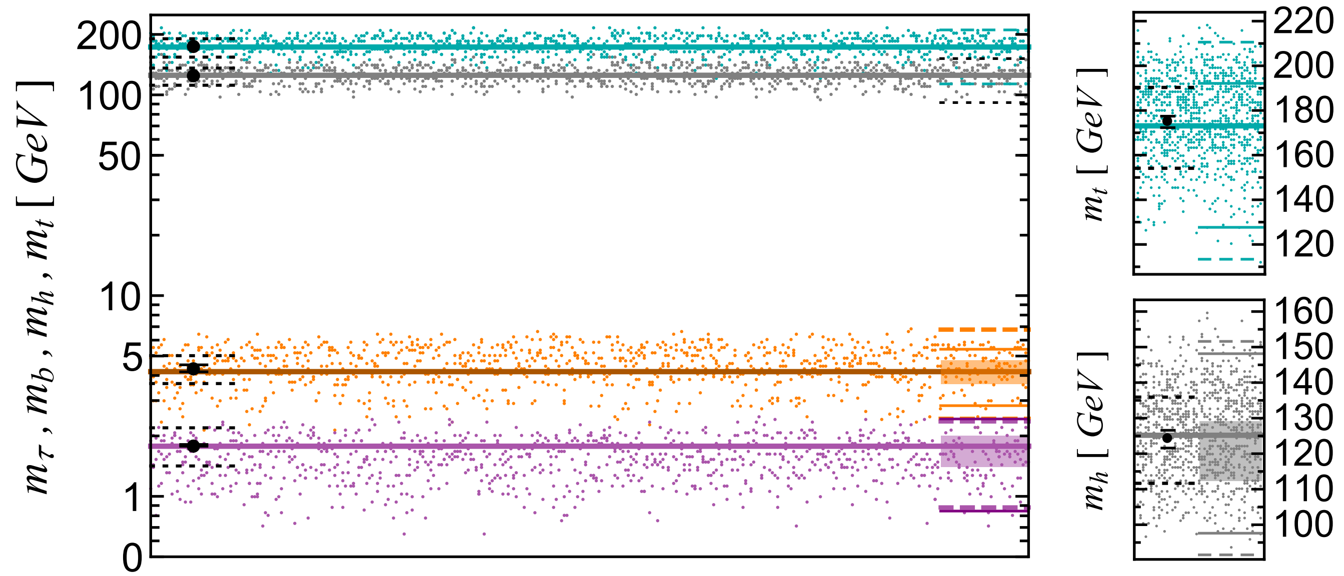


$$\lambda_h(M) \equiv \frac{g_2^2(M) + (3/5) g_1^2(M)}{4} \cos^2 2\beta\,.$$

Higgs quartic coupling



