Can the Cognitive Impact of Calculus Courses be Enhanced? †

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Abstract

I discuss the cognitive impact of introductory calculus courses after the initiation of the NSF's calculus reform program in 1987. Topics discussed are:

(a) What's calculus?
(b) Calculus, *language of nature* and gateway to science, technology, engineering, and mathematics.
(c) A typical calculus-course problem (even dogs can solve it).
(d) NSF's calculus reform effort, initiated in 1987.
(e) Assessments bemoan the lack of evidence of improved student learning.
(f) A glimmer of hope – the *Calculus Concept Inventory* (CCI).
(g) Typical question of the CCI type (dogs score at the random guessing level).
(h) Impact of the CCI on calculus education – early trials.
(i) Conclusion.
(j) Appendix #1: The Lagrange Approach to Calculus.
(k) Appendix #2: Math Education Bibliography.

I conclude that Epstein's CCI *may* stimulate reform in calculus education, but, judging from the physics education reform effort, it may take several decades before widespread improvement occurs - see the review "The Impact of Concept Inventories On Physics Education and Its Relevance For Engineering Education" [Hake (2011c)] at <http://bit.ly/nmPY8F> (8.7 MB).

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† The reference is: Hake, R.R. 2013. "Can the Cognitive Impact of Calculus Courses be Enhanced?" An update of 26 Dec 2013 of an invited talk of 24 April 2012, Department of Mathematics, University of Southern California, online as ref. 70 at <http://bit.ly/a6M5y0>.

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Prologue

"Mathematics is the gate and key of the sciences. . . .Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy."

- Roger Bacon (Opus Majus, bk. 1, ch. 4) <http://bit.ly/dzjbWv>

"To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature ... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in."

- Richard Feynman (1965, 1994) Ch. 2

A. What's Calculus?

1. From *Wikipedia (2013a) at <http://bit.ly/1cbtuDg>** (numbered references and some *covert* links have been eliminated):

"Calculus is the mathematical study of change, in the same way that geometry is the study of shape and algebra is the study of operations and their application to solving equations. It has two major branches, differential calculus <http://bit.ly/JC9KfK> (concerning rates of change and slopes of curves), and integral calculus <http://bit.ly/J5sG60> (concerning accumulation of quantities and the areas under curves); these two branches are related to each other by the fundamental theorem of calculus <http://bit.ly/1c13A3u>. Both branches make use of the fundamental notions of convergence of infinite sequences <http://bit.ly/1bdAI3s> and infinite series <http://bit.ly/1eoytlK> to a well-defined limit <http://bit.ly/1cR8EaQ>. Generally considered to have been founded in the 17th century by Isaac Newton <http://bit.ly/1hlBlOe> and Gottfried Leibniz <http://bit.ly/1kVEDHI>, today calculus has widespread uses in science, engineering and economics and can solve many problems that algebra alone cannot.

Calculus is a major part of modern mathematics education <http://bit.ly/19cYe6J>. A course in calculus is a gateway to other, more advanced courses in mathematics devoted to the study of functions and limits, broadly called mathematical analysis. Calculus has historically been called 'the calculus of infinitesimals', or 'infinitesimal calculus'. The word 'calculus' comes from Latin (*calculus*) and refers to a small stone used for counting. More generally, *calculus* (plural *calculi*) refers to any method or system of calculation guided by the symbolic manipulation of expressions. Some examples of other well-known calculi are propositional calculus, calculus of variations, lambda calculus, and process calculus."

2. From *Encyclopedia of Mathematics* [West et al. (1982)]

"Calculus is a branch of higher mathematics that deals with variable, or changing, quantities. . . . . based on the concept of infinitesimals (exceedingly small quantities) and on the concept of limits (quantities that can be approached more and more closely but never reached)."

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* Those who dismiss *Wikipedia* entries as a mere "opinion pieces," may not be aware that a study by *Nature* [Giles (2005)] indicated that *Wikipedia* comes close to *Britannica* in terms of the accuracy of its science entries – see e.g. "In Defense of Wikipedia" [Hake (2009c)].

3. "Calculus" Means Different Things to Different People

David Tall (1993) in "Students' Difficulties in Calculus" wrote [bracketed by lines "TTTTT. . . .":

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It should be emphasized that the Calculus means a variety of different things in different countries in a spectrum from:

a. *informal calculus* – informal ideas of rate of change and the rules of differentiation with integration as the inverse process, with calculating areas, volumes etc. as applications of integration, to

b. *formal analysis* – formal ideas of *completeness*, ε-δ definitions of limits, continuity, differentiation, Riemann integration, and formal deductions of theorems such as mean-value theorem, the fundamental theorem of calculus, etc., with a variety of more recent approaches including:

(1) infinitesimal ideas based on non-standard analysis,

(2) computer approaches using one or more of the graphical, numerical, symbolic manipulation facilities with, or without, programming. . . . . . .

In some countries the first of these is taught in secondary school and the second to mathematics majors in college. In others a subject somewhere along the spectrum between the two is taught as the first major college mathematics course. In a few countries (e.g. Greece), the formal ideas are taught from the beginning in secondary school.

The details of these approaches, the ***level of rigour*** . . . . [[my ***italics***, see section "**A3**" below]]. . . , the representations (geometric, numeric, symbolic, using functions or independent and dependent variables). . . . [[the reform calculus texts by Hughes-Hallet et al. (2008, 2009) 'use all strands of the 'Rule of Four' - graphical, numeric, symbolic/algebraic, and verbal/applied presentations - to make concepts easier to understand']]. . . . . , the individual topics covered, vary greatly from course to course. . . . .

The calculus represents the first time in which the student is confronted with the *limit* concept, involving calculations that are no longer performed by simple arithmetic and algebra, and infinite processes that can only be carried out by indirect arguments. Teachers often attempt to circumvent the problems by using an "informal" approach playing down the technicalities. . . . [[see e.g., *Calculus: An Intuitive and Physical Approach* (Kline(1967, 1998)]]. . . . However, whatever method is used, *a general dissatisfaction with the calculus course has emerged in various countries round the world in the last decade*. . . . . [[My *italics*.]]. . . . . .

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4. Level of Rigor As Judged By Definitions of Continuity in Some Standard Sources[†]

a. *Wikipedia** (2013b) <http://bit.ly/LoAz5S> - paraphrased [bracketed by lines WWWW. . . ."]:

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A function from the set of real numbers can be represented by a graph in the Cartesian plane; the function is continuous if, roughly speaking, the graph is a single unbroken curve with no "holes" or "jumps." There are several ways to make this intuition mathematically rigorous. These definitions are equivalent to one another, so the most convenient definition can be used to determine whether a given function is continuous or not.

a. Consider $f\colon\ \mathrm{I} \to \mathbf{R}$,

A function defined on a subset I of the set **R** of real numbers. This subset I is referred to as the *domain* of *f*. Possible choices include

(1) $\mathrm{I} = \mathbf{R}$, the whole set of real numbers;

(2) an open interval $\mathrm{I} = (a,b) = \{x \in \mathbf{R} \mid a < x < b\}$; or

(3) a closed interval $\mathrm{I} = [a,b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$;

where a and b are real numbers.

b. In terms of limits of functions: $\lim_{x \to c} f(x) = f(c)$.

In detail this means three conditions: first, *f* has to be defined at *c*. Second, the limit on the left hand side of that equation has to exist. Third, the value of this limit must equal *f*(*c*). The function *f* is said to be *continuous* if it is continuous at every point of its domain. If the point *c* in the domain of *f* is not a limit point of the domain, then this condition is vacuously true, since *x* cannot approach *c* through values not equal *c*. Thus, for example, every function whose domain is the set of all integers is continuous.

c. The Weierstrass definition[§] (epsilon-delta) of continuous functions:

Given a function $f\colon\ \mathrm{I} \to \mathbf{R}$ defined on a subset I of the real numbers, and an element *c* of the domain I, *f* is said to be continuous at the point *c* if the following holds: For any number $\varepsilon > 0$, however small, there exists some number $\delta > 0$ such that for all *x* in the domain of *f* with $c - \delta < x < c + \delta$, the value of $f(x)$ satisfies

$$f(c) - \varepsilon < f(x) < f(c) + \varepsilon$$

Alternatively written, continuity of $f : \mathrm{I} \to D$ at $c \in \mathrm{I}$ means that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that for all $x \in \mathrm{I}$:

$$|x - c| < \delta \to |f(x) - f(c)| < \varepsilon$$

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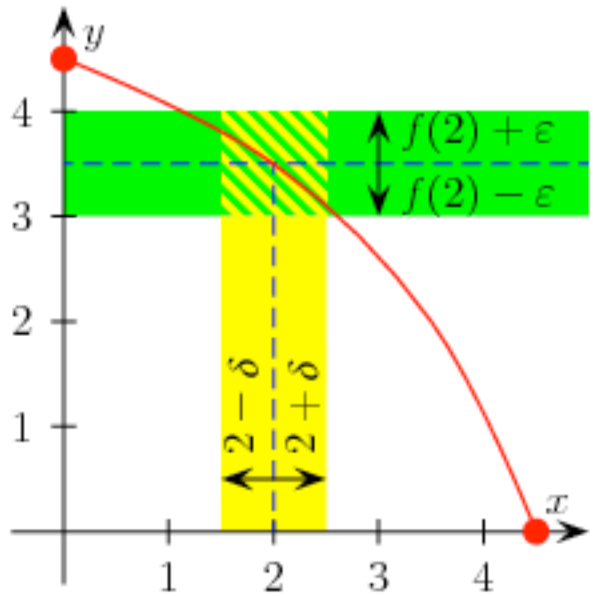
† Definitions of continuity not considered here are, among others, those in the following well regarded texts: Apostol (1967), Brown et al. (1991), Callahan et al. (1995), Hilbert et al. (1994), McCallum et al. (2002, 2008), Meridith et al. (2012)

* Those who dismiss Wikipedia entries as a mere "opinion pieces," may not be aware that a study by *Nature* [Giles (2005)] indicated that Wikipedia comes close to *Britannica* in terms of the accuracy of its science entries – see e.g. "In Defense of Wikipedia" [Hake (2009c)].

§ For a thorough treatment of Weierstrass' approach see e.g., Courant & John (1965, 1998).

More intuitively, we can say that if we want to get all the $f(x)$ values to stay in some small neighborhood around $f(c)$, we simply need to choose a small enough neighborhood for the x values around c, and we can do that no matter how small the $f(x)$ neighborhood is; f is then continuous at c.

Pictorially:



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b. ***Unified Calculus*** [Smith, Salkover, & Justice (1947)] [a traditional text used by Paul Halmos <http://en.wikipedia.org/wiki/Paul_Halmos> as a student and later admired by him [Halmos (1988)], page 4:

"A single-valued function $f(x)$ is said to be continuous for $x = a$ if $f(x)$ is defined for $x = a$, and for all values of x in a range extending on both sides of $x = a$, and if $\lim_{x \to a} f(x) = f(a)$."

c. ***Calculus: Single and Multivariable*** [Hughes-Hallet et al. (2008)] (a product of NSF's 1987 calculus reform program):

(1) page 47: "Roughly speaking, a function is said to be continuous on an interval if its graph has no breaks, jumps, or holes in that interval. . . . . A continuous function has a graph which can be drawn without lifting the pencil from the paper."

(2) page 56: "a function f is continuous at $x = c$ if $f(x)$ is defined at $x = c$ and if $\lim_{x \to c} f(x) = f(c)$. In other words, $f(x)$ is as close as we want to $f(c)$ provided x is close enough to c. The function is continuous *on an interval* $[a,b]$ if it is continuous at every point in the interval."

d. ***Basic Calculus: From Archimedes to Newton to its Role in Science*** [Hahn (1998)] (a historically oriented text for non-science/math majors), page 212:

"The algebraic conditions for a function $y = f(x)$ to be continuous at a number (or point) c on the x axis are: (i) $f(c)$ makes sense. In other words c is in the domain of f and
(ii) $\lim_{x \to c} f(x) = f(c)$. "

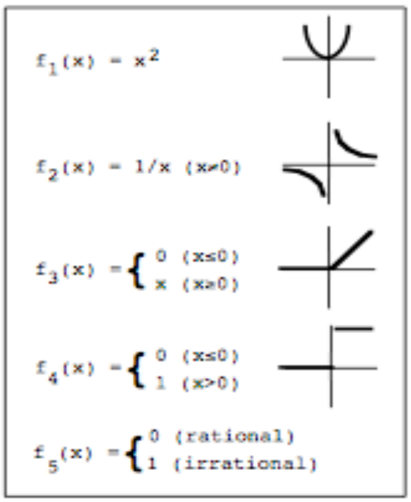
e. ***Calculus: An Intuitive and Physical Approach*** [Kline, 1967, 1989)] (an early reform text), page 116-117 of the 1967, 1989 edition):

"One of these properties [that $\Delta y/\Delta x$ approaches a limit] is that Δy must approach 0 as Δx does. This is known as the property of continuity. . . . .We can best understand what continuity means if we consider the graphs of functions. In graphical terms a function is continuous if the curve can be drawn with one uninterrupted motion of a pencil."

And on pages 117-118, Kline discuses the continuity of functions such as f_2 (see below) which are continuous *everywhere* ***except*** at one point (for f_2 at $x = 0$).

4. Students' Understanding of *Continuity*

David Tall (1990) in "Inconsistencies in the Learning of Calculus and Analysis" discusses student responses to this question:



	C	DC	NR
f_1	41	0	0
f_2	6	35	0
f_3	27	12	2
f_4	1	38	2
f_5	8	26	7

Which of the above functions are continuous?

Note that 35 out of 41 students (85%) thought that $f_2(x) = 1/x$ $(x \neq 0)$ was *discontinuous* (wrong); 12 out 41 (29%) thought that $f_3(x)$ was *discontinuous* (wrong); and 8 out of 41 students (20%) thought f_5 was continuous (wrong) :-(. Regarding f_2, Tall (1990, p. 6) wrote (correcting an apparent typo): "The function f_2 often causes dispute even amongst seasoned mathematicians. It is continuous according to the ε–δ definition *on the domain* $\{x \in I = \mathbf{R} \mid x \neq 0\}$."

But students blindly following Kline's, "a function is continuous if the curve can be drawn with one uninterrupted motion of a pencil" and not noticing the exclusion of f_2 at $x = 0$ might indicate that f_2 is *discontinuous*.

5. Calculus and Newtonian Mechanics

In "Helping Students to Think Like Scientists in Socratic Dialogue Inducing Labs" [Hake (2012b)] I wrote [bracketed by lines "HHHHH. . . . ."]:

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(a) We agree with *The Mechanical Universe* [MU (2012), Goodstein & Olenick (1998), Frautschi et al. (2008)] standpoint that it is almost impossible to understand terms such as "velocity" and "acceleration" without some knowledge of the basic ideas of differential calculus. Thus, in our view, the appellation "non-calculus physics text" is a contradiction in terms. Authors of effective "non-calculus" physics texts must negate their own "non-calculus" claims: most of them give an expression for instantaneous velocity in one dimension: $v = \lim_{\Delta t \to 0} (\Delta x/\Delta t) = (dx/dt))$ but omit the right-hand side of this equation (the identification of the derivative "dx/dt"), possibly because they fear it might frighten students and/or jeopardize their book's position as a "non-calculus' text."

(b) Although about 70% of students entering the non-calculus-based Indiana University (IU) introductory physics course have completed a university calculus course, *almost none seems to have the foggiest notion of the graphical meaning of a derivative or integral*, as addressed in this section. Similar calculus illiteracy is commonly found among students in calculus-based introductory physics courses at IU. In my judgment, these calculus interpretations are essential to the crucial operational definitions of instantaneous position, velocity, and acceleration: the term "substantive non-calculus-based mechanics course" is an oxymoron.

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6. Calculus and "Physics First"

In "Re: Do not pass this by: Seventeen very well-spent minutes with Conrad W" [Hake (2010a)], I wrote:

"Because of its computational complexity, calculus has traditionally been taught very late; but by using computers, calculus concepts are amenable to a much younger age group. In my opinion, programs such as Wolfram's 'Computer-based Math' <http://bit.ly/h7V2jX> and the Kaput Center's (2013) 'Simulations of various time-based models (e.g., Position/Velocity, Finance)' <http://bit.ly/YDxSTw>, if used in K-8, can pave the way for the education of ninth graders in the basic ideas of Newtonian mechanics - thus facilitating Leon Lederman's (2001) 'Physics First'. "

Regarding the Kaput Center's work see "The SimCalc Vision and Contributions: Democratizing Access to Important Mathematics" [Hegedus & Roschelle (2013)], "Democratizing access to Calculus: New routes using old routes" [Kaput (1994), and "The Evolution of Technology and the Mathematics of Change and Variation" [Tall (2013)]

B. Calculus – *Language of Nature** and Gateway to Science, Technology, Engineering, & Mathematics

1. High School

a. MAA/NCTM Recommends De-emphasis of Calculus

MAA/NCTM (2012) Joint Statement on Calculus [my *italics*]: "Although calculus can play an important role in secondary school, the ultimate goal of the K–12 mathematics curriculum *should not be to get students into and through a course in calculus by twelfth grade* but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college. The college curriculum should offer students an experience that is new and engaging, broadening their understanding of the world of mathematics while strengthening their mastery of tools that they will need if they choose to pursue a mathematically intensive discipline." For an assessment of high-school calculus see "Meeting The Challenge Of High School Calculus. . . . ." [Bressoud (2010a,b,c,d,e,f,g)]

2. College and University

a. Calculus Required for STEM Majors

Despite the MAA/NCTM de-emphasis of high-school calculus, as far as I'm aware (please correct me if I'm wrong) a college-level course in calculus (or equivalent) is required for nearly all students who major in STEM disciplines, as well it should be considering that *Calculus is the Language of Nature*.* Therefore the cognitive impact of university calculus courses should be a national concern.

b. PCAST Report - Suggests Undergraduate Math Course Not Be Taught by Mathematicians

Recommendation #3 of the PCAST (2012) report (page vi) is "Launch a national experiment in postsecondary mathematics education to address the mathematics-preparation gap." Among actions recommended (page vii) are "Support a national experiment in mathematics undergraduate education at NSF, the Department of Labor, and the Department of Education" [including, my *italics*]. . . . . college mathematics teaching and curricula developed and taught *by faculty from mathematics-intensive disciplines* ***other than mathematics***, including physics, engineering, and computer science."

(1) Note that there are *no* mathematicians among the "President's Council of Advisors on Science and Technology" as listed on the initial pages of PCAST (2012). According to a Notice <http://1.usa.gov/MtviIF> of 3 May 2012 on the Federal Register:

"The President's Council of Advisors on Science and Technology (PCAST) is an advisory group of the nation's leading scientists and engineers, appointed by the President to augment the science and technology advice available to him from inside the White House and from cabinet departments and other Federal agencies. See the Executive Order at <http://www.whitehouse.gov/ostp/pcast>. PCAST is consulted about and provides analyses and recommendations concerning a wide range of issues where understandings from the domains of science, technology, and innovation may bear on the policy choices before the President. PCAST is co-chaired by Dr. John P. Holdren, Assistant to the President for Science and Technology, and Director, Office of Science and Technology Policy, Executive Office of the President, The White House; and Dr. Eric S. Lander, President, Broad Institute of the Massachusetts Institute of Technology and Harvard."