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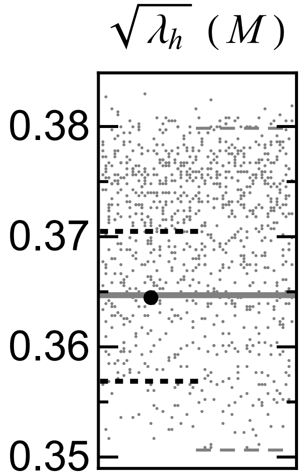
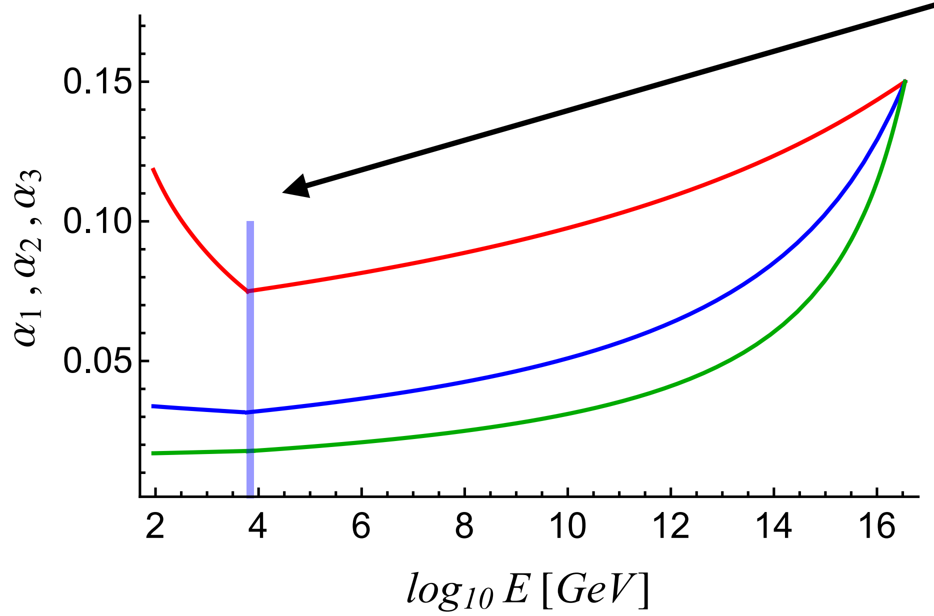
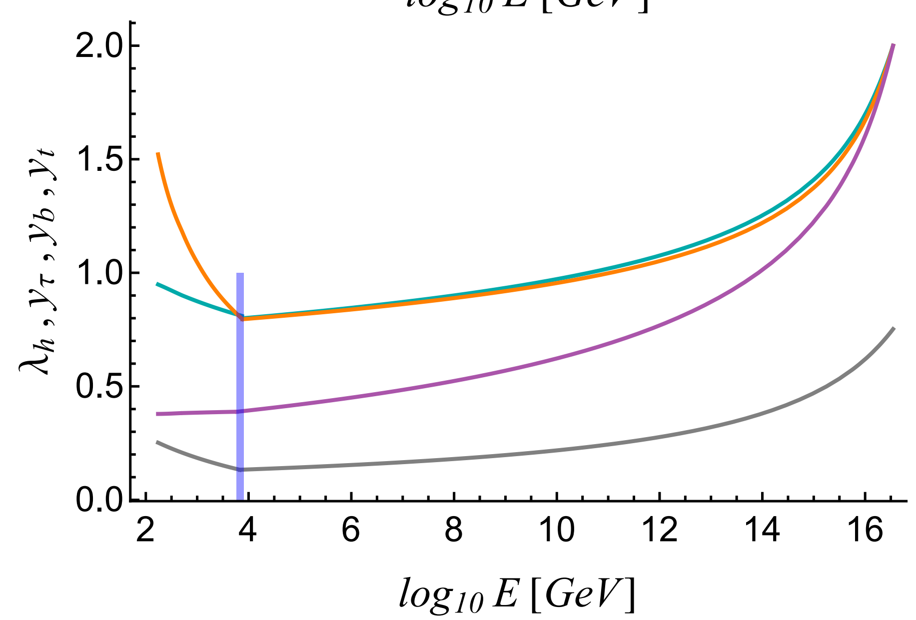