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* A sage designation borrowed from the Chapter 3 title of *The Mechanical Universe: Mechanics and Heat* [Frautschi et al. (2008)].

(2) Mathematician David Bressoud (2012), in his MAA Launchings column "On Engaging to Excel" summed up the significance of the PCAST report for the math community as follows:

> "But the nature of [the PCAST] recommendations combined with the other previously mentioned statements from this report suggest that PCAST does not trust the mathematics community to get right undergraduate mathematics education either in support of other STEM fields or in the preparation of K-12 mathematics teachers. In this report, there is a clear sense of frustration that despite its central role in STEM education, the *mathematics community appears to have been slow to rethink its undergraduate curricula or pedagogy* on a truly national scale." [My *italics*.]

c. Persistence in Math Studies

(1) David Bressoud (2013b) in his MAA Launchings entry "MAA Calculus Study: Persistence through Calculus" wrote [my *italics*]:

> "A successful Calculus program must do more than simply ensure that students who pass are ready for the next course. It also needs to support as many students as possible to attain this readiness. And it must encourage those students to continue on with their mathematics. As I wrote in my January 2010 column "The Problem of Persistence" Bressoud (2010h) just because a student needs further mathematics for the intended career and has done well in the last mathematics course is no guarantee that he or she will decide to continue the study of mathematics. *This loss between courses is a significant contributor to the disappearance from STEM fields of at least half of the students who enter college with the intention of pursuing a degree in science, technology, engineering, or mathematics*. Chris Rasmussen and Jess Ellis, drawing on data from MAA's Calculus Study, have now shed further light on this problem. This column draws on some of the results they have gleaned from our data. . . . .
>
> I am concerned by these good students who find calculus simply too hard. As I documented in my column from May 2011, 'The Calculus I Student' [Bressoud (2011a)], these students experienced success in high school, and an overwhelming majority had studied calculus in high school. They entered college with high levels of confidence and strong motivation. Their experience of Calculus I in college has had a profound effect on both confidence and motivation.
>
> *The solution should not be to make college calculus easier*. However, we do need to find ways of mitigating the shock that hits so many students when they transition from high school to college. We need to do a better job of preparing students for the demands of college, working on both sides of the transition to equip them with the skills they need to make effective use of their time and effort.
>
> Twenty years ago, I surveyed Calculus I students at Penn State and learned that most had no idea what it means to study mathematics. Their efforts seldom extended beyond trying to match the problems at the back of the section to the templates in the book or the examples that had been explained that day. *The result was that studying mathematics had been reduced to the memorization of a large body of specific and seemingly unrelated techniques for solving a vast assortment of problems. No wonder students found it so difficult.* ***I fear that this has not changed.***

(2) Benjamin Braun (2014) in “Persistent Learning, Critical Teaching: Intelligence Beliefs and Active Learning in Mathematics Courses” wrote [my *italics*]:

“One way to create a classroom environment that cultivates malleable intelligence beliefs, *supporting students through sequences of challenges and critical responses*, is the use of active learning techniques. These include many well-known methods: e.g., cooperative learning, peer-based instruction, guided discovery, and inquiry-based learning. While active learning techniques are not all identically effective and while they require persistence by teachers to be successfully applied, a growing body of evidence suggests that such methods generally have a positive effect on student learning and attitudes in mathematics [Laursen et al. (2011), engineering [Prince (2004)], and other STEM disciplines [Singer et al. (2012), Chapter 6]. Active learning has also been studied extensively at the K–12 level; hence these methods deserve attention from mathematicians teaching courses aimed at future K–12 teachers.

d. Why Have Mathematicians Lagged in Undergraduate Pedagogy

Herewith follows a **Galilean Dialogue** [updated and revised from “Re: Math Education Research” (Hake, 2003)]. According to the Wikipedia entry <http://bit.ly/1azo97y> on Gaileo's “Dialogue Concerning the Two Chief World Systems”: "Salviati argues for the Copernican position and presents some of Galileo's views directly. . . . .**Sagredo** is an intelligent layman who is initially neutral." In the version below Salviati’s role, is taken by **Hake** who argues for *interactive engagement* methods† of education and **Sagredo** is still an intelligent layman who is initially neutral on educational methods.

Sagredo: In what important respects is MER different from PER?

Hake: It appears to me that Mathematics Education Research (MER) of quality and quantity comparable to that in Physics Education Research PER - [see e.g. McDermott & Redish (1999), Redish (2003), Heron & Meltzer (2005), Meltzer & Thornton (2012)] exists but overall:

(a) MER groups are more apt to be found in graduate and undergraduate *Schools Of Education*, while PER groups are found primarily in *Physics Departments* [see the listing at <http://www.compadre.org/per/programs/>.]. The location of PER groups in physics departments gives them a distinct advantage for research on undergraduate education because student subjects take courses in physics departments (Redish, 1999), and physicists tend to be more knowledgeable in physics than are the faculty of Ed Schools.

(b) MER has yet to:

(1) with the exception of the *Calculus Concept Inventory* [Epstein (2007; 2012, 2013)] devise standardized tests of important concepts in undergraduate math courses (such as the *Force Concept Inventory* for introductory mechanics) that would be useful in rigorous pre/post testing of thousands of students so as to access the need for and the effectiveness of reform math pedagogy [see “Lessons from the Physics Education Reform Effort” Hake (2002a - Lesson #3), Stockstad (2000)], Wood & Gentile (2003), Michael (2006)].

(2) awaken from near total ignorance of the ground-breaking work of Louis Paul Benezet (1935/36) - see the Benezet Centre at <http://bit.ly/926tiM>.

†“Interactive Engagement methods” are defined by Hake (1998a) as “those designed at least in part to promote conceptual understanding through *active engagement* of students in heads-on (always) and hands-on (usually) activities which yield *immediate feedback* through discussion with peers and/or instructors.”

Sagredo: As I recall, in a discussion-list post titled "Re: Math Education Research" [Hake (2003)] there was a Galilean Dialogue in which Hake opined that another important difference between MER and PER is that MER had focused more on *K-12* education than on *higher* education.

Hake: After surveying the MER literature more thoroughly – see e.g.. the present REFERENCE list below and the references in APPENDIX #2 "Math Education Bibliography" - **I'VE *CHANGED MY MIND*!** There's been a tremendous amount of math education research on higher education of which I had been unaware. Nevertheless I think it should be realized that (a) colleges and universities supply the K-12 teachers, (b) teachers tend to teach math and science in the way they were taught - presently in the ineffective passive-student lecture mode - even despite all the Ed School methods courses pre-service teachers may take, (c) a crucial problem in K-12 education is the severe dearth of ***effective*** science/math teachers [PhysTec (2012), PMET (2012), AAAS (2012), Hake (2002b, 2011a), Meltzer, Plisch, & Vokos (2013a,b). MER seems to be late in realizing this problem [Jackson (2003), Lewis (2001), Cohen & Krantz (2001), Katz & Tucker (2003)].

Sagredo: OK, after scanning your reference lists, I agree that there's been a tremendous amount of math education research on higher education. So why then does the PCAST Report suggest Undergraduate Math Course be taught by non-Mathematicians? What's so difficult about reforming undergraduate math education? There the professors generally have subject expertise (unlike many teachers in K-12). So all that's needed is to inform the professors of pedagogical methods more effective than the ones they're using.

Hake: My 25-year stint at a large research university (typical of the locations where most future teachers are educated) suggests that research mathematicians are even less concerned with undergraduate education than research physicists, and are even more convinced than physicists that the lecture method is the *only* effective method. (It's certainly the easiest, and after all, it worked for them.) In my response "Whence Do We Get the Teachers?" (Hake 2002b) to the MAA's Bernie Madison (2002) at a PKAL Assessment Roundtable of 2002, I opined that Sherman Stein (1997) <http://bit.ly/JCQbDT> hit the nail on the head [my *italics*]:

> "*The first stage in the reform movement should have been to improve the mathematical knowledge of present and prospective elementary teachers*. Unfortunately, the cart of curriculum reform has been put before the horse of well-prepared teachers. In fact, not a single article on the subject of the mathematical preparation of teachers has appeared in *The Mathematics Teacher* since the second Standards volume was published. . . . [[but to be fair one should survey articles in other journals such as the *Journal of Mathematics Teacher Preparation*, and *The Journal for Research in Mathematics Education*]]. . . . Because the AMS and MAA presumably agree with those twelve most crucial pages . . pages 132-143 of "Professional Standards for Teaching Mathematics" (PSSM (1991), ". . . *these organizations should persuade mathematics departments to implement the recommendations made there*. If all teachers were mathematically well prepared, I for one would stop worrying about the age-old battle still raging between 'back to basics' and 'understanding'. On the other hand, *if mathematics departments do nothing to improve school mathematics, they should stop complaining that incoming freshmen lack mathematical skills*."

Sagredo: Why do most mathematics departments do nothing? Certainly Stein has made a good case that its to their own advantage to do something.

Hake: I think Herb Clemens (1988) <http://bit.ly/1bPYYJ9> explained it perfectly (my *italics*): "Why don't mathematicians from universities and industry belong in math education? *The first reason is that it is self-destructive*. The quickest way to be relegated to the intellectual dustbin in the mathematics departments of most research universities today is to demonstrate a continuing interest in secondary. . .(or tertiary). . . mathematics education. Colleagues smile tolerantly to one another in the same way family members do when grandpa dribbles his soup down his shirt. Math education is certainly an acceptable form of retiring as a mathematician, like university administration (unacceptable forms being the stock market, EST. . .[[Erhard Seminar Training?]]. . ., or a mid-life love affair). *But you don't do good research and think seriously about education*."

(Clemens' comments apply as well to physicists and physics education.)

C. Typical Calculus Course Problem – Even Dogs Can Solve It!

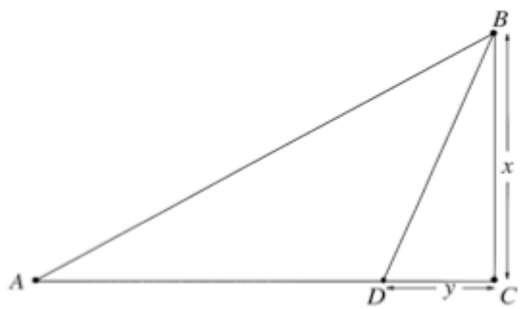


Fig. 1. Paths to a ball thrown from A on the shore to B in the water. (the "shore" is a straight line running through A and C)

A man standing at A with his dog on the edge of a straight shoreline running through points A and C, throws a ball that lands in the water at point B, a perpendicular distance x from point C. His dog's running speed on land is r and her swimming speed in the water is s (less than her land speed r). The distance from A to C is z. The dog wishes to minimize the time T taken to reach the ball:

$$T = (z - y) / r + [(x^2 + y^2)^{0.5}] / s \quad \ldots\ldots\ldots\ldots\ldots (1)$$

At what value of y is T a minimum?

In "Do Dogs Know Calculus?" mathematician Tim Pennings (2003) reports the solution of the above problem by his:

Welsh Corgi dog, Elvis,

who invariably follows the path A ---> D ---> B, which minimizes the time taken by Elvis to reach the ball, as verified by Pennings' careful measurements!

D. NSF's Calculus Reform Effort Initiated in 1987

Undergraduate Curriculum Development In Mathematics: Calculus

According to "Calculus: Catalyzing a National Community for Reform" [NSF (1995)]: "From 1987-1995, NSF invested *more than $44 million dollars in calculus reform efforts*. Over 350 awards were made during that period to a very broad spectrum of institutions. See, e.g., Przemyslaw Bogacki's (2012) "Calculus Resources On-line" listing at <http://bit.ly/Je9xrk> of over 30 U.S. universities that took part.

E. Assessments Bemoan Lack of Evidence of Improved Student Learning

1. William Haver (1998)

In *Calculus: Catalyzing a National Community for Reform*, Haver wrote:

"The NSF Calculus Program has had widespread impact on Calculus courses, on introductory collegiate mathematics instruction, and indeed on collegiate mathematics at all levels. . . . . . Nationwide, content of the Calculus course has been modified to include:
(a) substantially more applications of mathematics,

(b) the use of technology to improve the understanding of concepts, to encourage the formulation of conjectures, and to perform calculations that are normally too difficult to do by hand, and

(c) *a deeper understanding of Calculus from a geometric and numerical as well as analytic point of view* (my *italics*). Calculus students today are making extensive use of modem technology; regularly completing long-term assignments; and frequently participating actively as members of study groups and activity teams. Ten years ago these activities were virtually unheard of in college mathematics classes. . . . . It should be acknowledged, however, that some college and university mathematicians believe that the increased use of technology, the introduction of more applications, and the increased emphasis on student communication is a change in the wrong direction. In addition, ***there are others who believe that more evidence of improved student learning is necessary before a final decision can be made concerning the ultimate value of the change***." [My ***italics***.]

2. Susan Ganter (1999)

In "An Evaluation of Calculus Reform: A Preliminary Report of a National Study" Susan Ganter wrote:

"A number of reports that present programmatic information and indicators of success in the efforts to incorporate technology and sound pedagogical methods in calculus courses have indeed been written. Reform has received mixed reviews, with students seemingly faring better on some measures, while lagging behind students in traditional courses on others However, these reports ***present only limited information on student learning in reform courses*** [my ***italics***], primarily because the collection of reliable data is an enormous and complicated task and concrete guidelines on how to implement meaningful evaluations of reform efforts simply do not exist. The need for studies that determine the impact of these efforts, in combination with the increase in workload brought on by reform, is creating an environment of uncertainty. Funding agencies, institutions, and faculty require the results of such studies to make informed decisions about whether to support or withdraw from reform activities."

F. A Glimmer of Hope† for Calculus Education: The *Calculus Concept Inventory* (CCI) - Development and Validation of the *Calculus Concept Inventory* by:

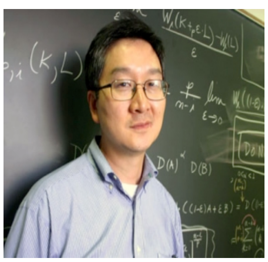
Jerry Epstein (2007; 2012; 2013)

and a panel of widely respected calculus educators* plus psychologist Howard Everson <http://bit.ly/Or9puu>, nationally known for development and validation of standardized tests:

Dan Flath
MacAlaster College

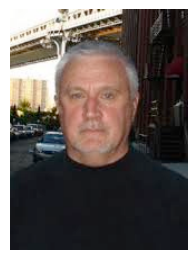
Maria K. Robinson
Seattle University

Maria Terrell
Cornell



Deane Yang
NYU Polytechnic

Kimberly Vincent
Washington State



Howard Everson
CUNY Grad Center

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†Might Concept Inventories <http://en.wikipedia.org/wiki/Concept_inventory> also be a "Glimmer of Hope" for Higher Education generally? See, e.g., "The Physics Education Reform Effort: A Possible Model for Higher Education?" [Hake (2005)] and the 10,600 Google hits at <http://bit.ly/17iXpGx> on that title (on 19 Dec 2013 08:58-0800). See also "U.S. Colleges Put Low Priority on Student Learning" [Hake (2012a)].

*Mahendra C. Shah of NYU Polytechnic (deceased) is not shown.

Epstein (2013) wrote (see his paper for the references): "The Calculus Concept Inventory (CCI) is a test of conceptual understanding (and only that—there is essentially no computation) of the most basic principles of differential calculus. The idea of such a test follows the Mechanics Diagnostic Test and its successor the Force Concept Inventory (FCI) in physics, the last a test which has spawned a dramatic movement of reform in physics education and a large quantity of high quality research. The MDT and the FCI showed immediately that a high fraction of students in basic physics emerged with little or no understanding of concepts that all faculty assumed students knew at exit and that a semester of instruction made remarkably little difference. . . . . . . Mathematics education is often mired in "wars"* between "back-to-basics" advocates and "guided- discovery" believers. There seems to be no possibility of any resolution to this contest without hard, scientific evidence of what works. Such evidence requires widespread agreement on a set of very basic concepts that *all* sides agree students should—must—be expected to master in, for example, first semester calculus. The CCI is a first element in such a development and is an attempt to define such a basic understanding."

G. Typical Question§ of the CCI Type - Dogs Score at the Random Guessing Level

Figure 2.27 shows position as a function of time for two sprinters running in parallel lines. Which of the following is true?

(a) At time A, both sprinters have the same velocity

(b) Both sprinters continually increase their velocity.

(c) Both sprinters run at the same velocity at some time before A.

(d) At some time before A, both sprinters have the same acceleration.

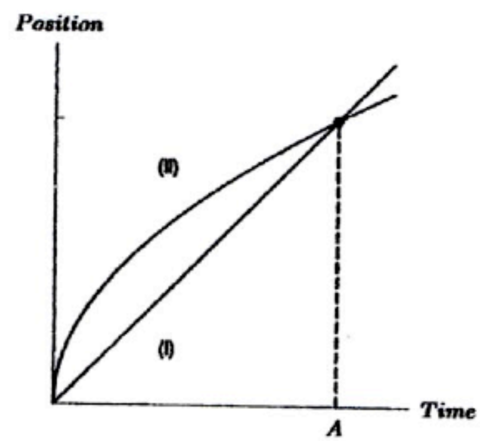

Figure 2.27

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* See "The Math Wars" [Schoenfeld (2004)].

§ From Lomen & Robinson (2004). In Socratic Dialogue Inducing (SDI) Lab #0.2, "Introduction To Kinematics," online at <http://bit.ly/xIil7c>, students use an acoustic position detector to plot position x vs time t of their own bodies and to become familiar with the graphical relationship of x (t), $v = dx/dt$, and $a = d^2x/dt^2$. See "A Microcomputer-Based SDI Lab Emphasizing the Graphical Interpretation of the Derivative and Integral" [Hake (1998c)].

H. Impact of the CCI on Calculus Education

1. Pre 2013

Epstein (2007) wrote:

"The [CCI] shows good performance characteristics and exposes exactly what the [FCI] showed. . . . . Both show that traditional instruction has remarkably little effect on basic conceptual understanding, and this has been the greatest shock to faculty. . . . . The most optimistic results were from Uri Treisman <http://bit.ly/KM0t1a>. He did not expect much, he said, because he was stuck with a large class of some 85 students. Nevertheless, he came in with <g> = 0.30 which is well outside the range of all the standard lecture based sections (0.15 to 0.23), though significantly lower than what was seen in physics.* Obviously the amount of data from good alternative instruction is far too small for any final conclusions, and the foundational question of whether teaching methodology strongly affects gain (on the CCI) as it does for physics (on the FCI) will have to await further data."