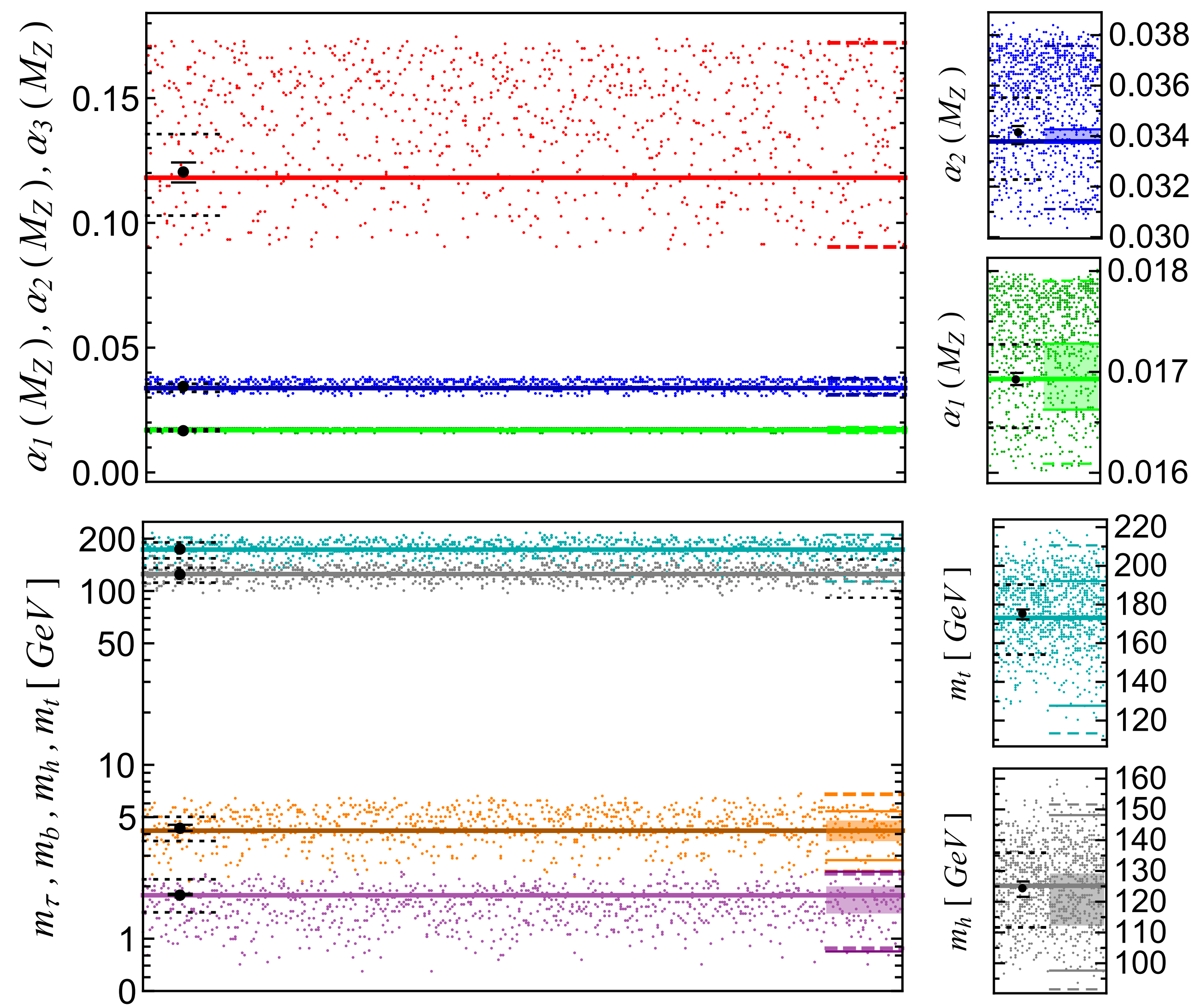
sharp prediction at the matching scale

In the MSSM+1VF

For large range of b.c. there is a narrow range of M within which all the couplings in the MSSM+1VF meet the corresponding parameters in the SM:







Optimizing parameters related to scales

For random unrelated (or unified) parameters:

$$\alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1, 0.3]$$

$$y_t(M_G), y_b(M_G), y_\tau(M_G), Y_V(M_G) \in [1, 3]$$

three parameters,

$$M_G,\ M,\ \tan\beta,$$

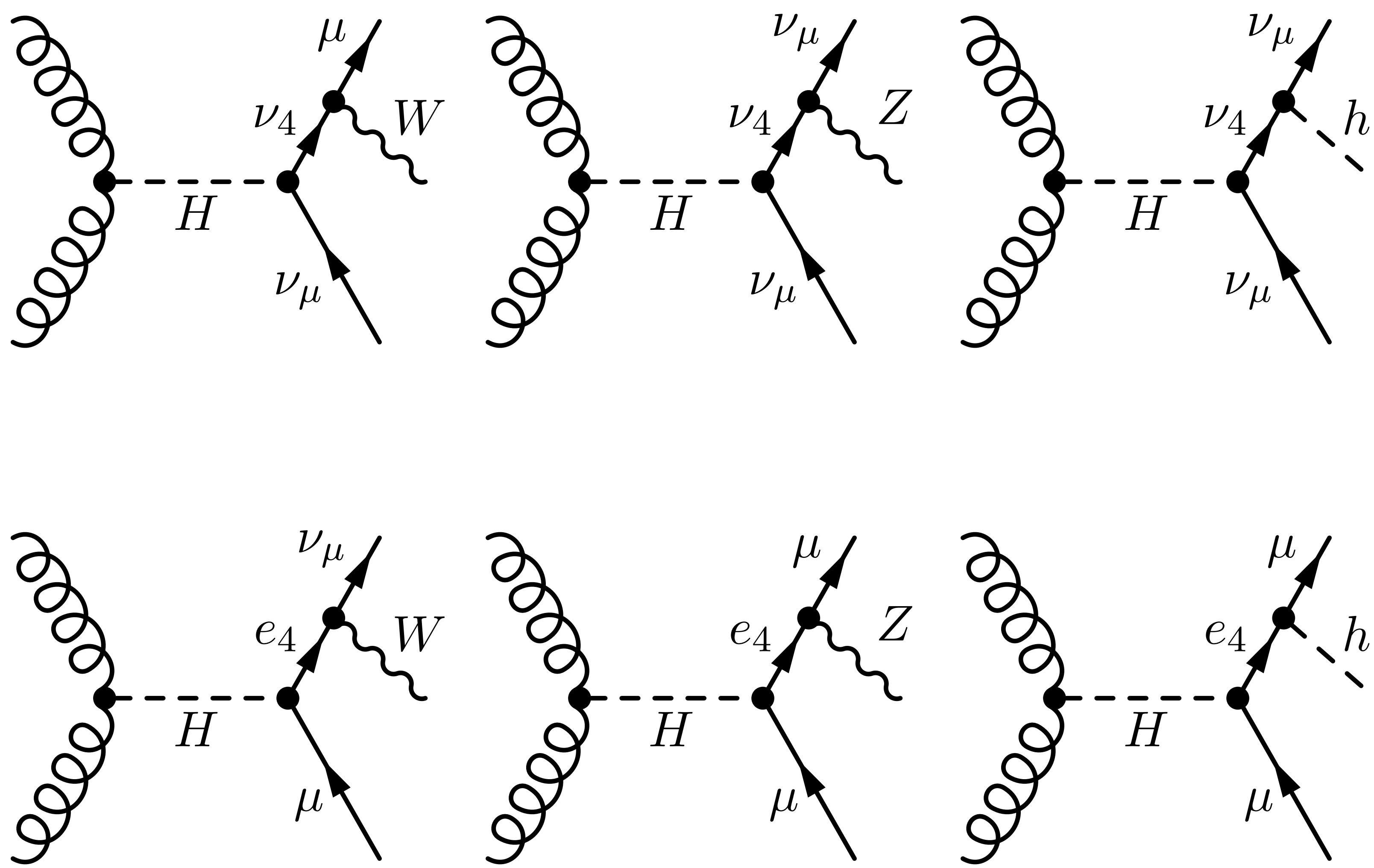
can be optimized so that none of the seven observables is more than 25% (or 15%) from the measured values.

Further optimizing Y_V to obtain the required overall scale of Yukawa couplings, all 7 observables are within 11% (or 7.5%) from their measured values.