More recently Epstein (2012) wrote (paraphrasing):

"The Calculus Concept Inventory (CCI) [has been given] to about 1000 university students in Shanghai, China. The classes are, I think, a bit larger than typical American calculus classes, and are ***totally teacher centered lectures***. . . . . Preliminary analysis indicates that the Chinese calculus students are overall at the same level as the University of Michigan students. . . . . "
Epstein's "at same level as the University of Michigan students" evidently means that the Chinese students attained about the same average **normalized** pre-to-posttest gains <g> = (<%post> - <%pre>) / (100% - <%pre>) – as the students reported by the University of Michigan's Karen Rhea (2000) and ***two standard deviations*** above traditional U.S. first-year calculus students. . . .

How could a Chinese traditional "teacher centered" course result in such relatively large <g>'s ? Epstein suggests it might be due to the habit of Chinese students to form after-class "study gangs" [Treisman (1992) Effect"] but more rigorous K-12 math preparation could also be a factor.

2. Post 2013

Section **H1** above, from the talk "Can the Cognitive Impact of Calculus Courses be Enhanced?" of 24 April 2012, at the University of Southern California, indicates the CCI situation as it was on that date. For the present CCI picture see (a) Epstein's (2013) recent publication "The Calculus Concept Inventory - Measurement of the Effect of Teaching Methodology in Mathematics" in the *Notices of the AMS* of September 2013; and (b) David Bressoud's (2013a) laudatory remarks on Epstein's CCI in his MAA Launchings piece "Evidence of Improved Teaching." Bressoud wrote [bracketed by lines "BBBBB. . . . ."; slightly edited; my *italics*]:

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
Last December I discussed the NRC report, *Discipline-Based Education Research: Understanding and Improving Learning in Undergraduate Science and Engineering*. . . . .[[Singer, Nielsen, & Schweingruber (2012)]]. . . . One of its themes is the importance of the adoption of "evidence-based teaching strategies." It is hard to find carefully collected quantitative evidence that certain instructional strategies for undergraduate mathematics really are better. I was pleased to see two articles over the past month that present such evidence for active learning strategies.

One of the articles is the long-anticipated piece by Jerry Epstein (2013), “The Calculus Concept Inventory—Measurement of the Effect of Teaching Methodology in Mathematics” which appeared in the September 2013 *Notices of the AMS*. Because this article is so readily available to all mathematicians, I will not say much about it. Epstein’s Calculus Concept Inventory (CCI) *represents a notable advancement in our ability to assess the effectiveness of different pedagogical approaches to basic calculus instruction*.

Epstein presents strong evidence for the benefits of Interactive Engagement (IE) over more traditional approaches. As with the older *Force Concept Inventory* developed by Halloun & Hestenes (1985a,b), CCI has a great deal of surface validity. It measures the kinds of understandings we implicitly assume our students pick up in studying the first semester of calculus, and *it clarifies how little basic conceptual understanding is absorbed under traditional pedagogical approaches*.

Epstein claims statistically significant improvements in conceptual understanding from the use of Interactive Engagement, stronger gains than those seen from other types of interventions including plugging the best instructors into a traditional lecture format. Because CCI is so easily implemented and scored, it should spur greater study of what is most effective in improving undergraduate learning of calculus.

The second paper is “Assessing Long-Term Effects of Inquiry-Based Learning: A Case Study from College Mathematics” [Kogan & Laursen (2013)]. This was a carefully controlled study of the effects of Inquiry-Based Learning (IBL) on persistence in mathematics courses and performance in subsequent courses. They were able to compare IBL and non-IBL sections taught at the same universities during the same terms. . . . . . Most striking is the very clear evidence that IBL does no harm, despite the fact that spending more time on interactive activities inevitably cuts into the amount of material that can be “covered.” In fact, it was the course with the densest required syllabus, G1, where IBL showed the clearest gains in terms of preparation of students for the next course.

IBL is often viewed as a luxury in which we might indulge our best students. In fact, as this study demonstrates, it can have its greatest impact on those students who are most at risk. BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

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* The average normalized gain is defined in Hake (1998a) as <g> = (<%post> - <%pre>) / (100% - <%pre>) = <% Gain> / (max. possible <% Gain>). In Hake (1998a) the 48 “Interactive Engagement” introductory physics courses achieved <<g>> = 0.48 ± 0.14 (std dev), about a 2 std. deviation superiority to the 14 “Traditional Courses” with <<g>> = 0.23 ± 0.04 (std dev).

I. Conclusion

Q. Can the Cognitive Impact of Calculus Courses Be Enhanced?
A. *Possibly*, But It May Take Several Decades.

Judging from the physics education reform effort – see "The Impact of Concept Inventories On Physics Education and Its Relevance For Engineering Education" [Hake (2011c)]:

The cognitive impact of calculus courses might be increased, especially if further effort is made to:

(a) continue Jerry Epstein's (2007; 2012, 2013) development of the CCI into a test that is widely accepted as valid and consistently reliable,

(b) administer the CCI to many different traditional and reform courses, and

(c) subsequently meta-analyze the results.

2. But even then, it may take several decades before widespread improvement occurs.

Epilogue

"The academic area is one of the most difficult areas to change in our society. We continue to use the same methods of instruction, particularly lectures, that have been used for hundreds of years. Little scientific research is done to test new approaches, and little systematic attention is given to the development of new methods. Universities that study many aspects of the world ignore the educational function in which they are engaging and from which a large part of their revenues are earned."

- Richard M. Cyert, former president of Carnegie Mellon University in Tuma & Reif (1980)

APPENDIX #1 – The Lagrange Approach to Calculus

Judith Grabiner (2010) in her introduction to *A Historian Looks Back: The Calculus as Algebra and Selected Writings* wrote bracketed by lines "GGGGG. . . .":

GGGGGGGGGGGGGGGGGGGGGGGGGGG

In *The Calculus as Algebra: J.-L. Lagrange, 1736–1813*, I show what Lagrange's mathematical practice was like, in order to understand the genesis of the rigorous analysis of Cauchy, Bolzano, and Weierstrass. For Lagrange, the calculus was not about rates of change or ratios of differentials, or even about limits as then understood. Lagrange thought that the calculus should be reduced to "the algebraic analysis of finite quantities." This sounds as though he was about to introduce deltas and epsilons. But instead he believed that there was an algebra of infinite series, and that every function had a power-series expansion except perhaps at finitely many isolated points. Lagrange *defined* the derivative as the coefficient of the linear term in the function's power-series expansion. Why he thought this was justified tells us both about his philosophy of mathematics and about the way many mathematicians practiced their subject in the eighteenth century. Euler, for example, did marvelous things by what we would now call the carefree formal manipulation of infinite series, infinite products, and infinite continued fractions. But Lagrange found something else in infinite series as well. He imported what we now call delta-epsilon techniques from the 18th-century study of approximations into some of his proofs about the concepts of the calculus. He was the first to attempt to prove, let alone to use inequalities in so doing, statements like "a function with a positive derivative on an interval is increasing there." He justified many results of calculus using inequalities, including the mean-value theorems for derivatives and integrals, and the Lagrange remainder of the Taylor series. This, and much more, helped build Cauchy's work in the 1820s on the foundations of analysis.

GGGGGGGGGGGGGGGGGGGGGGGGGGG

See also Alain Schremmer's (1998) "A Story Line for Calculus" and Carl Boyer's (1959) *The History of the Calculus and Its Conceptual Development*.

REFERENCES

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Notes:

1. The present reference formatting is a blend of the *best* formatting features from the style manuals of the AIP (American Institute of Physics) <http://bit.ly/d8hJgp> [this URL was dead on 20 Dec 2013 but it may recover], APA (American Psychological Association) <http://apastyle.apa.org/>, and CSE (Council of Science Editors) <http://bit.ly/1hx4pox>. Such formatting is not commonly employed *but should be*.

2. Some of the references are to posts on Academic Discussion Lists (ADLs) – see "A Guide to the ADLsphere" [Hake (2010b)].

3. A few references are to posts on the archives <http://bit.ly/nG318r> of the physics education discussion list PhysLrnR. To access the archives of PhysLnR one needs to subscribe : - (, but that takes only a few minutes by clicking on <http://bit.ly/nG318r> and then clicking on "Join or Leave PHYSLRNR-LIST." If you're busy, then subscribe using the "NOMAIL" option under "Miscellaneous." Then, as a subscriber, you may access the archives and/or post messages at any time, while receiving NO MAIL from the list!

4. Most URL's were accessed on 19-26 Dec 2013; most are shortened by <http://bit.ly/>.

5. At the sacrifice of some readability, I use *overt* URLs <http://bit.ly/19cB6VT> rather than *covert* URLs in the text because: (a) *covert* URLs do not work in my pdfs and in some mail/server systems, and (b) *overt* URLs are compatible with a standard alphabetically-ordered academic REFERENCE list that provides the author(s), title, year of publication, URL access date, file size (for pdfs), and other information on the references.

6. Math Education Acronyms:
   CUPM: Committee on the Undergraduate Program in Mathematics
   PMET Preparing Mathematicians to Educate Teachers
   PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies
   PSSM: Principles and Standards for School Mathematics
   SAUM: Supporting Assessment in Undergraduate Mathematics
   UME Trends: Newsletter on Undergraduate Math Education Trends
      According to "UME Trends: Observing a Decade of Change and Preparing for the Future" [Dubinsky (2000)], that newsletter ceased publication in the late 1990s. Since my Goggle searches fail to bring up *UME Trends* articles [with the exception of articles saved on their own websites by authors] I assume *UME Trends* left no archive **:-(** . (Please correct me if I'm wrong.)

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Anderson, M., V. Katz, & R. Wilson, eds. 2004. *Sherlock Holmes in Babylon*. MAA. Amazon.com information at <http://amzn.to/1cqnscG>, note the searchable "Look Inside" feature. An expurgated Google book preview is online at <http://bit.ly/1dtSenk>. Publisher's information at <http://bit.ly/1dtzK6j>, wherein it's stated:

> "Sherlock Holmes in Babylon is a collection of 44 articles on the history of mathematics, published in MAA journals over the past 100 years. Covering a span of almost 4000 years, from the ancient Babylonians to the eighteenth century, it chronicles the enormous changes in mathematical thinking over this time, as viewed by distinguished historians of mathematics from the past (Florian Cajori, Max Dehn, David Eugene Smith, Julian Lowell Coolidge, and Carl Boyer etc.) and the present."

Anderson, M., V. Katz, & R. Wilson, eds. 2009. *Who Gave you the Epsilon?: & Other Tales of Mathematical History*. MAA, Amazon.com information at <http://amzn.to/MywafO>, note the searchable "Look Inside" feature. Publisher's information at <http://bit.ly/LgEz7q>, wherein it's stated:

> "*Who Gave You the Epsilon?* is a sequel to the MAA bestselling book, *Sherlock Holmes in Babylon*. . . . . .[[Anderson, Katz, & Wilson (1994) – see above]]. . . . Like its predecessor, this book is a collection of articles on the history of mathematics from the MAA journals, in many cases written by distinguished mathematicians (such as G H Hardy and B.van der Waerden), with commentary by the editors. Whereas the former book covered the history of mathematics from earliest times up to the 18th century and was organized chronologically, the 40 articles in this book are organized thematically and continue the story into the 19th and 20th centuries. The topics covered in the book are analysis and applied mathematics, Geometry, topology and foundations, Algebra and number theory, and Surveys. Each chapter is preceded by a Foreword, giving the historical background and setting and the scene, and is followed by an Afterword, reporting on advances in our historical knowledge and understanding since the articles first appeared."

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Bressoud. D.M. 2010f. “Meeting the Challenge of High School Calculus, V: The Need for Guidelines,” MAA Launchings, August; online at <http://bit.ly/14dS8jL>.

Bressoud. D.M. 2010g. “Meeting The Challenge Of High School Calculus, VII: Our Responsibilities,” MAA Launchings, September; online at <http://bit.ly/18xV3zs>.

Bressoud, D.M. 2010h. “The Problem of Persistence,” MAA Launchings Column of January online at <http://bit.ly/1gaT6lr>.

Bressoud. D.M. 2010i. “The Common Core State Standards,” MAA Launchings, October; online at <http://bit.ly/1iAEMo4>.

Bressoud. D.M. 2010j. “Historical Reflections on Teaching the Fundamental Theorem of Integral Calculus,” online as a 942 kB pdf at <http://bit.ly/1kiGuca>. See also “Teaching the Fundamental Theorem of Calculus: A Historical Reflection” [Lopez & Rodriguez (2012)]

Bressoud, D. 2011a. "The Calculus I Student," in the MAA Launchings Column of May & December; online at <http://bit.ly/1bIXbeD>. Bressoud wrote:

"I am distressed by how poorly these students do in Calculus I: Over a quarter essentially fail, and only half earn the A or B that is the signal that they are likely to succeed in further mathematics. I know the frustration of high school teachers who see what they consider to be the best and brightest of their students run into mathematical roadblocks in college. I recognize that much of the fault lies on the high school side of the transition. Many students who consider themselves well prepared for college mathematics in fact are not. We need to do a better job of communicating what these students really need and working with their teachers so that they can provide it. I also know that we in the colleges and universities can do a better job of supporting these students after they have arrived on our campuses, moving them forward with challenging and engaging mathematics while bringing them up to the level they need to be at to succeed."

Bressoud, D. 2011b. MAA Launchings Blog; blog entries from "Introduction of Feb 2005 to "The Worst Way to Teach" of July 2011; online at <http://bit.ly/18mBMls>.

Bressoud, D. 2012. "On Engaging to Excel," *MAA Launchings*, 1 March; online at <http://bit.ly/GWTX80>. Bressoud wrote:

"The President's Council of Advisors on Science and Technology (PCAST) has just released its report to President Obama on undergraduate Science, Technology, Engineering, and Mathematics (STEM) education: *Report to the President, Engage to Excel: producing one million additional college graduates with degrees in Science, Technology, Engineering, and Mathematics*, online as a 3 MB pdf at <http://1.usa.gov/GZmbzq>."

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Cohen, A. & S.G. Krantz. 2001. "Two Reactions to 'The Mathematical Education of Teachers'," . . . . . .[[CBMS (2001)]]. . . . *Notices of the AMS* **48**(9): 985-991; online as a 74 kB pdf at <http://bit.ly/LKskxw>.

Amy Cohen's reaction is generally positive. She concludes:

> "I recommend that my colleagues read this report looking for ideas to adopt (or at least adapt) rather than looking for excerpts to disparage. What I like best is the report's underlying assumption that we need an alliance of *content* and *process*, not a victory of one over the other. I hope we can agree that students and their teachers must understand as well as remember mathematics in order to use it well. It would follow that education for prospective teachers, like all college-level education, should aim for deep understanding not just coverage of material. I hope that the ideas and arguments of the MET report will both spur and assist the mathematical community to improve the mathematical education of teachers and thus to improve the status of mathematics in twenty-first-century America."

On the other hand Steven Krantz, editor of the *Notices of the AMS*, had a negative reaction, writing:

> "I had a dream last night. I dreamed that I was teaching a class on pseudodifferential operators. On the first day I asked the class what they thought a pseudodifferential operator should be. No good. I got nothing but blank stares. I then said, 'OK. Who can tell us what a singular integral should be? *Hint:* Think Calderón and Zygmund.' Still I got no response. . . . . . . We spent the rest of the class time discussing how we felt about mathematical analysis, about the role of the mathematician in society at large, and about what kind of teacher David Hilbert was. It was a rewarding hour. The reader who has stuck with me so far is probably thinking that old Steve Krantz has finally gone around the bend. But no, I am portraying a teaching process that is being purveyed by well-meaning individuals who have set themselves up as the arbiters of teaching standards for the next generation of school mathematics teachers. Students are supposed to cogitate and interact with each other and generate - hit or miss - the ideas for themselves. The volume under review is an instance of this new *Weltanschauung*."

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DeHaan, R.L. 2005. "The Impending Revolution in Undergraduate Science Education," *Journal of Science Education and Technology* **14**(2): 253-269; online as a 152 kB pdf at <http://bit.ly/ncAuQa>. DeHaan wrote:

> "There is substantial evidence that scientific teaching in the sciences, i.e., teaching that employs instructional strategies that encourage undergraduates to become actively engaged in their own learning, can produce levels of understanding, retention and transfer of knowledge that are greater than those resulting from traditional lecture/lab classes. But widespread acceptance by university faculty of new pedagogies and curricular materials still lies in the future."

Docktor, J.L. & J.P. Mestre. 2011. “A Synthesis of Discipline-Based Education Research in Physics,” online at <http://bit.ly/18wSoKD> along with other papers commissioned for the Committee on the Status, Contributions, and Future Directions of Discipline-Based Education Research through support from the National Science Foundation. These papers were presented at committee meetings between June 2010 and July 2011. See also NAP (2013).

In “Problematic Pronouncements on the Normalized Gain” [Hake (2011d)] I wrote: Docktor & Mestre (2011), in an otherwise excellent review, make some problematic pronouncements regarding the "normalized gain" – see "The Impact of Concept Inventories On Physics Education and Its Relevance For Engineering Education" [Hake (2011c)], wherein I wrote:

> Docktor & Mestre (2011) wrote: “The ‘normalized gain’ is a commonly reported measure for comparing pretest and posttest scores across populations (Hake, 1998a), but the statistical origin for this measure is unclear and alternatives have been suggested (such as ‘normalized change’). It is unclear why normalized gain is still favored, and the PER community should reach an agreement about how to analyze and report scores from concept inventories.”
>
> > The *statistical origin is unclear*? As indicated in Section IIC11 above, “Experimental Justification of <g> as a *Comparative Measure of Course Effectiveness,”* the “average normalized gain” <g> was utilized in Hake (1998a,b) as a strictly *empirically-based* parameter with no claim to a statistical pedigree. Nevertheless <g> provided a consistent analysis of pre/post test results – see Fig. 1 and 2 above - over diverse student populations in high schools, colleges, and universities. Furthermore, similar consistent analyses using <g> have been obtained in over 25 physics education research papers as listed above in Section IIC9 “Research Consistent With the Above Meta-analysis.”
> >
> > > In my opinion, it is unclear why Docktor & Mestre (2011): (1) choose to discount the above evidence for the value of <g>, and (2) think that “It is unclear why normalized gain is still favored.”

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> "The movement to change the nature of the calculus course at the undergraduate and secondary levels has sparked discussion and controversy in ways as diverse as the actual changes. The first years of the calculus reform movement were characterized by a whirlwind of ideas concerning the organization of the course and the associated curriculum. The papers contained [in this book] will spark a renewed interest in the endeavor embarked upon over 10 years ago when the first calculus grants were awarded by the National Science Foundation (NSF). This book intends to address: relating mathematics to other disciplines; determining the appropriate mathematical skill for students exiting first-year collegiate mathematics courses; determining the appropriate role of technology; determining the appropriate role of administrators in the change process; and evaluating the progress and impact of curricular change."

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" This book grew out of a report by a subcommittee of the Committee on the Undergraduate Program in Mathematics, chaired by Bernie Madison, and out of a concern by Sandy Keith that mathematicians take assessment seriously. The introductions by Lynn Steen (1999) and Bernie Madison (1999) set the context more fully."