Signatures of heavy Higgses and vectorlike quarks and leptons

Vectorlike leptons in Higgs decays

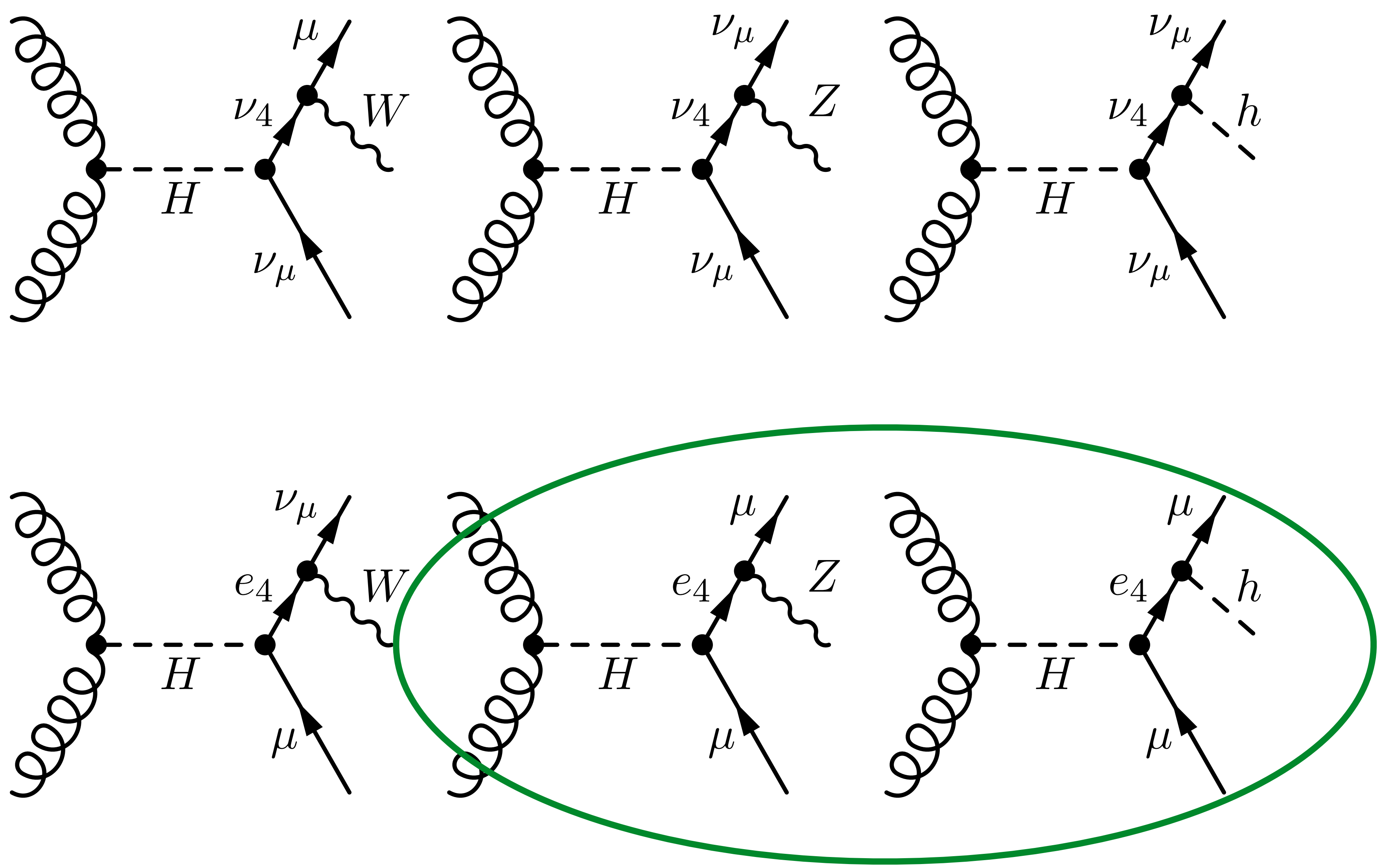
The flavor changing couplings lead to new decay modes of heavy Higgses:



can be repeated with e or tau or quarks

Vectorlike leptons in Higgs decays

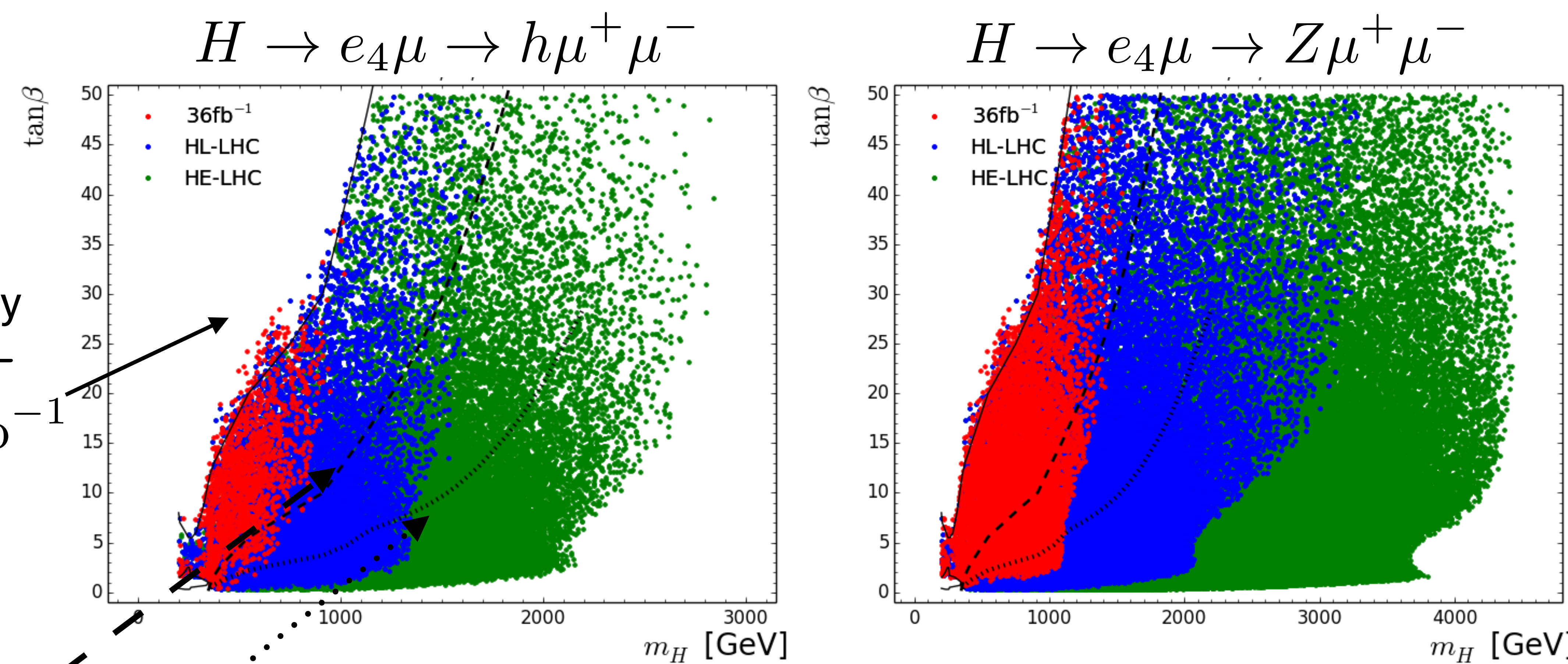
The flavor changing couplings lead to new decay modes of heavy Higgses:



some have negligible SM background!

Sensitivity to $H \to h\mu\mu,\ Z\mu\mu$ at HL/HE-LHC

Scenarios satisfying all the limits that can be seen at 95% C.L.:



excluded by $H \to \tau\tau$ with $36\,\text{fb}^{-1}$

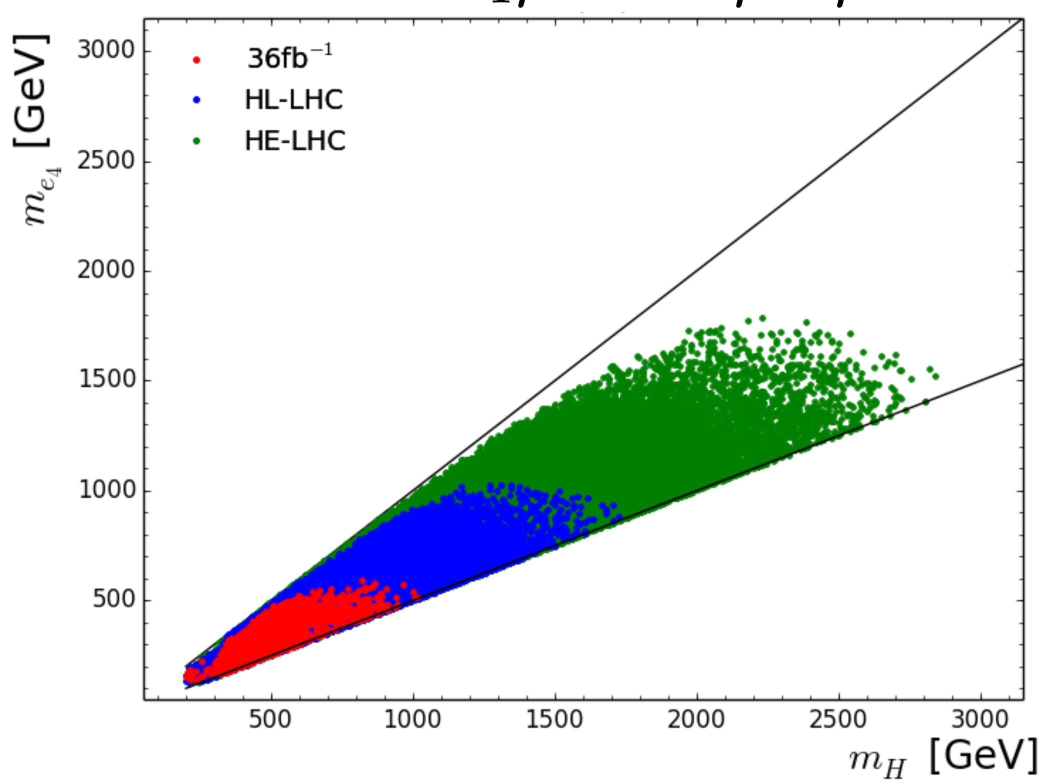
projected $H \to \tau\tau$ at HL-LHC and at HE-LHC