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"This volume contains student and instructor material for the delivery of a two-semester calculus sequence at the undergraduate level. It can be used in conjunction with any textbook. It was written with the view that students who are actively involved inside and outside the classroom are more likely to succeed, develop deeper conceptual understanding, and retain knowledge, than students who are passive recipients of information."

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Kline, M. 1967, 1998. *Calculus: An Intuitive and Physical Approach*. Dover. Amazon.com information at <http://amzn.to/LsOYKC>, note the searchable "Look Inside" feature. First published in 1967.

Kogan, M. & S.L. Laursen. 2013. "Assessing Long-Term Effects of Inquiry-Based Learning: A Case Study from College Mathematics," *Innov High Educ,* 17 August, online as a 430 kB pdf at <http://bit.ly/1d7eC5j>; note the "Look Inside" feature. The authors wrote [see the article for references other than Fullilove & Treisman (1990); Hake (1998a); Springer, Stanne & Donovan (1999); Prince & Felder (2007); Froyd (2008); Kuh (2008); and Watkins & Mazur (2013)]:

> "Student-centered or 'active' forms of instruction have been shown to improve student learning and affective outcomes in the sciences and in other fields (Ambrose et al., 2010; Froyd, 2008; Hake, 1998a; Prince & Felder, 2007; Springer, Stanne & Donovan, 1999). Yet these proven, "high-impact" educational practices are not typical of what students experience in college (Kuh, 2008). Especially crucial are mathematics courses, key prerequisites that may regulate students' access to many majors and careers, or to any college degree at all (Carlson et al., 2011; Seymour & Hewitt, 1997; Stigler, Givvin & Thompson, 2010). Thus the use of active learning methods in college mathematics may help to attract and retain students, including students of diverse backgrounds (Ellis, Rasmussen & Duncan, 2013; Fullilove & Treisman, 1990; Watkins & Mazur, 2013)."

Kuh, G. 2008. "*High-impact educational practices: What they are, who has access to them, and why they matter*." American Association of Colleges and Universities; online as a 1.5 MB pdf at <http://bit.ly/18kZFiI>

> With an introduction by Carol Geary Schneider and findings on student success from AAC&U's LEAP initiative – see <http://bit.ly/1fi3O6M>.

Laursen, S., M. Hassi, M. Kogan, A. Hunter, & T. Weston. 2011. "Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics," online as a 4.8 MB pdf at <http://bit.ly/1ddM9Ls>.

Lederman, L. 2001. "Revolution in Science Education: Put Physics First." *Physics Today* **54**(9): 11-12, September; online at <http://bit.ly/anddW1>.

Lewis, J. 2001. "Spotlight on Teachers," *Notices of the AMS* **48**(4):396-403' online as a 78 kB pdf at <http://bit.ly/LNJwSC>: Lewis wrote:

"But despite all the other suggestions for how to improve our schools, one idea recurs frequently - good teachers matter. This idea is often combined with the viewpoint that our colleges and universities are not doing enough to produce high-quality teachers." For two diametrically opposed reactions to Lewis's article see Cohen & Krantz (2001).

Lomen, D.O. & M. K. Robinson. 2004. “Using Conceptests in Single and Multivariable Calculus.” *Proceeding of the 16th Annual International Conference on Technology in Collegiate Mathematics*, Addison Wesley; online as a 311 kB pdf at <http://bit.ly/JLtsoL>. They wrote (slightly edited):

ConcepTests were developed by Eric Mazur [1997, 2009] as a method of improving student conceptual understanding and scores on concept examinations in physics. This has been replicated in other science areas (see <http://bit.ly/1dpRuzj>), now including calculus (see Epstein (2013)].

Lopez, F.J.M. & O.A.H. Rodriguez. 2012. “Teaching the Fundamental Theorem of Calculus: A Historical Reflection” *Loci* (January), online at <http://bit.ly/LWiImT>. See also “Historical Reflections on Teaching the Fundamental Theorem of Integral Calculus” [Bressoud (2010j)].

MAA Focus, online at <http://bit.ly/1d8yODM>.

MAA Advanced Search, online at <http://www.maa.org/search/node/>.

MAA/NCTM. 2012. “Calculus,” Joint Position Statement of the Mathematical Association of America and the National Council of Teachers of Mathematics. Online as a 102 kB pdf at <http://bit.ly/Kes74F>, where in it’s stated [my *italics*]:

“Although calculus can play an important role in secondary school, *the ultimate goal of the K–12 mathematics curriculum should not be to get students into and through a course in calculus by twelfth grade* but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college. The college curriculum should offer students an experience that is new and engaging, broadening their understanding of the world of mathematics while strengthening their mastery of tools that they will need if they choose to pursue a mathematically intensive discipline.”

Madison, B.L. 2001. “Assessment: The Burden of a Name,” online as a 25 kB pdf at <http://bit.ly/1bBGeBr>; a similar paper was delivered at a PKAL conference “Assessment in the Service of Student Learning,” Duke University, 1-3 March 2002. For a response see “Whence Do We Get the Teachers?” [Hake (2002b)].

Mahajan, S. & R.R. Hake. 2000. “Is it time for a physics counterpart of the Benezet/Berman math experiment of the 1930's?” Physics Education Research Conference 2000: Teacher Education, online at <http://arxiv.org/abs/physics/0512202>. We wrote:

“Students in Manchester, New Hampshire were not subjected to arithmetic algorithms until grade 6. In earlier grades they read, invented, and discussed stories and problems; estimated lengths, heights, and areas; and enjoyed finding and interpreting numbers relevant to their lives. In grade 6, with 4 months of formal training, they caught up to the regular students in algorithmic ability, and were far ahead in general numeracy and in the verbal, semantic, and problem-solving skills they had practiced for the five years before. We conjecture that implementation of the ‘Benezet Method’ in early grades would drastically improve the effectiveness of high-school and university physics, science, and mathematics instruction.”

Mazur, E. 1997. *Peer instruction: a user's manual*. Prentice Hall; information online at <http://bit.ly/n73k9g>.

Mazur, E. 2009. “Confessions of a Converted Lecturer” talk at the University of Maryland on 11 November 2009. That talk is now on UTube at <http://bit.ly/dBYsXh>, and the abstract, slides, and references - sometimes obscured in the UTube talk - are at <http://bit.ly/9qzDIq> as a 4 MB pdf.
As of 18 December 12:53-0800 Eric's talk had been viewed 104,748 times. In contrast, serious articles in the education literature (or even articles such as this one) are often read only by the author and a few cloistered specialists, creating tsunamis in educational practice equivalent to those produced by a pebble dropped into the middle of the Pacific Ocean.

McCallum, W.G. 1991. “Rigor in the Undergraduate Calculus Curriculum.” *Notices of the AMS* **38**(9): 1131-1132. I’ve not found an online version .

McCallum, W.G. 2000. “The Goals of the Calculus Course,” Chapter 2 in *Calculus Renewal: Issues for Undergraduate Mathematics Education in the Next Decade* [Ganter (2000)] at <http://bit.ly/YtcYSt>. See also the publisher’s information on “The Goals of the Calculus Course” at <http://bit.ly/18amG3U>.

McCallum,W.G., D. Hughes-Hallett, A.M. Gleason, D.O. Lomen, D. Lovelock, J. Tecosky-Feldman, T.W. Tucker, D.E. Flath, J. Thrash, K.R. Rhea, A. Pasquale, S.P. Gordon, D. Quinney, & P.F. Lock. 2008. *Calculus: Multivariable*, 5th ed. John Wiley. Amazon.com information at <http://amzn.to/1hPGlgD>, note the searchable “Look Inside” feature.

McCallum, W.G. 2013a. Curriculum Vitae; online as a 86 kB pdf at <http://bit.ly/1gtrPee>.

McCallum, W.G. 2013b. “Engaging students in mathematics,” *Arizona Daily Star*, 28 October; online at <http://bit.ly/IIkNE1>. McCallum wrote [bracketed by lines “MMMM. . . .]:

MMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM

What if we lived in a world where people knew, used and enjoyed mathematics? What if mathematics made sense to students? What if teachers teaching fractions in Grade 3 could tell the story of how fractions flow naturally from whole numbers in Grade 2?

Until recently, they had to tell that story from incoherent textbooks based on a cacophony of voices from 50 different sets of state standards. *Now that 45 states, including Arizona, have adopted the Common Core State Standards in mathematics, teachers can share ideas for conveying the coherence of mathematics across state lines*.. . . .[[My *italics*.]]. . . .

Improvement in mathematics education ultimately depends on teachers. If teachers have focused, coherent and rigorous standards to work from, they can portray the way mathematical ideas build over time, for example by connecting addition of fractions to addition of whole numbers.

If teachers can show students the ways in which mathematics is used in science, engineering, and finance, they can reveal to students the beauty of mathematical ideas.

Illustrative Mathematics, at <http://www.illustrativemathematics.org/>, gives teachers the tools to do this. *Illustrative Mathematics* arose out of a promise I made while working on the standards, to provide illustrative problems illuminating the meaning of the standards. But it has grown into much more: a community of 20,000 teachers and mathematicians who explore the standards and comment on grade-appropriate mathematics tasks.

MMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM

Regarding the CCSS, compare the above with (a) “The Common Core State Standards” [Bressoud (2010i); (b) “*Next Generation Science Standards*: Good or Bad for Science Education?” [Hake (2013a)]; (c) “The future of high school math education” [Strauss (2013) in APPENDIX #2]; (d) “Mathematics and Education” [Kessel (2013) in APPENDIX #2]; (e)“Why I Cannot Support the Common Core Standards” [Ravitch (2013a) in APPENDIX #2; and (f) “Study supports move toward common math standards” [Schmidt (2012b)] in APPENDIX #2.

Hake, R.R. 2013a. “*Next Generation Science Standards*: Good or Bad for Science Education?” 18 March, online as a 221 kB pdf at <http://bit.ly/147K6qY> and as ref. 68 at <http://bit.ly/a6M5y0>.

Regarding the Common Core State Standards (CCSS), compare this with (a) “The Common Core State Standards” [Bressoud (2010i)]; (b “The future of high school math education” [Strauss (2013) in APPENDIX #2]; (c) “Mathematics and Education” [Kessel (2013) in APPENDIX #2]; (d) “Engaging students in mathematics” [McCallum (2013)]; (e)“Why I Cannot Support the Common Core Standards” [Ravitch (2013a) in APPENDIX #2; and (f) “Study supports move toward common math standards” [Schmidt (2012b)] in APPENDIX #2.

McDermott L.C. & E.F. Redish. 1999. “RL-PER1: Resource letter on physics education Research,” *Am. J. Phys*. **67**(9): 755-767 ; online at <http://bit.ly/baDTtQ>. See also the Univ. of Washington PER group at <http://bit.ly/jWvlqS>.

Meltzer, D. & R. Thornton. 2012. “Resource Letter ALIP-1: Active-Learning Instruction in Physics,” *Am. J. Phys*. **80**(6): 478-496; online to subscribers at <http://bit.ly/1gnBhQ6>.

Meltzer, D.E., M. Plisch, & S. Vokos, eds. 2013a. "Transforming the Preparation of Physics Teachers: A Call to Action - A Report by the Task Force on Teacher Education in Physics (T-TEP)"; online as a 2 MB pdf at <http://bit.ly/17gMhcZ>.

Meltzer, D.E., M. Plisch, & S. Vokos. 2013b. "The Role of Physics Departments in High School Teacher Education," *APS News* **22**(8), Back Page; online at <http://bit.ly/1arpo3C>.

Meridith, D., K. Black, & K. Marrongelle. 2012. University of New Hampshire Studio Calculus/Physics Course' online at <http://bit.ly/LYVYiP>.

Michael, J. & H, I. Modell. 2003. *Active Learning in Secondary and College Science Classrooms: A Working Model for Helping the Learner To Learn*. Routledge. Amazon.com information at <http://amzn.to/ok03wD>, note the searchable "Look Inside" feature. See also Michael (2006).

Michael, J. 2006. "Where's the evidence that active learning works?" *Advances in Physiology Education* **30**: 159-167, online at <http://bit.ly/9x4l7g>, a masterful review by a medical education researcher/developer. See also Michael & Modell (2003). Michael wrote:

> "One of the most striking findings [came from comparison of the learning outcomes (as measured by the FCI and a related inventory on mechanics) from 14 traditional courses (2,084 students) and 48 courses using "interactive-engagement" (active learning) techniques (4,458 students). The results on the FCI assessment showed that students in the interactive engagement courses outperformed students in the traditional courses by 2 SDs. Similarly, students in the interactive-engagement courses outperformed students in the traditional courses on the Mechanics Baseline Test, a measure of problem-solving ability. This certainly looks like evidence that active learning works! Research in physics education is having a profound effect on the development of instructional materials."

MU. Mechanical Universe. 2013. At MU website <http://bit.ly/h3EVkN> it's stated that:

> "The Mechanical Universe...and Beyond is a critically acclaimed series of 52 thirty-minute videotape programs covering the basic top- ics of an introductory university physics course. The series was originally produced as a broadcast telecourse by the California Institute of Technology and Intelecom, Inc. with program funding from the Annenberg/CPB Project." See also "Making 'The Mechanical Universe,' " [Goodstein & Olenick (1988)].
>
> Some of these videotapes were shown to students in the "lecture" portion of the course that included "Socratic Dialogue Inducing" (SDI) Labs [Hake (1992, 2008, 2012b), but some students, thinking that the videotape material would not be covered on the tests, headed for the doors when the lights dimmed! To counter this tendency I started to use a few test questions based on historical or literary details discussed in the videotapes. Some students were outraged: "What is this, a poetry class?"

NAP. 2013. National Academies Press. "Commissioned Papers: Status, Contributions, and Future Direction of Discipline-Based Education Research (DBER)," online at <http://bit.ly/17VWIVu>. These papers were presented at committee meetings between June 2010 and July 2011.

Notices of the AMS, homepage <http://bit.ly/1aHBVjf>.

PCAST. 2012. “Report To The President: Engage To Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics”; President's Council of Advisors on Science and Technology, online as a 3 MB pdf at <http://1.usa.gov/GZmbzq>.
A Summary is online as a 45 kB pdf at <http://1.usa.gov/H3up7x>. An Executive Report is online as a 668 kB pdf at <http://1.usa.gov/GZmF4U>. The Agenda of Public Briefing, chaired by Carl Wieman, Associate Director for Science, White House Office of Science and Technology Policy, is online at <http://1.usa.gov/H0XmEd>. See also “Helping Students to Think Like Scientists in Socratic Dialogue Inducing Labs” [Hake (2012b)] and posts "PCAST Report" on the RUME discussion list archives for March <http://bit.ly/HyaY6p> and April <http://bit.ly/HYoz3t>.

Pennings, T.J. 2003. “Do Dogs Know Calculus?” *The College Mathematics Journal* **34**(3):178-182; online as a 1.7 MB pdf at <http://bit.ly/19AH3Gq>. See also "Dog Plays Fetch With Calculus" on YouTube at <http://bit.ly/HZEnrL> (14,184 views as of 18 Dec 2013 12:58-0800.

PhysTec. 2012. “Physics Teacher Education Coalition: Improving and promoting the "PhysTEC is a program to improve the science preparation of future physics and physical science teachers; online at <http://www.phystec.org/>.

PMET. 2012. “Preparing Mathematicians to Educate Teachers,” [Katz & Tucker (2013)], online at <http://bit.ly/SMcwTr>.

PSSM. 1991. Principles and Standards for School Mathematics; online at <http://bit.ly/Le1vky>. the more recent PSSM (2000).

PSSM. 2000. Principles and Standards for School Mathematics; online at <http://bit.ly/KUlV5p>. See also . “Inductive Teaching and Learning Methods: Definitions, Comparisons, and Research Bases” [Prince & Felder (2006)].

Prince, M. 2004. "Does Active Learning Work? A Review of the Research," *Journal of Engineering Education* **93**(3): 223-231; online as a 752 kB pdf at <http://bit.ly/rkiBjq>. See also:
(a) “Inductive Teaching and Learning Methods: Definitions, Comparisons, and Research Bases [Prince & Felder (2006)];
(b) “The Many Faces of Inductive Teaching and Learning” [Prince & Felder (2007)];
(c) “Does Faculty Research Improve Undergraduate Teaching? An Analysis of Existing and Potential Synergies” [Prince, Felder, & Brent (2007);
(d) "Engineering Education - Training" [Hake (2009a)];
(e) "Engineering Education – Training #2" [Hake (2009b)], and
(f) “The Impact of Concept Inventories On Physics Education and Its Relevance For Engineering Education” [Hake (2011c)].

Prince, M., & R. Felder. 2006. “Inductive Teaching and Learning Methods: Definitions, Comparisons, and Research Bases,” *Journal of Engineering Education* **95**(2):123-138; online as a 148 kB pdf at <http://bit.ly/qheyVH>.

Prince, M., & R. Felder. 2007. “The Many Faces of Inductive Teaching and Learning,” *Journal of College Science Teaching* **36**(5): 14-20, March/April. online as a 324 kB pdf at <http://bit.ly/om3rcg> (324 kB).

Prince, M., R. Felder, & R. Brent. 2007. "Does Faculty Research Improve Undergraduate Teaching? An Analysis of Existing and Potential Synergies." *Journal of Engineering Education* **96**(4): 283-294; online as a 116 kB pdf at <http://bit.ly/mVl5mf>.

Redish, E.F. 1999. "Millikan lecture 1998: building a science of teaching physics." *Am. J. Phys.* **67**(7): 562-573; online as a 258 kB pdf at <http://bit.ly/KMqgIx>.

Redish, E.F. 2003. *Teaching Physics With the Physics Suite*, John Wiley. This book is online at <http://bit.ly/gdE3Tu>. Note the crucial correction of Fig. 5.2 and its caption on page 100 of the online version.

Schoenfeld, A.H. 1994. *Mathematical Thinking and Problems Solving*. Routledge. Amazon.com information at <http://amzn.to/1fk7iVK>, note the searchable "Look Inside" feature.

Schoenfeld, A.H. 2004. "The Math Wars," *Educational Policy* **18**(1): 253-286; online as a 270 kB pdf at <http://bit.ly/19QhRvI>.

Schremmer, A. 1997. "Math Knowledge vs Math Research?" Notes from Mathematical Underground, *AMATYC Review* 19(1) online at <http://bit.ly/KtQKsY>. Other "notes from mathematical underground" by Schremmer are at <http://bit.ly/YGYEpY>.

Schremmer, A. 1998. "A Story Line for Calculus," Notes from the Mathematical Underground, *AMATYC Review* **20**(1); online as a 29 kB pdf at <http://bit.ly/LKxwBa>. Other "notes from mathematical underground" by Schremmer are at <http://bit.ly/YGYEpY>.

Singer, S.R., N.R. Nielsen, & H.A. Schweingruber, eds. *Discipline-Based Education Research: Understanding and Improving Learning in Undergraduate Science and Engineering,* National Academies Press; online at <http://bit.ly/LCkSDm>. See also the commissioned papers at NAP (2013) <http://bit.ly/17VWIVu>, among them Docktor & Mestre (2011).