HL(HE)-LHC sensitive to heavy Higgses up to ~3(4.5) TeV

Sensitivity to e_4 in $H \to h\mu\mu,\ Z\mu\mu$

Scenarios satisfying all the limits that can be seen at 95% C.L.:

$H \to e_4\mu \to h\mu^+\mu^-$



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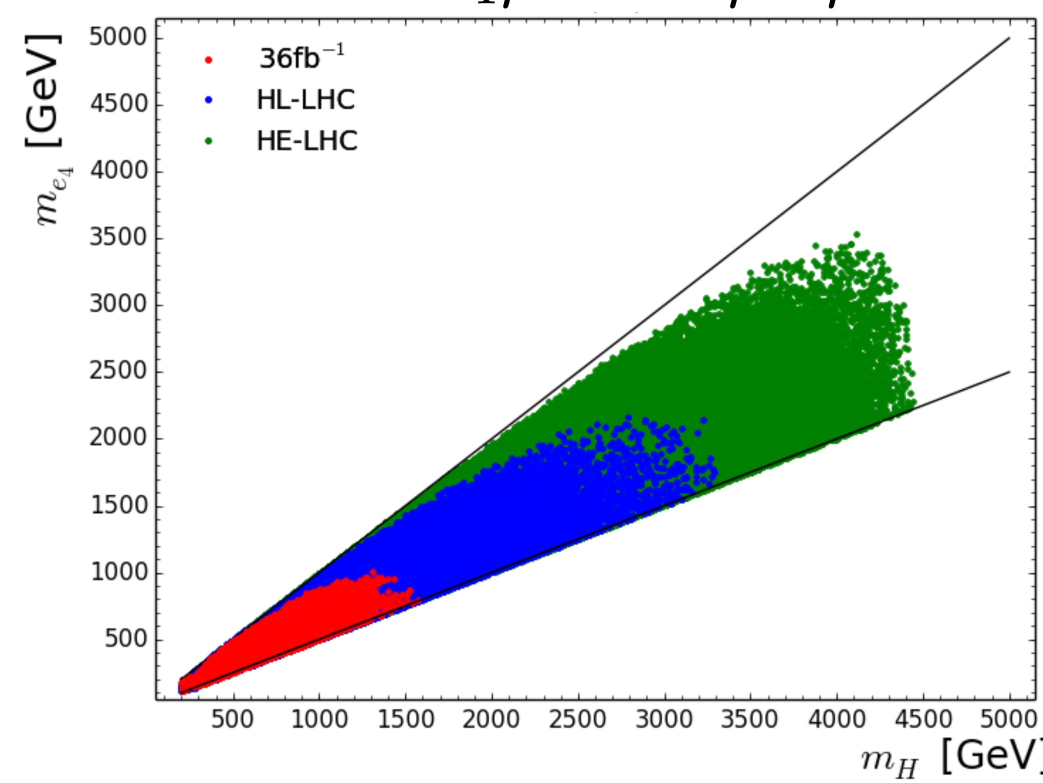
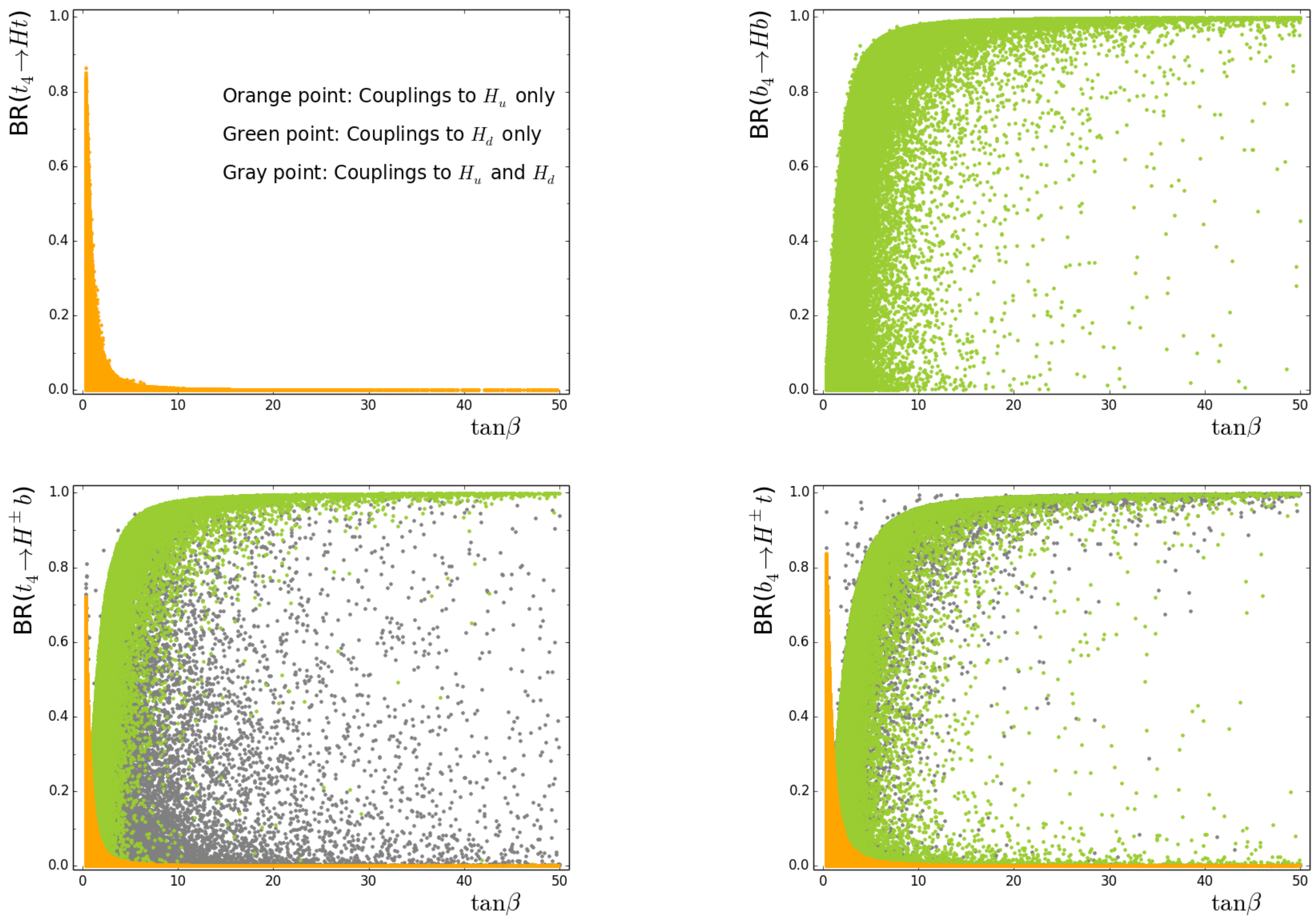


HL(HE)-LHC sensitive to vectorlike leptons up to ~2(3.5) TeV

Heavy Higgses in vectorlike quark decays



close to 100% BRs, unusual final states: 6t, 4t2b, 2t4b, 6b

Conclusions

In the **MSSM+1VF** with vectorlike matter and superpartners at a multi-TeV scale:

$$\alpha_1, \ \alpha_2, \ \alpha_3, \ y_t, \ y_b, \ y_\tau, \ \lambda_h$$

can be understood as a consequence of the particle content of the model!

Conclusions

In the **MSSM+1VF** with vectorlike matter and superpartners at a multi-TeV scale:

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Although the typical scale of new physics is beyond the reach of LHC, part of the spectrum might be within the reach and many clean signatures can be probed up to several TeV.