Smith, E.S., M. Salkover, & H.K. Justice. 1947. *Unified Calculus*. Amazon.com informatiion at <http://amzn.to/13R4H0q>. For editions between 1938 and 1962 see <http://amzn.to/15D3GNC>.

Springer, L., Stanne, M. E., & Donovan, S. S. 1999. "Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis," *Review of Educational Research* **69**(1), 21- 51; the first page is online at <http://bit.ly/n077Nd>.

Stein, S. 1997. "Preparation of Future Teachers," *Notices of the AMS* **44** (3): 311-312; online at <http://bit.ly/LF4Snx> / "Letters to the Editor, " and scroll down, where "/"means "click on."

Stokstad, E. 2001. "Reintroducing the Intro Course," *Science* 293: 1608-1610, 31 August; an abstract, online at <http://bit.ly/16WTEHl>, reads:

> "Physicists are out in front in measuring how well students learn the basics, as science educators incorporate hands-on activities in hopes of making the introductory course a beginning rather than a finale. Figuring out what works is vitally important to the country, say U.S. educators. Each year, hundreds of thousands of U.S. students get their only exposure to science in an intro class--and most leave without understanding how science works or with any desire to take further courses."

See also "Teaching in a research context" [Wood & Gentile (2003) and "The Impending Revolution in Undergraduate Science Education"[DeHaan (2005)].

Stroyan, K.D. 2006. "The changing face of calculus: Engineering math at the University of Iowa," *MAA FOCUS*, February; I've not found an online version.

Stroyan, K. 2011. “Why Do So Many Students Take Calculus?” *Notices of the AMS* **58**(8), Docemaus section: 1122-1124; online as 115 kB pdf at <http://bit.ly/Lj4yrB>. See also Stroyan (2006, 2012a,b). Stroyan wrote:

“Calculus is one of the great achievements of the human intellect. It has served as the language of change in the development of scientific thought for more than three centuries. The contemporary importance of calculus includes applications in economics, psychology, and the social sciences and continues to play a key role in its traditional areas of application. Our students’ interests and preparation are changing—see Bressoud (2004, 2010a-g), Stroyan (2006)—but calculus deserves a place in the curriculum of educated people in many walks of life, not only as technical preparation for careers in math and the physical sciences. Here I suggest a method to improve reasoning skills, promote teamwork, and capture the interest of a broad spectrum of college students. Student projects can engage students in realistic problems they find interesting but, more importantly, they can help students synthesize and apply the knowledge gained by working template exercises and can send a message that the subject can solve real problems.

Most students at the University of Iowa take calculus for one semester or less (with AP credit), so I believe we should strive—in the first course—to really convince students that the subject speaks to their interests. A number of texts do this in different ways, such as Callahan et al. (1995), Brown et al. (1991), Smith & Moore (2010), Hilbert et al. (1994), Hughes-Hallett et al. (2002), McCallum et al. (2002)], Gaughan et al. (1992)], and Cohen (2009), but I believe courses often fall short of showing students how calculus might affect their lives. It is easy to get sidetracked by algebra or trig skills and boil the course down to template exercises. That ends up reinforcing students’ impression that math doesn’t solve real problems.”

See also Stroyan (2012a,b).

Stroyan, K. 2012a. *Calculus: The Language of Change (third edition)* “A new approach to calculus - available FREE as pdf files with accompanying Mathematica or Maple programs,” online at <http://bit.ly/MUIJ4m>.

According to Stroyan, Armstrong & Hendrix (1999) have shown that for 2286 Brigham Young University students taking calculus, those using *Calculus: The Language of Change* received higher average grades in later courses that those using The Harvard Consortium Calculus (HCC) text, or a traditional text.

Stroyan, K. 2012b. References to teaching articles, online at <http://bit.ly/1bXoemJ> [None are hot-linked **: -** (.]

Tall, D & S. Vinner. 1981. “Concept image and concept definition in mathematics, with special reference to limits and continuity,” *Educational Studies in Mathematics* **12:** 151–169; online as a 53 kB pdf at <http://bit.ly/11DlAOK>.]

Tall, D. 1990. “Inconsistencies in the Learning of Calculus and Analysis,” *Focus* **12**(3&4): 49–63; online as a 74 kB pdf at <http://bit.ly/IfernI>.

Tall, D. 1993. “Students’ Difficulties in Calculus,” in *Proceedings of Working Group 3 on Students’ Difficulties in Calculus,* ICME-7, Québec, Canada; online as a 176 kB pdf at <http://bit.ly/K9ZlTs>.

Tall, D. 2013a. “The Evolution of Technology and the Mathematics of Change and Variation: Using Human Perceptions and Emotions to Make Sense of Powerful Ideas,” in Hegedus & Roschelle (2013, pp. 449–561).

Tall, D. 2013b. Homepage, online at <http://bit.ly/14NaE0Y>.

Tall, D. 2013c. *How Humans Learn to Think Mathematically: Exploring the Three Worlds of Mathematics (Learning in Doing: Social, Cognitive and Computational Perspectives)*. 2013. Cambridge University Press, publisher's information at <http://bit.ly/1dZvwId>, note the "Look Inside" feature. Amazon.com information at <http://amzn.to/1d3WzNl>, note the searchable "Look Inside" feature.

Treisman, P. Uri. 1992. "Studying Students Studying Calculus: A Look at the Lives of Minority Mathematics Students in College," *College Math Journal* **23**(5): 362-373; online as a 1.5 MB pdf at <http://bit.ly/yzS1tw>. See also Fullilove & Treisman (1990).

Tuma, D.T. & F. Reif, eds. 1980. "Problem Solving and Education: Issues in Teaching and Research," Lawrence Erlbaum. Amazon.com information at <http://amzn.to/muxMCA>.

Watkins, J., & E. Mazur 2013. "Retaining students in science, technology, engineering and mathematics (STEM) majors," *Journal of College Science Teaching* **42**(5): 36–41; online as a 380 kB pdf at <http://bit.ly/1gXDEqt>.

West, B.H., E.N. Griesbach. J.D. Taylor, & L.T. Taylor. 1982. Prentice Hall. *Encyclopedia of Mathematics*. Amazon.com information at <http://amzn.to/170byFq>.

Wood, W.B., & J.M. Gentile. 2003. "Teaching in a research context," *Science* **302**: 1510; 28 November; online as a 209 kB pdf at <http://bit.ly/oK46p7>. Wood and Gentile wrote:

> "Physics educators have led the way in developing and using objective tests to compare student learning gains in different types of courses, and chemists, biologists, and others are now developing similar instruments. These tests provide convincing evidence that students assimilate new knowledge more effectively in courses including active, inquiry-based, and collaborative learning, assisted by information technology, than in traditional courses."

Woods, D.R. 2011. "Re: Interactive Engagement Typically Lowers Student Evaluations of Teaching?" STLHE-L post of 20 Mar 2011 16:23:36 -0400; online on the OPEN! STLHE-L archives at <http://bit.ly/gpVdGx>. Woods wrote:

> "Any new intervention to improve learning usually, in my experience, creates a drop in SETs. Students dislike change and usually respond via the SET. However, forewarned means that action can to taken to prevent the lower SET and indeed to result in higher SETs. At least that's my experience."

Similar pro-SET opinions have expressed by Felder (1992).

APPENDIX #2 – Math Education Bibliography (with some exceptions these references are (a) related to Calculus and (b) *not* among those in the ***REFERENCE*** list above.) This listing gives only a small window into the vast math education literature. For example, a Google <http://www.google.com/) search for "Math Education" (with the quotes) yielded 1.48 million hits at <http://bit.ly/1fPrQX9> on 21 Dec 13:10-0800.

Abbey, K.A. 2008. "Students' Understanding of Deriving Properties of a Function's Graph from the Sign Chart of the First Derivative," Master's Thesis, online as a 1.1 MB pdf at <http://bit.ly/1bPj1e1>.

Ahlgren, A. & M. Harper. 2011. "Assessment and Placement through Calculus I at the University of Illinois," Doceamus section, *Notices Of The AMS* **58**(10): 1460-1461; online as at 86 pdf at <http://bit.ly/MUj38Y>.

Alarcon, F.E., & R.E. Stoudt. 1997. "The rise and fall of a Mathematica-based calculus curriculum reform movement," *Primus* 7(1): 73-88. The first page is online at <http://bit.ly/189MFvU>.

Alexander, E.H. 1997. "An Investigation of the results of a change in calculus instruction at the University of Arizona," Unpublished doctoral dissertation, University of Arizona. An abstract is online at <http://bit.ly/1bbualX>. See also "Reform through Grade Analysis" [Alexander & Madden (1998)].

Alexander, E. & D.J. Madden. 1998. "Reform through Grade Analysis," *Focus on Calculus, A Newsletter for the Calculus Consortium Based at Harvard University*; online at <http://bit.ly/1cNiiv1>.

Aline R. & N. Speer. 2002. "Research on the Teaching and Learning of Calculus/Elementary Analysis," pages 283–300 in *Teaching and Learning of Mathematics at the University Level* [Holton et al (2001) at <http://bit.ly/KPPbWq>. An abstract is online at <http://bit.ly/Ir6l2r>.

Armstrong, G.M. & L.J. Hendrix. 1999. "Does Traditional or Reformed Calculus Prepare Students Better for Subsequent Courses? A Preliminary Study." *Journal of Computers in Mathematics and Science Teaching* **18**(2), 95-103. AACE. An ERIC abstract is online at <http://1.usa.gov/1kEKcKp>.

Arcavi, A., C. Kessel, L. Meira, & J. Smith. 1998. "Teaching mathematical problem solving: A microanalysis of an emergent classroom community," in *Research in Collegiate Mathematics Education III* (pp. 1–70) in Schoenfeld et al. (1998)]; online as a 250 kB pdf at <http://bit.ly/RmuEyg>.

Artigue, M. 1999. "The Teaching and Learning of Mathematics at the University Level: Crucial Questions for Contemporary Research in Education," *Notices of the AMS* **46**(11): 1377- 1385; online as a 94 kB pdf at <http://bit.ly/KJCVJ3>.

Artigue, M. 2000. "Teaching and learning calculus: What can be learned from education research and curricular changes in France?" in *Research in Collegiate Mathematics Education IV,* in Dubinsky et al. (2000)] at <http://bit.ly/MNQbwI>. An abstract is online at <http://bit.ly/18FNrfg>.

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Bonfert-Taylor, P., D.M. Bressoud, & H. Diamond. 2014. “Musings on MOOCs,” *Notices of the AMS **61***(1): 69-71; online as a 106 kB pdf at <http://bit.ly/1hDjt40>. This report begins:

> “MOOCs (massive open online courses) are causing a revolution in higher education today. What will be the impact of this revolution on mathematics teaching in colleges and universities? The *Notices* is hosting a discussion of MOOCs, which began in the November 2013 issue with the Opinion column “MOOCs and the future of mathematics” by Robert Ghrist (2013) of the University of Pennsylvania. The following three articles continue the discussion.”
>
> > Compare these musings with (a) "Is Higher Education Running AMOOC?" [Hake (2013b)]; (b) "The Darwinization of Higher Education" [Devlin (2012)]; and "MOOCs and the Myths of Dropout Rates and Certification" [Devlin (2013)].

Bonsangue, M.V. 1994 “An efficacy study of the calculus workshop model,” in *Research in Collegiate Mathematics Education I* in *Research in Collegiate Mathematics Education I* in [Dubinsky et al. (1994)] at <http://bit.ly/NWJisn>.

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Bookman, J. & C.P. Friedman. 1998. “Student Attitudes and Calculus Reform,” *School Science and Mathematics*, March 1998: 117-122. The abstract, online at <http://bit.ly/MUJAAq>, reads:

> “This paper compares the attitudes about mathematics of students from traditionally taught calculus classes and those from a ‘reformed’ calculus course. The paper is based on three studies, which together present a consistent picture of student attitudes about calculus reform. The reformed course appeared to violate students' deeply held beliefs about the nature of mathematics and how it should be learned. Although during their first months in the reformed course most students disliked it, their attitudes gradually changed. One and 2 years after, reform students felt significantly more than the traditionally taught students that they better understood how math was used and that they had been required to understand math rather than memorize formulas.”

Bookman, J. Publications of Jack Bookman, online at <http://bit.ly/1gylRbY>.

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Bransford, J.D., D.J. Stipek, N.J. Vye, L.M. Gomez, & D, Lam. 2009. “The Role of Research in Educational Improvement.” Harvard Education Press, publisher’s information at <http://bit.ly/1duvMtU>.Amazon.com information at <http://amzn.to/JPCC45>.

Bressoud, D.M. 1992. "Why do we teach calculus?," *American Mathematical Monthly* **99**(7): 615–617; online as a 131 kB pdf at <http://bit.ly/MI8ghY>. Bressoud wrote (paraphrasing):

"The chimera of a course in discrete mathematics to replace freshman calculus raised its head briefly in the early 1980's and drew forth defenders of the calculus. Ronald Douglas (1986), Daniel Kleitman, Peter Lax, Saunders MacLane,. . . . .[[Kleitman, Lax, and MacLane in responses to "Will discrete mathematics surpass calculus in importance?" (Ralston, 1984); Lax's comment is also reprinted in Douglas (1986)]]. . . . . and others have eloquently defended the necessity of placing calculus at the heart of the college mathematics curriculum. The issue seem settled, witness the Committee on the undergraduate Program in Mathematics [CUPM(1981)] report reprinted in *Reshaping College Mathematics* [Steen (1989)] - see <http://amzn.to/LnNJ0m>.

Bressoud, D.M. 1999. "The One-Minute Paper," in "Assessment Practices in Undergraduate Mathematics" [Gold, Keith, & Marion (1999, pp. 87-88), online at <http://bit.ly/1e6ShtL>. Bressoud wrote:

"The one-minute paper (described in Angelo and Cross, *Classroom Assessment Techniques*) is a quick and easy assessment tool that helps alert us when this disjuncture occurs, while it also gives the timid student an opportunity to ask questions and seek clarification."

Unfortunately Angelo and Cross fail to credit Berkeley physicist Charles Schwartz with the invention of Minute Papers – see, e.g., "Schwartz Invented Minute Papers" [Hake (2001b)].

Bressoud, D.M. 2004a, "The Changing Face of Calculus: First-Semester Calculus as a High School Course," *Focus*, August/September; online at <http://bit.ly/MA6zPB>.

Bressoud, D.M. 2004b. "The changing face of calculus: First- and second-semester calculus as college courses," *MAA FOCUS*, November; online at <http://bit.ly/14lhxCR>.

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Bressoud. D.M. 2010d. "Meeting the Challenge of High School Calculus, IV: Recent History," MAA Launchings, June; online at <http://bit.ly/15yEeTp>.

Bressoud. D.M. 2010e. "Meeting the Challenge of High School Calculus, V: Overcoming Ignorance" MAA Launchings, July; online at <http://bit.ly/14ljMGg>.

Bressoud. D.M. 2010f. "Meeting the Challenge of High School Calculus, V: The Need for Guidelines," MAA Launchings, August; online at <http://bit.ly/14dS8jL>.

Bressoud. D.M. 2010g. "Meeting The Challenge Of High School Calculus, VII: Our Responsibilities," MAA Launchings, September; online at <http://bit.ly/18xV3zs>.

Bressoud, D.M. 2010h. "The Problem of Persistence," MAA Launchings Column of January online at <http://bit.ly/1gaT6lr>.

Bressoud. D.M. 2010i. "The Common Core State Standards," MAA Launchings, October; online at <http://bit.ly/1iAEMo4>.

Bressoud. D.M. 2010j. "Historical Reflections on Teaching the Fundamental Theorem of Integral Calculus," online as a 942 kB pdf at <http://bit.ly/1kiGuca>. See also "Teaching the Fundamental Theorem of Calculus: A Historical Reflection" [Lopez & Rodriguez (2012)]

Bressoud, D. 2011a. "The Calculus I Student," in the MAA Launchings Column of May & December; online at <http://bit.ly/1bIXbeD>. Bressoud wrote:

> "I am distressed by how poorly these students do in Calculus I: Over a quarter essentially fail, and only half earn the A or B that is the signal that they are likely to succeed in further mathematics. I know the frustration of high school teachers who see what they consider to be the best and brightest of their students run into mathematical roadblocks in college. I recognize that much of the fault lies on the high school side of the transition. Many students who consider themselves well prepared for college mathematics in fact are not. We need to do a better job of communicating what these students really need and working with their teachers so that they can provide it. I also know that we in the colleges and universities can do a better job of supporting these students after they have arrived on our campuses, moving them forward with challenging and engaging mathematics while bringing them up to the level they need to be at to succeed."

Bressoud, D. 2011b. "The Calculus I Instructor," in the MAA Launchings Column of June; online at <http://bit.ly/IOTOXA>. Bressoud wrote:

> "The emphasis in exams is on computational technique, but almost all instructors have some points devoted to graphical interpretation of central ideas, and most include some complex or unfamiliar problems as well as proofs or justifications. Most instructors see themselves as fairly traditional. *They view lecture as the best way to teach students* and believe that procedural fluency precedes conceptual understanding." [My *italics*.]

Bressoud, D. 2011c. MAA Launchings Blog; blog entries from "Introduction of Feb 2005 to "The Worst Way to Teach" of July 2011; online at <http://bit.ly/18mBMls>.

Bressoud, D.M. 2011d. “The Worst Way to Teach,” in the MAA Launchings Column of 1 July; online at <http://bit.ly/18amy8x>. Bressoud wrote:

“This common belief . . . .[[in the efficacy of lectures]]. . . is also contradicted by the evidence that we have, the most recent and dramatic of which is 'Improved Learning in a Large-Enrollment Physics Class' [Deslauriers et al. (2011)] from the Carl Wieman Science Education Initiative (CWSEI) at the University of British Columbia (UBC). The CWSEI study compared lecture format with interactive, clicker-based peer instruction in two large (267 and 271 students, respectively) sections of introductory physics for engineering majors. The results were published in *Science* and made enough of a splash that they were reported by Carey (2011) in *The New York Times*, Mervis (2011) in *ScienceNOW*, and by *The Economist* (2011). What is most impressive is how well controlled the study was - ensuring that the two classes really were comparable - and how strong the outcome was: The clicker-based peer instruction class performed 2.5 standard deviations above the control group.”

But see my comments in “Physics Education Research (PER) Could Use More PR”[Hake (2011b) and “Re: Lecture Isn’t Effective: More Evidence” [Hake (2011a)].

Bressoud, D. 2011e. “The Best Way to Learn,” in the MAA Launchings Column of 1 August; online at <http://bit.ly/1bQ4D6j >. Bressoud wrote:

“Last month, in ‘The Worst Way to Teach’ [Bressaud (2011a)] I wrote about some of the problems with instruction delivered by lecture. It stirred up a fair amount of discussion. Richard Hake (2011f) started a thread on the Math Forum list. . . . .[[more specifically the MathEdCC list]]. . . . He added several references to my own list and sparked a discussion that produced some heat and a lot of light. I do want to clarify that I recognize how important what I say in the classroom can be, as I will expound a bit later in this column. Nevertheless, I stand by my statement that ‘sitting still, listening to someone talk, and attempting to transcribe what they have said into a notebook is a very poor substitute for actively engaging with the material at hand, for doing mathematics.’ ”

Bressoud. D.M. 2011f. “The Greatest Problems Facing Math Departments,” MAA Launchings Column of September; online at <http://bit.ly/1bh6DTQ>.

Bressoud. D.M. 2011g. “Quantitative Literacy versus Mathematics,” MAA Launchings Column of October; online at <http://bit.ly/18DktCp>.

Bressoud, D. 2011h. MAA Launchings Column; blog entries from “Introduction” of Feb 2005 to “The Worst Way to Teach” of July 2011; online at <http://bit.ly/18mBMls>.

Bressoud, D. 2012. “On Engaging to Excel,” *MAA Launchings*, 1 March; online at <http://bit.ly/GWTX80>. Bressoud wrote:

“The President’s Council of Advisors on Science and Technology (PCAST) has just released its report to President Obama on undergraduate Science, Technology, Engineering, and Mathematics (STEM) education: *Report to the President, Engage to Excel: producing one million additional college graduates with degrees in Science, Technology, Engineering, and Mathematics*, online as a 3 MB pdf at <http://1.usa.gov/GZmbzq>.”

Bressoud, D. 2013a. “MAA Calculus Study: Good Teaching” in the MAA Launchings Column of 1 March, online at <http://bit.ly/1eTA67o>.

Bressoud, D. 2013b. “Evidence of Improved Teaching,” *MAA Launchings*, 1 Oct; online at <http://bit.ly/191FtNw>.

Bressoud, D. 2013c. “An International Comparison of Adult Numeracy,” in the MAA Launchings Column of 1 November; online at <http://bit.ly/1it9gIM>.

Bressoud, D. 2013d. “MAA Calculus Study: Persistence through Calculus,” in the MAA Launchings Column of 1 December; online at <http://bit.ly/1eIhs21>.

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Budd, K., E. Carson, B. Garelick, D. Klein, R.J. Milgram, R. A. Raimi, M. Schwartz, S. Stotsky, V. Williams, & W.S. Wilson. 2005. “Ten Myths About Math Education And Why You Shouldn't Believe Them,” online at <http://bit.ly/M95386>.

Burmeister, S.L., P.A. Kenney, and D.L. Nice. 1996. “Analysis of effectiveness of supplemental instruction (SI) sessions for college algebra, calculus, and statistics,” in *Research in Collegiate Mathematics Education II*. in *Research in Collegiate Mathematics Education II* [Kaput et al. (1996, pp. 1-32).

Cargal, J.M. 1997. “The reform calculus debate and the psychology of learning mathematics,” *Humanistic Mathematics Network Journal **16***: 26-29. An ERIC abstract online at <http://1.usa.gov/OeqSn4> reads:

> “Highlights a debate concerning reform calculus between Professor Thomas W. Tucker on the reform side and Professor Howard Swann on the traditional side.”

Chappell. K.K. & K. Killpatrick. 2003. “Effects of concept-based instruction on students' conceptual understanding and procedural knowledge of calculus,” *Primus* **13**(1): 17-37. An abstract, online at <http://bit.ly/QO9JE3> reads:

> “An original study, involving 305 college-level calculus students and 8 instructors, and its replication study, conducted at the same university and involving 303 college-level calculus students and 8 instructors, investigated the effects of instructional environment (concept-based vs. procedure-based) on students' conceptual understanding and procedural knowledge of calculus. Multiple achievement measures were used to determine the degree to which students from different instructional environments had mastered the concepts and the procedures inherent to first semester calculus. Achievement measures included two skills examinations designed to evaluate procedural competence and a midterm examination and a final examination both designed to evaluate procedural skills as well as conceptual understanding. The learning environment resulted in no significant differences in the students' abilities to employ procedural skills, as measured by the skills examinations. The students enrolled in the concept-based learning environment scored significantly higher then the students enrolled in the procedure-based learning environment on assessments that measured conceptual understanding as well as procedural skills ($p < .001$). The results of the replication study are consistent with the results of the original study, increasing the generalizability of the results. These results provide post-secondary level evidence that concept-based instructional programs can effectively foster the development of student understanding without sacrificing skill proficiency.”

Chappell, K.K. 2006. "Effects of Concept-Based Instruction on Calculus Students’ Acquisition of Conceptual Understanding and Procedural Skill,” in *Research in Collegiate Mathematics Education VI* in *Research in Collegiate Mathematics Education VI* [Seldon et al. (2006)] at <http://bit.ly/Ne91NS>.

Cipra, B.A. 1987. “Recent Innovations in Calculus Instruction,” in *Calculus for a New Century: A Pump, Not a Filter* [Steen (1987)] at <http://bit.ly/1c8cvlr> (5.2 MB).

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Cipra, B.A. 1995. "The Bumpy Road to Reform" *UME Trends* **6**(6): 16,19. I’ve not found an online version. *UME Trends* evidently died without leaving a trace that I can find on the internet.

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Coppin, C.A. W.T. Mahavier, E.L. May, & E. Parker. 2009. *The Moore Method — A Pathway to Learner-Centered Instruction*. MAA. Amazon.com information at <http://amzn.to/KOa5fo>, note the searchable “Look Inside” feature. Publisher’s information at <http://bit.ly/LEdug3>:

> “[This book] offers a practical overview of the method as practiced by the four co-authors, serving as both a ‘how to’ manual for implementing the method and an answer to the question, ‘what is the Moore method?’. Moore is well known as creator of The Moore Method (no textbooks, no lectures, no conferring) in which there is a current and growing revival of interest and modified application under inquiry-based learning projects. . . .[[see the Wikepedia entry at <http://bit.ly/LElQzB> and the “Legacy of R.L. Moore” site at <http://bit.ly/NbapQi>.]]. . . . . . Beginning with Moore's Method as practiced by Moore himself, the authors proceed to present their own broader definitions of the method before addressing specific details and mechanics of their individual implementations. Each chapter consists of four essays, one by each author, introduced with the commonality of the authors' writings. Topics include the culture the authors strive to establish in the classroom, their grading methods, the development of materials and typical days in the classroom. Appendices include sample tests, sample notes, and diaries of individual courses. With more than 130 references supporting the themes of the book the work provides ample additional reading supporting the transition to learner-centered methods of instruction.”
>
> Paul Halmos <http://bit.ly/Jb2hE5> endorsed the Moore Method:
>
> > “Some say that the only possible effect of the Moore method is to produce research mathematicians - but I don't agree. The Moore method is, I am convinced, the right way to teach anything and everything — it produces students who can understand and use what they have learned. It does, to be sure, instill the research attitude in the student — the attitude of questioning everything and wanting to learn answers actively — but that's a good thing in every human endeavor, not only in mathematical research.”
> >
> > - Paul R. Halmos, Santa Clara University

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DeLange, J. 2007. "Large-scale Assessment of Mathematics Education" in *Second Handbook of Research on Mathematics Teaching and Learning* [Lester (2007)] at <http://bit.ly/NgiHfk>.

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Devlin, K. 2006. "Letter to a calculus student." Devlin's Angle; online at <http://bit.ly/196e2bf>. Devlin wrote:

> "Let me begin with a quotation from the great philosopher Bertrand Russell. He wrote, in *Mysticism and Logic* (1918): 'Mathematics, rightly viewed, possesses not only truth, but supreme beauty-a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.' . . . . The beauty of calculus is primarily one of ideas. And there is no more beautiful idea in calculus than the formula for the definition of the derivative f'(x) = lim_{h -> 0} [f(x+h) - f(x)]/h . For this to make sense, it is important that h is not equal to zero. For if you allow h to be zero, then the quotient in the above formula becomes [f(x+0) - f(x)]/0 = [f(x) - f(x)]/0 = 0/0 and 0/0 is undefined. Yet, if you take any nonzero value of h, no matter how small, the quotient [f(x+h) - f(x)]/h will not (in general) be the derivative. So what exactly is h? The answer is, it's not a number, nor is it a symbol used to denote some unknown number. It's a variable."

Devlin, K. 2006. *The Language of Mathematics: Making the Invisible Visible*. Holt. Amazon.com information at <http://amzn.to/MQ9RPn>.

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Devlin, K. 2012. "The Darwinization of Higher Education," MAA, Devlin's Angle, 4 Dec.; online at <http://bit.ly/14440kt>." Mathematician Keith Devlin is the Executive Director of the Human-Sciences and Technologies Advanced Research Institute (H-STAR) <http://stanford.io/15NEBzI> on NPR's "Weekend Edition." See also Devlin (2013a,b).

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> "*Ronald Douglas (the 'father' of Calculus Reform*), received funding from the Sloan Foundation for a conference on calculus reform to "*Develop Curriculum and Teaching Methods for Calculus at the College Level*." The conference was held at Tulane University in January of 1986, and is *often credited with being the birthplace of calculus reform*. The conference was to the reform movement what Euclid's The Elements was to mathematics. The Tulane conference served as a focal point for the reform movement which started in the early eighties via many different sources. The focus of the conference was overhauling both the content and pedagogy of calculus. The result was a report, "*Toward a Lean and Lively Calculus*" [Douglas (1986c)] Highlights of the conference:
>
> (a) a workshop created annotated syllabi for a general, "lean" two-semester sequence in single-variable calculus;
> (b) another workshop established pedagogical techniques to achieve the goals, which included the use of computers;
> (c) the team also discussed how to widely implement the new instruction.

Douglas, R.G. 1986b. "The Importance of Calculus in Core Mathematics," *Journal of College Science Teaching* **15**: 250-251, 397; reprinted in *Toward a Lean Lively Calculus* [Douglas (1986c)].

Douglas, R.G.. ed. 1986c. *Toward a Lean and Lively Calculus: Report of the Conference/Workshop to Develop Curriculum and Teaching Methods for Calculus At the College Level*. MAA. The first page is online at <http://bit.ly/KKrgrq>. The first paragraph reads:

> "Deep concerns in mathematical education have converged, like currents in the ocean, to generate both a certain amount of froth and a strong force for reform of calculus instruction. First there are the dual concerns of adapting to the needs of a burgeoning number of computer science majors and of making use of new technologies in microcomputers and hand-held calculators. Second is alarm at the decline of the presentation of calculus into an arcane study of detailed techniques of differentiation, integration, and tests for convergence of series, with artificial set piece problems that may be checked by making sure the answer is simple. Students see little of the towering intellectual achievement of the subject, and they cannot see how to formulate physical problems of change and constancy as mathematical ones involving differentiation and integration. Moreover, even many of the best students remain unable to unite English expressions and mathematical symbolism in a single coherent sentence, much less in an acceptable student paper on a mathematical subject."

Douglas, R.G. 1987. “Castles in the Sand,” in “*Calculus Problems for a New Century*,” *MAA Notes* **8**: 4-5 - see <http://amzn.to/LybEK3>.

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Douglas, R.G. 2000. *Calculus Renewal: Issues for Undergraduate Mathematics Education in the Next Decade*, Springer, publisher’s information at <http://bit.ly/YtcYSt>. Amazon.com information at <http://amzn.to/MBz0g5>, note the searchable “Look Inside” feature.

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Dubinsky, E. & K.E. Schwingendorf. 1990. “Calculus, Concepts and Computers - Innovations in Learning Calculus, in Tucker (1990)

Dubinsky, E. 1992. “A Learning Theory Approach to Calculus,” in Karian (1992, pp. 48-55)).

Dubinsky, E. A.H. Schoenfeld, & J. Kaput, eds. 1994. *Research in Collegiate Mathematics Education I*. AMS & CBMS, publisher’s information at <http://bit.ly/NWJisn>. Amazon.com information at <http://amzn.to/OX2BXE>, note the searchable “Look Inside” feature. [Also referenced as *CBMS Issues in Mathematics Education*, vol. 4].

Dubinsky, E., K.E. Schwingendorf, & D.M. Mathews. 1994. *Calculus, Concepts, and Computers*. MAA. Pubisher’s information at <http://bit.ly/NWJisn>. Amazon.com information at <http://amzn.to/HWA8iT>.

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Dubinsky, E., A.H. Schoenfeld, & J. Kaput, eds. 2000. *Research in Collegiate Mathematics Education IV*. AMS & CBMS, publisher's information at <http://bit.ly/MNQbwI>. Amazon.com information at <http://amzn.to/OcS3lM>, note the searchable "Look Inside" feature. [Also titled *CBMS Issues in Mathematics Education*, vol. 8].

Dubinsky, E. & M. McDonald. 2002. "APOS: A Constructivist Theory of Learning In Undergraduate Mathematics Education Research," online as a 78 kB pdf at <http://bit.ly/196l5Rs>.

Dubinsky, E. 2013. Homepage, online at <http://bit.ly/18ckdYu>.

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English, L.D. 2008. *Handbook of International Research in Mathematics Education*. Routledge, 2 edition, publisher's information at <http://bit.ly/M7AqlX>. Amazon.com information at <http://amzn.to/LMx2Mw>.

Felder, R.M. 1992. "What Do They Know Anyway?" *Chem. Engr. Education* **26**(3): 134-135 online at <http://bit.ly/1drhBpE>.

Ferrini-Mundy, J. & K.G. Graham. 1991. "An Overview of the Calculus Curriculum Reform Effort: Issues for Learning, Teaching, and Curriculum Development," *American Mathematical Monthly* **98** (7): 627-635; the first paragraph is online at <http://bit.ly/1erhawp>. An ERIC abstract is online at <http://1.usa.gov/196ebpe>.

Ferrini-Mundy, J. YouTube video "Joan Ferrini-Mundy on STEM Education Research Areas," online at <http://bit.ly/IOcTca>, 315 views as of 18 Dec 16:24-0800.

Fisher, N.D., H.B. Keynes, & P.D. Wagreich, eds.1990. *Mathematicians and Education Reform, Proceedings of the July 6-8, 1988, Workshop*. A Google book preview is online at <http://bit.ly/196l5Rs>.

Fisher, N.D., H.B. Keynes, & P.D. Wagreich, eds. 1991. *Mathematicians and Education Reform 1989-1990*. AMS. Amazon.com information at <http://amzn.to/Nygx4m>, note the searchable "Look Inside" feature. [Also referenced as *CBMS Issues in Mathematics Education, vol. 2*.] A Google book preview is online at <http://bit.ly/196l5Rs>.

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Frid, S. 1994. "Three approaches to undergraduate calculus instruction: Their nature and potential impact on students' language use and sources of conviction," in *Research in Collegiate Mathematics Education I* in [Dubinsky et al. (1994)] at <http://bit.ly/NWJisn>.

Gaughan, E.D., D.J. Pengelley, A. Knoebel, & D. Kurtz, *Student Research Projects in Calculus* (Spectrum Series), MAA, 1992. Amazon.com information at <http://amzn.to/MSB05e>, note the searchable "Look Inside" feature. [Recommended by Stroyan (2011) as speaking to students' interests.]

Gavosto, E.A., S.G. Krantz, & W. McCallum, eds. 1999. *Contemporary Issues in Mathematics Education*. Cambridge University Press, publisher's information at <http://bit.ly/1gw1zNr>. Amazon.com information at <http://amzn.to/IzsM5s>, note the searchable "Look Inside" feature.

George, M.D. 2000. "Calculus Renewal in the Context of Undergraduate SMET," Chapter 1 in Ganter (2000). The first two pages are online at http://bit.ly/1hcEEad>.

Ghrist, R. 2013. "MOOCs and the Future of Mathematics," *Notices of the AMS* **60**(1): 1277, November, online as a 45 kB pdf at <http://bit.ly/18JAo27>. Ghrist wrote:

"In 2012 I created a calculus MOOC (massive open online course) for Penn's partnership with Coursera. The course ran in the spring of 2013 and again in summer 2013. You are welcome to view the materials for this course (and other free courses) on Coursera's website. Contrary to apocalyptic fears, this is not the end of calculus instruction. Rather, this and other open-access online courses herald a time of experimentation and rapid improvement in how we communicate mathematics to the world. . . . .. The rise of MOOCs is our profession's moment of opportunity to communicate its truths skillfully and artfully and to promote our core insights and modes of thought to an eager worldwide audience through this medium of rapid innovation."

Compare Grist's assessment of MOOCs with (a) "Is Higher Education Running AMOOC?" [Hake (2013b)]; (b) "The Darwinization of Higher Education" [Devlin (2012)]; and "MOOCs and the Myths of Dropout Rates and Certification" [Devlin (2013)].

Gleason, A.M., & D. Hughes-Hallett. 1992. "Introducing the calculus consortium based at Harvard University." *Focus On Calculus Newsletter*; online as a 168 kB pdf at <http://bit.ly/12tZbNQ>.

Goodfriend, J. 2006. *Calculus: A gateway to higher mathematics*. Jones and Bartlett. Referenced by Windham (2011) but I've found no other reference on the internet.

Gold, B., S.Z. Keith, & W.A. Marion, eds. 1999. *Assessment Practices in Undergraduate Mathematics,* Publisher's information at <http://bit.ly/1hNLupA>. Online as a 1.4 MB pdf at <http://bit.ly/1e6ShtL>. Amazon.com information at <http://amzn.to/LZa7RE>, note the searchable "Look Inside" feature. The preface reads:

" This book grew out of a report by a subcommittee of the Committee on the Undergraduate Program in Mathematics, chaired by Bernie Madison, and out of a concern by Sandy Keith that mathematicians take assessment seriously. The introductions by Lynn Steen (1999) and Bernie Madison (1999) set the context more fully."

Green, K.H. 2002. "Creating Successful Calculus Writing Assignments." Fsher Digital Publications, online at <http://bit.ly/1fdPV91>.

Bonfert-Taylor, P., D.M. Bressoud, & H. Diamond. 2014. "Musings on MOOCs," *Notices of the AMS* ***61***(1): 69-71; online as a 106 kB pdf at <http://bit.ly/1hDjt40>. This report begins:

> "MOOCs (massive open online courses) are causing a revolution in higher education today. What will be the impact of this revolution on mathematics teaching in colleges and universities? The *Notices* is hosting a discussion of MOOCs, which began in the November 2013 issue with the Opinion column ''MOOCs and the future of mathematics' by Robert Ghrist (2013) of the University of Pennsylvania. The following three articles continue the discussion.

Devlin, K. 2012. "The Darwinization of Higher Education," MAA, *Devlin*'s Angle, 4 Dec.; online at <http://bit.ly/14440kt>."Mathematician Keith *Devlin* is the Executive Director of the Human-Sciences and Technologies Advanced Research Institute (H-STAR) <http://stanford.io/15NEBzI> on NPR's "Weekend Edition." See also Devlin (2013).

Devlin, K. 2013a. "MOOCs and the Myths of Dropout Rates and Certification," Huffington Post, 02 March; online at <http://huff.to/16IFdWP>.

Devlin, K. 2013b. "MOOCtalk" blog, online at <http://mooctalk.org/>.

Gallistel, C.R. & R. Gelman. 2005. "Mathematical Cognition," in *The Cambridge Handbook of Thinking and Reasoning* [Holyoak & Morrison (2005, pp 559-588)].

Gleick. J. 1987. "New Directions; The Hand-Held Calculus." New York Times, 08 Nov.; online at <http://nyti.ms/1fh8rgO>.

Gonick, L. 2011. *The Cartoon Guide to Calculus*. William Morrow Paperbacks, publisher's information at <http://bit.ly/LhO9Fp>. Amazon.com information at <http://amzn.to/LGpVaO>.

Graham, K. G., and J. Ferrini-Mundy. 1989. "An Exploration of Student Understanding of Central Concepts in Calculus," paper presented at the Annual Meeting of the American Educational Research Association. I've not found an online version.

Gray, S.B, Venit, & R. Abbott. 2012. "National Curve Bank: Math on the Web"; online at <http://bit.ly/LATtU1>.

Grouws, D.A., ed. 1992. *Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics*. Macmillan Library Reference. Amazon.com information at < http://amzn.to/N3dr8k>. See also Grouws (2006).

Grouws, D.A., ed. 2006. *Handbook of Research on Mathematics Teaching and Learning*, Information Age.Amazon.com information at <http://amzn.to/KO0GxO>. This is evidently a paper back version of Grouws (1992).

Hake, R.R. 1998. "A Microcomputer-Based SDI Lab Emphasizing the Graphical Interpretation of the Derivative and Integral," *AAPT Announcer* **28**(2). This is the Socratic Dialogue Inducing (SDI) lab #0.2, "Introduction to Kinematics" online along with other SDI labs at <http://bit.ly/9nGd3M>.

Hake, R.R. 2001. "Schwartz Invented Minute Papers,"online on the OPEN AERA-J archives at <http://bit.ly/19451zC>. Post of 5 May 2001 19:59:40 -0700 to AERA-J and various other discussion lists.

Hake, R.R. 2002a. “Physics First: Opening Battle in the War on Science/Math Illiteracy,” submitted to the *American Journal of Physics* on 27 June 2002; online as a 220 kB pdf at <http://bit.ly/bEEwBa>. Contains cartoon versions of Leon Lederman's 9th grade physics *cliff* and Ken Ford's K-12 science/math *ramp*.

Hake, R.R. 2002b. “Physics First: Precursor to Science/Math Literacy for All?” Summer 2002 issue of the American Physical Society *Forum on Education Newsletter*; online at <http://bit.ly/JzlVrH> and as ref. 19 at <http://bit.ly/a6M5y0>.

Hake, R.R. 2002c. “Whence Do We Get the Teachers? (Response to Madison, 2002), PKAL Roundtable on the Future: Assessment in the Service of Student Learning, Duke University, March 1-3; online at <http://bit.ly/w9M6dc>.

Hake, R.R. 2003. "Re: Math Education Research," online on the CLOSED! PhysLrnR archives at <http://bit.ly/LKpg4G>. To access PhysLrnR posts see Note #3 under ***REFERENCES***. Post of 12 Feb 2003 21:01:38-0800. [NOTE – this post is also on OPEN! Math-Teach archives at <http://bit.ly/Ir3MN1>, but most of the URL’s have been obliterated :-(.]

Hake, R.R. 2011a. “Re: Lecture Isn’t Effective: More Evidence,” online on the OPEN! MathEdCC archives at <http://bit.ly/r80W5i> along with 12 responses (as of 12 Jan 2012). Post of 15 July, shamelessly cross-posted to Math-Teach, Math-Learn, MathEdCC, and RUME.

Hake, R.R. 2011b. “Physics Education Research (PER) Could Use More PR,” online on the OPEN! AERA-L archives at <http://bit.ly/uQ7X5U> Post of 7 Dec 2011 13:45:18-0800 to AERA-L and Net-Gold. The abstract and link to the complete post are being transmitted to several discussion lists and are also on my blog “Hake’sEdStuff” at <http://bit.ly/uLyLzz> with a provision for comments.

Hake, R.R. 2011c. “SETs Are Not Valid Gauges of Students' Higher-Level Learning #2,” online on the OPEN! AERA-L archives at <http://bit.ly/jLZaz5>. Post of 17 May 2011 09:47:36-0700 to AERA-L and Net-Gold. The abstract and link to the complete post were distributed to various discussion lists and are also on my blog “Hake'sEdStuff? at <http://bit.ly/ixcQxs>. Stimulated by Don Woods’ (2011) statement that SETs could be used for measuring teaching “productivity.”

Hake, R.R. 2012. "Engage To Excel: Producing One Million Additional College Graduates with STEM Degrees," online on the OPEN! AERA-L archives at <http://bit.ly/H0d2BE>. Post of 29 Mar 2012 15:19:02-0700 to AERA-L and Net-Gold. The abstract and link to the complete post are also being transmitted to several discussion lists and are on my blog “Hake'sEdStuff” at <http://bit.ly/GZazvO> with a provision for comments.

Hake, R.R. 2013a. “*Next Generation Science Standards*: Good or Bad for Science Education?” 18 March, online as a 221 kB pdf at <http://bit.ly/147K6qY> and as ref. 68 at <http://bit.ly/a6M5y0>. Regarding the Common Core State Standards (CCSS), compare this with (a) “The Common Core State Standards” [Bressoud (2010i)]; (b) “The future of high school math education” [Strauss (2013); (c) “Mathematics and Education” [Kessel (2013)]; (d) “Engaging students in mathematics” [McCallum (2013)] in ***REFERENCES***]; and (e) “Why I Cannot Support the Common Core Standards” [Ravitch (2013a)]; and (f) “Study supports move toward common math standards” [Schmidt (2012b)].

Hake, R.R. 2013b. "Is Higher Education Running AMOOC?" online on the OPEN! AERA-L archives at <http://bit.ly/J7mOcX>. Post of 19 May 2013 18:47:06-0700 to AERA-L and Net-Gold. The abstract and link to the complete post are also on my blog "Hake'sEdStuff" at <http://bit.ly/18YJjt9>.

Hake, R.R. 2013c."Hake'sEdStuff" blog; online at <http://bit.ly/9yGsXh>.

Halmos, P.R. 1988. *I Want to Be a Mathematician: An Automathography in Three Parts*. MAA, publisher's information at <http://bit.ly/KEnTIL>. Amazon.com information at <http://amzn.to/oImPVB>.

Halpern, D.F. 1992. *Enhancing Thinking Skills in the Sciences and Mathematics*. Routledge. Amazon.com information at <http://amzn.to/1c5Phwf>. An expurgated Google book preview is online at <http://bit.ly/1fyGbpW>. According to Wikipedia <http://bit.ly/JMdE6s> Diane Halpern "is an American psychologist and past-president of the American Psychological Association (APA). . . . She is currently Professor of Psychology and Director of the Berger Institute for Work, Family, and Children at Claremont McKenna College."

Hastings, N.B. & B.E. Reynolds. 1999. *Workshop Calculus with Graphing Calculators: Guided Exploration with Review*. Springer, vol. 1, publisher's information at <http://bit.ly/KBizzy>, note the "Read Online" feature. Contributing authors: Christa Fratto, Priscilla Laws, Kevin Callahan, & Mark Bottorff.

Hastings, N.B., C. Fratto, P. Laws, K. Callahan, & M. Bottorff. 1999. *Workshop Calculus with Graphing Calculators: Guided Exploration with Review*. Springer, vol. 2, publisher's information at <http://bit.ly/JZq0AV>. Amazon.com information at <http://amzn.to/L3iBX2>, note the "Look Inside" feature.

Hastings, N.B. 2006. *A Fresh Start for Collegiate Mathematics*, Mathematical Association of America, publisher's information at <http://bit.ly/LIVIGp>. Amazon.com information at <http://amzn.to/LgrICh>.

Hastings, N.B. 2008. *Workshop Precalculus: Discovery with Graphing Calculators*. Wiley. Amazon.com information <http://amzn.to/LN8NxP>, note the searchable "Look Inside" feature.

Hastings, N.B. 2009. "Workshop Calculus: Assessing Student Attitudes and Learning Gains," in [Gold et al. (1999), pp. 241-244; online as a 1.4 MB pdf at <http://bit.ly/1e6ShtL>.

Hegedus, S. 2013. "Young Children Investigating Advanced Mathematical Concepts With Haptic Technologies: Future Design Perspectives," Mathematical Enthusiast 10(1&2): 87-108; online at <http://bit.ly/1exUKxB>.

Heggen, J. 2008. "Report Critical of Math Teachers' Preparation" *Inside Higher Ed*. 30 June; online at <http://bit.ly/IMhdIn>.

Heid, K.M. 1988). "Resequencing Skills and Concepts in Applied Calculus Using the Computer as a Tool," *Journal for Research in Mathematics Education* **19**(1): 3-25. The first page is online at <http://bit.ly/19t0WTu>.

Herzig, A. & D.T. Kung. 2003. "Cooperative learning in calculus reform: What have we learned?" in *Research in Collegiate Mathematics Education V* [Selden et al. 2003] at <http://bit.ly/KOquKC>.

Hiebert, J.S. & D.A. Grouws. 2007. "The Effects of Classroom Mathematics Teaching on Students' Learning," in in *Second Handbook of Research on Mathematics Teaching and Learning* [Lester (2007)] at <http://bit.ly/NgiHfk>.

Hilbert, S., J. Maceli, E. Robinson, D.D. Schwartz, and S. Seltzer. 1994. *Calculus, An Active Approach with Projects*, Wiley [Recommended by Stroyan (2011) as speaking to students' interests.] Amazon.com information at <http://amzn.to/JZBrbY>. The publisher states:

"This volume contains student and instructor material for the delivery of a two-semester calculus sequence at the undergraduate level. It can be used in conjunction with any textbook. It was written with the view that students who are actively involved inside and outside the classroom are more likely to succeed, develop deeper conceptual understanding, and retain knowledge, than students who are passive recipients of information."

Hillel, J. 2001. "Trends In Curriculum: Working Group Report," pages 59-69 in *Teaching and Learning of Mathematics at the University Level* [Holton et al (2001) at <http://bit.ly/KPPbWq>. Hillel wrote:

"There is a general trend towards reducing the mathematical content of courses, both for programme and client students. This is, in part, due to client disciplines reducing the amount of mathematics required by their students. Coupled with this reduction of content there is also a trend towards undergraduate teaching which is less formal, more open-ended and which tries to build on students' intuitions, visualizations, and experimentation. This trend is perhaps best exemplified by the 'reform calculus' movement that offers an alternative approach to the teaching of the beginning calculus sequence. Though the approach is not without its detractors, it has been adopted by a good number of institutions in North America, South Africa, Australia, and other countries as well."

Hitt, F., D. Holton, & P.W. Thompson, eds. 2010. *Research in Collegiate Mathematics Education VII* – AMS & CBMS, publisher's information at <http://bit.ly/KOn6zx>. Amazon.com information at <http://amzn.to/Mos6R4>, note the searchable "Look Inside" feature. [Also titled *CBMS Issues in Mathematics Education*, vol. 16].

Holton, D., M. Artigue, U. Kirchgraber, J. Hillel, M. Niss, & A. H. Schoenfeld, eds. 2001. *The Teaching and Learning of Mathematics at the University Level*, New ICMI Study Series, volume 7, Kluwer Academic Publishers; (ICME = International Commission on Mathematical Instruction). Publisher's information at <http://bit.ly/KPPbWq> where it's stated:

"This book arose from the ICMI Study into the teaching and learning of mathematics at university level that began with a conference in Singapore in 1998. The book looks at tertiary mathematics and its teaching from a number of aspects including practice, research, mathematics and other disciplines, technology, assessment, and teacher education. Over 50 authors, all international experts in their field, combined to produce a text that contains the latest in thinking and the best in practice. It therefore provides in one book a state-of-the-art statement on tertiary teaching from a multi-perspective standpoint. No previous book has attempted to take such a wide view of the topic." Note the "Read Online" (for a price) feature and the *free* downloads at <http://bit.ly/KPPbWq>.

Holyoak, K. & R. Morrison, eds. 2005. *The Cambridge Handbook of Thinking and Reasoning*. Cambridge University Press. Amazon.com information at <http://amzn.to/19kReVq>, note the searchable "Look Inside" feature. The entire book is online as a 9MB pdf at <http://bit.ly/J7ynku>.

Hsaio, F. S. 1984/85. "A New CAI Approach to Teaching Calculus" in *Computers in Mathematics and Science Teaching* **4** (2): 29-36. An ERIC abstract is online at <http://1.usa.gov/Jz3PIZ>.

Hughes-Hallett, D. 2013. Free PDF Downloads, online at <http://bit.ly/1c5T10y>.

Jackson, M.B. & J.R. Ramsay, eds. 1993. Resources for Calculus Collection, vol. 4: *Problems for Student Investigation*. MAA. Amazon.com information at <http://amzn.to/MURQn5>.

Jackson, A. 2003. "Presidential Views: Interview with Hyman Bass," *Notices of the AMS* **50**(2): 232-234; online as a 90 kB pdf at <http://bit.ly/MT4Yeh>.

Judson, P. 2001. "High school calculus in the U.S. and Japan." November, online at <http://bit.ly/z13oSx>.

Kaput, J., A.H. Schoenfeld, & E. Dubinsky, eds. 1996. *Research in Collegiate Mathematics Education II*. AMS & CBMS, publisher's information at <http://bit.ly/NWFHKU>. Amazon.com information at <http://amzn.to/NeAwXD>, note the searchable "Look Inside" feature. [Also titled as *CBMS Issues in Mathematics Education*, vol. 6].

Kasten, M. & Others. 1988. "The Role of Calculus in College Mathematics," ERIC/SMEAC Mathematics Education Digest No. 1; online at <http://bit.ly/OEtI6V>. They write:

> "While there is some agreement regarding the breadth and conceptual orientation of a desirable calculus course, there is evidence to suggest that the calculus that is actually taught is 'the moral equivalent of long division.' An examination of final examination questions in collegiate calculus courses (Steen, 1987) revealed that 90 percent of the items focused on calculation and only 10 percent on higher order challenges. Steen suggests that the curriculum of collegiate calculus has changed dramatically in the last two or three decades and that the change has not been a good one. He feels that the movement has been away from conceptual understanding about the nature of calculus and toward more 'plug and crank' exercises, with undue emphasis on computation and manipulative skills. Whether or not one accepts this view, it is certainly the case that far too much time is spent in most calculus courses doing things that are best done by machines."

Karian, Z., ed. 1992 *Symbolic Computation in Undergraduate Mathematics Education*, MAA Notes #24, Mathematical Association of America. Amazon.com information at <http://amzn.to/LdOQQ4>, note the searchable "Look Inside" feature.

Katz, V.J. & A. Tucker. 2003. "Preparing Mathematicians to Educate Teachers (PMET)," *Focus*, March; online at <http://bit.ly/SMcwTr> thanks to the Univ. of Arkansas. The authors wrote:

> "The Glenn Report . . . . .[[Glenn Commission Report (2000)]]. . . . made only a few straightforward points, but it made them urgently and insistently. In particular, the report concluded that "the most powerful instrument for change, and therefore the place to begin, lies at the very core of education - with teaching itself."

Kessel, C. 1990. "Why I Quit My Job," *UME Trends* **2**(5): 7-8; online as a 53 kB pdf at <http://bit.ly/UYdpbC>.

Kessel, C. 2013. "Mathematics and Education" blog, online at <http://bit.ly/17JGziK>, wherein it's stated (slightly edited):

"Cathy Kessel was educated as a mathematician, specializing in mathematical logic. She taught for three years after her PhD, then quit for a variety of reasons. Some of these are given in her 1990 *UME Trends* article 'Why I Quit My Job' [Kessel (1990) – see above]. During the 1990s, she made the shift from being a mathematician to being a researcher in mathematics education, auditing courses, and working on research projects at the School of Education at the University of California, Berkeley. She now works as a mathematics education consultant. What a mathematics education consultant does may not be completely obvious. She has listed some of the projects she's worked on at <http://bit.ly/17JGziK>. These tend to involve various combinations of mathematical knowledge, education expertise, and editing skills. From 2007 to 2009, she served as president of the Association for Women in Mathematics <http://bit.ly/1hERFfn>. . . . . .Unsurprisingly, she's interested in research on mathematics and gender. She's written about that as well as about mathematics education. One of her current projects is 'Progressions for the Common Core State Standards' <http://bit.ly/1gft6Fw>. See <http://bit.ly/17JGziK> for more about her involvement with the CCSS. . . . .[[*wherein it's stated that Kessel is the editor of the penultimate version (as of March 2010) of the CCSS for mathematics*]]. . . . . ."

Regarding the CCSS, compare the above with (a) "The Common Core State Standards" [Bressoud (2010i); (b) "*Next Generation Science Standards*: Good or Bad for Science Education?" [Hake (2013a)] in ***REFERENCES***; (c)"The future of high school math education" [Strauss (2013) and (d) "Engaging students in mathematics" [McCallum (2013) in ***REFERENCES***.]

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Keynes, H. & A. Olsen. 2001."Professional Development For Changing Undergraduate Mathematics Instruction," pages 113-126 in *Teaching and Learning of Mathematics at the University Level* [Holton et al (2001) at <http://bit.ly/KPPbWq>. The authors wrote:

"With these assumptions in mind, one of the objectives of the Calculus Initiative (CI) at the University of Minnesota, a project which successfully revitalized the undergraduate calculus sequence for engineering students, was to introduce changes in pedagogy and practice that made faculty aware of the value of such efforts. The CI emphasized (i) how the active learning approaches enhanced the faculty's own success as teachers; and (ii) how these methods improved student motivation and learning of important classical calculus topics. In this sense, many of the Initiative's efforts were devoted to innovative ways of providing professional development for the diverse members of the CI instructional teams—senior faculty, post-doctoral fellows, visiting faculty, graduate students, teaching specialists (many of whom were outstanding high school teachers on sabbatical), and undergraduate teaching assistants. A major objective was to provide a mentoring environment that helped each of these groups to be accepting of and successful in both short- and long-term implementation of these changes, which incorporated modem instructional approaches. The results of a four-year study of the CI are given in Keynes, Olson, O'Loughlin and Shaw (2000)."

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“*Mathematically Correct* is a website created by educators, parents, mathematicians, and scientists who were concerned about the direction of reform mathematics curricula based on NCTM standards. Created in 1997, it was a frequently cited website in the so-called Math wars, and was actively updated until 2003. The website went offline sometime in late 2012 or early 2013 but has been preserved on the Internet Archive <http://bit.ly/1aNt6KM>.” But I’ve found no online version of this essay.

Klein, D., & J. Rosen. 1997. Calculus reform—For the $millions. *Notices of the AMS, 44*(10), 1324-1325; online as a 74 kB pdf at <http://bit.ly/17eu25k>. A Critique of “Calculus Reform—For the Millions” [Mumford (1997)].

Klein, D. 2003. “A Brief History of American K-12 Mathematics Education in the 20th Century,” a preprint version is online at <http://bit.ly/1fJcJyc>.

Klein, D. 2007. "School math books, nonsense, and the National Science Foundation," *Am. J. Phys.* **75**(2): 101-102; online at <http://bit.ly/ck0p8Z>. Klein wrote: "Parents' organizations such as, 'Mathematically Correct,' 'New York City Honest and Open Logical Debate,' and 'Where's the Math?' among dozens of others, continue to resist the imposition of 'fuzzy math' in their schools." Letters critical of Klein (2007) by Leslie Atkins and Terry Millar, and a response by Klein are online as a 74 kB pdf at <http://bit.ly/1fPVs9a>.

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(1) ”The Irrelevance of Calculus Reform; Ruminations of a Sage-on-the-stage” by George Andrews;
(2) “Mathematical Content” by Richard Askey;
(3) “Personal Thoughts on Mature Teaching” by David Bressoud;
(4) “Remember the Students” by William Davis;
(5) “Reflections on Krantz’s How to Teach Mathematics: A Different View” by Ed Dubinsky [also online at <http://bit.ly/18wHmHN>;
(6) “Are We Encouraging Our Students to Think Mathematically” by Deborah Hughes Hallett”;
(7) “Big Business, Race, and Gender in Mathematics Reform” by David Klein [also online at <http://bit.ly/1fXxyso>;
(8) “Will This be on the Exam” by William McCallum;
(9) “Teaching or Appearing to Teach: What’s the Difference” Kenneth Millett;
(10) “Why (and How) I Teach without Long Lectures” by J.J. Uhl;
(11) “The Joy of Lecturing – with a Critique of the Romantic Tradition in Education Writing” by H. Wu.online as a 139 kB pdf at <http://bit.ly/1be6OvE>
(12) Teaching Freshmen to Learn Mathematics” by Steven Zucker.

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Lester, F.K, 2007. *Second Handbook of Research on Mathematics Teaching and Learning*. Information Age Publishing, publisher's information at <http://bit.ly/NgiHfk>. Amazon.com information at <http://amzn.to/L5HrEf>. An expurgated Google book preview is online at <http://bit.ly/KVnfqo>.

Lewis, J. 2001. "Spotlight on Teachers," *Notices of the AMS* **48**(4):396-403' online as a 78 kB pdf at <http://bit.ly/LNJwSC>: Lewis wrote:

> "But despite all the other suggestions for how to improve our schools, one idea recurs frequently - good teachers matter. This idea is often combined with the viewpoint that our colleges and universities are not doing enough to produce high-quality teachers."

For two diametrically opposed reactions to Lewis's article see Cohen & Krantz (2001).

Li, Y. & J.N. Moschkovich, eds. *Proficiency and Beliefs in Learning and Teaching Mathematics: Learning from Alan Schoenfeld and Günter Törner*. Sense Publishers, publisher's information at <http://bit.ly/Iu6LFr>, including the first 29 pages online as a 516 kB pdf at <http://bit.ly/1eAJfBA>. Amazon.com information at <http://amzn.to/1b3UX7L>.

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> The MAA is conducting a study of Calculus I instruction in American colleges and universities sponsored by NSF (DRL REESE #0910240). The goals of this study are:
>
> 1. To improve our understanding of the demographics of students who enroll in calculus, and
> 2. To measure the impact of the various characteristics of calculus classes that are believed to influence student success.
>
> The PI and co-PI's are David Bressoud (Macalester College), Marilyn Carlson (Arizona State University), Vilma Mesa (University of Michigan), Michael Pearson (MAA), and Chris Rasmussen (San Diego State University). Institutional Review Boards of Arizona State University and San Diego State University have provided IRB approvals.

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"Students in Manchester, New Hampshire were not subjected to arithmetic algorithms until grade 6. In earlier grades they read, invented, and discussed stories and problems; estimated lengths, heights, and areas; and enjoyed finding and interpreting numbers relevant to their lives. In grade 6, with 4 months of formal training, they caught up to the regular students in algorithmic ability, and were far ahead in general numeracy and in the verbal, semantic, and problem-solving skills they had practiced for the five years before. We conjecture that implementation of the 'Benezet Method' in early grades would drastically improve the effectiveness of high-school and university physics, science, and mathematics instruction."

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> "*Calculus & Its Origins* is an overview of calculus as an intellectual pursuit having a 2,000-year history. Author David Perkins examines the extent to which mathematicians and scholars from Egypt, Persia, and India absorbed and nourished Greek geometry, and details how the scholars wove their inquiries into a unified theory. Chapters cover the story of Archimedes'discovery of the area of a parabolic segment; Ibn Al-Haytham's calculation of the volume of a revolved area; Jyesthadeva's explanation of the infinite series for sine and cosine; Wallis's deduction of the link between hyperbolas and logarithms; Newton's generalization of the binomial theorem; Leibniz's discovery of integration by parts—and much more. Each chapter also contains exercises by such mathematical luminaries as Pascal, Maclaurin, Barrow, Cauchy, and Euler. Requiring only a basic knowledge of geometry and algebra—similar triangles, polynomials, factoring—and a willingness to treat the infinite as metaphor—*Calculus & Its Origins* is a treasure of the human intellect, pearls strung together by mathematicians across cultures and centuries."

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> “In the August/September 2005 issue of FOCUS there was a brief summary [Pearson (2005)] of a document entitled “Finding Common Ground in K-12 Mathematics Education” (hereafter CG), [Ball et al. (2005)] The authors of CG are two research mathematicians, three mathematics educators and the convener of the group, who is a senior vice-president and math and science policy advisor for a major American technology corporation and who has a Ph. D. in applied mathematics. . . . . . . the examples below are of issues that will surely elicit disagreement with CG among a substantial number of readers of FOCUS. . . . .[[and example relevant to calculus is]]. . . . (i) ‘By the time they leave high school, a majority of students should have studied calculus.’ Leave aside the fact that this— or anything close to it — cannot be achieved in any foreseeable future. Leave aside also the fact that many students who now study calculus in high school come away from it with little understanding and little more than an ability to perform mechanically various algorithms, all of which can be done better on a calculator. But, anyhow, why would you wish half the students to have studied calculus? Too much of the mathematics community has failed to come to terms with the fact that discrete mathematics is (almost?) as good an entrée to college mathematics as calculus. Not to recognize this in a document such as this is to arouse the suspicion that too many of the authors are living in the past. If they had said ‘…a majority of students should have studied first year college mathematics’, that would at least have been a defensible aspiration. I would still not have agreed with it on the grounds of unattainability but, at least, the document would have sounded like it had had input from some younger mathematicians.”

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> “Ralston proposes that the decrease in the importance of calculus in the world of mathematics is accelerating and the world of applied mathematics is changing rapidly. He briefly presents arguments for discrete mathematics. Then follow reactions from McLane, Wagner, Hilton, Woodriff, Kleitman, and Lax, and a response by Ralston.”

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RRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRR

I have come to the conclusion that the Common Core standards effort is fundamentally flawed by the process with which they have been foisted upon the nation.

The Common Core standards have been adopted in 46 states and the District of Columbia without any field test. They are being imposed on the children of this nation despite the fact that no one has any idea how they will affect students, teachers, or schools. We are a nation of guinea pigs, almost all trying an unknown new program at the same time.

Maybe the standards will be great. Maybe they will be a disaster. Maybe they will improve achievement. Maybe they will widen the achievement gaps between haves and have-nots. Maybe they will cause the children who now struggle to give up altogether. Would the Federal Drug Administration approve the use of a drug with no trials, no concern for possible harm or unintended consequences?

President Obama and Secretary Duncan often say that the Common Core standards were developed by the states and voluntarily adopted by them. This is not true.

They were developed by an organization called Achieve and the National Governors Association, both of which were generously funded by the Gates Foundation. There was minimal public engagement in the development of the Common Core. Their creation was neither grassroots nor did it emanate from the states.

In fact, it was well understood by states that they would not be eligible for Race to the Top funding ($4.35 billion) unless they adopted the Common Core standards. Federal law prohibits the U.S. Department of Education from prescribing any curriculum, but in this case the Department figured out a clever way to evade the letter of the law. Forty-six states and the District of Columbia signed on, not because the Common Core standards were better than their own, but because they wanted a share of the federal cash. In some cases, the Common Core standards really were better than the state standards, but in Massachusetts, for example, the state standards were superior and well tested but were ditched anyway and replaced with the Common Core. The former Texas State Commissioner of Education, Robert Scott, has stated for the record that he was urged to adopt the Common Core standards before they were written.

. . . . Another reason I cannot support the Common Core standards is that I am worried that they will cause a precipitous decline in test scores, based on arbitrary cut scores, and this will have a disparate impact on students who are English language learners, students with disabilities, and students who are poor and low-performing. A principal in the Mid-West told me that his school piloted the Common Core assessments and the failure rate rocketed upwards, especially among the students with the highest needs. He said the exams looked like AP exams and were beyond the reach of many students. When Kentucky piloted the Common Core, proficiency rates dropped by 30 percent. The Chancellor of the New York Board of Regents has already warned that the state should expect a sharp drop in test scores. What is the purpose of raising the bar so high that many more students fail? . . . .

RRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRR

Ravitch, D. 2013b.“Diane Ravitch's blog: A site to discuss better education for all,” online at <http://dianeravitch.net/>.

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Schoenfeld, A.H. 2000b. "Models of the teaching process," *Journal of Mathematical Behavior* **18**(3): 243-26. An abstract is online at <http://bit.ly/KNbLQd>.

Schoenfeld, A.H. 2001a. "Reflections on an Impoverished Education," from *Mathematics and Democracy: The Case for Quantitative Literacy,* online as a 1.6 MB pdf at <http://bit.ly/1bULLD7>.

Schoenfeld, A.H. 2001b. "Purposes And Methods Of Research In Mathematics Education," pages 221 –236 in *Teaching and Learning of Mathematics at the University Level* [Holton et al (2001) at <http://bit.ly/KPPbWq>. Schoenfeld wrote [bracketed by lines "SSSSS. . . . ."; see Holton et al. (2001) for the references:

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Initial efforts in calculus reform grew from pragmatic concerns and the design of solutions was pragmatic as well. This approach was reflected in the publications about reform. Having designed and implemented a project, the people involved then communicated their ideas and shared materials with others. This dissemination of information was one of the ways in which the movement spread from the small collection of original NSF sites to other schools. People also sought out opportunities to express their opinions about specific projects or issues of reform more generally. To date, the vast majority of publications related to calculus reform are opinions or descriptions of projects, curricula and software where the objective is to inform others of what was done and how. Relevant examples can be found in Solow (1994), Douglas (1995), and Schoenfeld (1995).

People were also interested in the impact that the reform projects were having on students. This led to the design and implementation of evaluation studies. The emphasis of these evaluations tended to be very pragmatic. In essence, frequently the main question asked was "Has it worked?" The specifics of how this question was pursued differed from project to project, but the experimental design of comparing students from traditional classes with students from reform classes was predominant.

In many instances, the evaluations used experimental designs borrowed from physical or biological sciences (or early educational research). They used 'experimental' and 'control' groups, typically comparing the test scores of classes that used the reform approach and those that did not. Sometimes the tests were designed specifically to assess particular types of student performance such as proficiency with traditional algorithms or ability on conceptual problems. (For examples, see Armstrong, Garner and Wynn, 1994; Bookman and Friedman, 1994.) The *limitations* of such comparative studies is well known (see, e.g., Schoenfeld, 1994). . . . .[[**but the *advantages* are less well known** – see e.g. "The Physics Education Reform Effort: A Possible Model for Higher Education?" (Hake, 2005)]]. . . . . . . Comparative studies of this sort are poorly-suited to the study of phenomena as complex as teaching and learning. These studies often did a reasonable job of answering a very specific question for a particular course or programme, however, instructional situations are complex and differ so dramatically from project to project that answering a project-specific question usually cannot contribute much to our understanding of teaching and learning as a whole.

Even if comparative methods were appropriate and we had ideal control and experimental groups, knowing that one group out-performed the other does not tell us what brought about the differences. Because of this, many of these evaluation studies are best described (in K-12 research) as a 'pilot' study. In many cases, the research identified what may be interesting phenomena but the results and conclusions fall short of contributing to our understanding of the processes of teaching and learning. In most cases, having identified an interesting occurrence (perhaps differential performance on exam questions by students in different courses), a study utilizing a different experimental design could be carried out. For example, problem solving interviews and/or observation of students' in-class behaviour might generate results with explanatory power for the differences in performance. In a small number of cases, those methods have been used to examine research questions of a non-comparative nature. For examples of non-comparative studies, see Palmiter (1991); Selden, Selden, and Mason (1994); Bonsangue and Drew (1995); Park and Travers (1996). Unfortunately, studies that yield results with strong explanatory power are scarce and the comparative studies undertaken in connection with calculus reform have not significantly advanced our understanding of how students learn calculus and how different instructional circumstances influence that learning.

3.5 Research

Some research on calculus has gone beyond comparative studies and evaluation of particular courses. Such work has not been primarily about calculus *reform*, but instead looked at student understanding of calculus concepts. This work examined student understanding in ways that have explanatory power and utility – the aim was not to answer some yes/no question but to explore some of the underlying mechanisms through which learning occurs. The researchers often made connections between existing research and their current research and the nature of the work also permits future researchers to build upon it.

Research of this sort examined how student understanding of particular concepts interacts with their understanding of calculus. In particular, researchers examined how calculus understanding is influenced by student understanding of variables, functions and limits. It is accepted in much of the educational research community that students' understanding of one concept influences their learning and understanding of related concepts. In terms of calculus learning, this means that the understandings of the concept of a variable, functions and limits will influence the development of their understanding of derivatives and other calculus concepts. Research has revealed that what may appear to be weaknesses in students' understanding of calculus concepts can really be just manifestations of their pre-existing understanding of a related concept. For example, students may understand the concept of function in ways that served them well in certain contexts but that are incompatible with, or do not support the development of, a robust understanding of derivative. Examples of this research can be found in Monk (1987), Williams (1991), Tall (1992), Ferrini-Mundy and Graham (1994), and White and Mitchelmore (1996).
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Schoenfeld, A.H. 2004. "The Math Wars," *Educational Policy* **18**(1): 253-286; online as a 270 kB pdf at <http://bit.ly/19QhRvI>.

Schoenfeld, A.H. 2006. "What Doesn't Work: The Challenge and Failure of the What Works Clearinghouse to Conduct Meaningful Reviews of Studies of Mathematics Curricula," *Educational Researcher*- 35(2): 13–21, March; an abstract is online at <http://bit.ly/1fzpkE8>.

Schoenfeld, A.H., ed. 2007. *Assessing Mathematical proficiency*. Cambridge University Press, publisher's information at <http://bit.ly/MMQXgi>. Amazon.com information at <http://amzn.to/LekFwp>, note the searchable "Look Inside" feature.

Schoenfeld, A.H. 2007 "Method," in *Second Handbook of Research on Mathematics Teaching and Learning* [Lester (2007)] at <http://bit.ly/NgiHfk>.

Schoenfeld, A.H. 2009. "Instructional Research and the Improvement of Practice," chapter 7 in *The Role of Research in Educational Improvement* in Bransford et al. (2009)

Schoenfeld, A.H. 2011. *How We Think*. Routledge, publisher's information at <http://bit.ly/12pN20Y>. Author's information at <http://bit.ly/Iuabs3>, including Preface and Chapter One as a 287 kB pdf at <http://bit.ly/IxDKIW>. Amazon.com information at <http://amzn.to/1inJYvs>, note the searchable "Look Inside" feature. A version delivered at the 12th International Congress on Mathematical Education 8-15 July 2012 is online as a 5.2 MB pdf at <http://bit.ly/1bUOe0x>.

Schoenfeld, A.H. 2012. "A Modest Proposal," *Notices of the AMS* (Doceamus Section) **59**: 312-319; online as a 270 kB pdf at <http://bit.ly/LDf5Qs>.

Schoenfeld, A.H. 2013a. Berkeley Websites:
Functions Research Group <http://bit.ly/18Aod7R>,
Algebra Teaching Study <http://bit.ly/IJD7g8>,
Mathematics Assessment Project <http://bit.ly/1bNlL8e>,
Formative Assessment with Computational Technologies (FACT) <http://bit.ly/187WA5r>.
A former Berkeley website, containing links to many of Schoenfeld's articles, seems to have vanished from the web **:-(**.

Schoenfeld. A.H. 2013b. "Reflections on Problem Solving Theory and Practice," *Mathematical Enthusiast* **10**(1&2): 9-34; online at <http://bit.ly/1jxSdpo>.

Schwingendorf, K.E. 1999. "Assessing the Effectiveness of Innovative Educational Reform Efforts," in MAA (1999); in Gold, Keith, & Marion (1999) online at <http://bit.ly/1e6ShtL>, pp. 249- 252
"This study explains the creation of a calculus reform program, its objectives, and philosophy and provides an in-depth comparison of reform-trained students with traditional students. The 'Calculus, Concepts, Computers and Cooperative Learning,' or C4L Calculus Reform Program is part of the National Calculus Reform Movement. The initial design of the C4L program began in 1987 under the leadership of Ed Dubinsky and Keith Schwingendorf on the West Lafayette campus of Purdue University."

Schwingendorf, K.E., G.P. McCabe, & J. Kuhn. 2000. "A longitudinal study of the C^4L calculus reform program: Comparisons of C^4L and traditional students," in *Research in Collegiate Mathematics Education IV* [Dubinsky et al. (2000)] at <http://bit.ly/MNQbwI>.

Selden, J., Mason, A. & Selden, A. 1989. "Can average calculus students solve non routine problems?" *Journal of Mathematical Behavior* ***8***: 45-50. I've not found an online version.

Selden, J. & Selden, A. 1990. "Constructivism in mathematics education: A view of how people learn," *UME Trends* **2**(2): 8. I've not found an online version. *UME Trends* evidently died without leaving a trace that I can find on the internet.

Seldon, A. & J. Seldon. 1996-2005. *Research Sampler*:
"This column contains brief expositions of research on undergraduate mathematics education and is linked to a bibliography <http://bit.ly/M5Tcrz>, a glossary <http://bit.ly/O4V1p4>, and a list of research questions <http://bit.ly/MJ8xmh>. For archival purposes, entries will be dated and remain unaltered subsequent to their initial publication. Occasionally articles will be written by guest authors. Potential authors should contact us at <js9484@usit.net> before proceeding."

Selden, A., J. Selden, S. Hauk, & A. Mason. 2000. "Why can't calculus students access their knowledge to solve non-routine problems? in *Research in Collegiate Mathematics Education IV* in [Dubinsky et al. (2000)] at <http://bit.ly/MNQbwI>.

Seldon, A. & J. Seldon. 2002. "Tertiary Mathematics Education Research and Its Future," pages 237 –254 in *Teaching and Learning of Mathematics at the University Level* [Holton et al (2001) at <http://bit.ly/KPPbWq>. A 1999 version is online as a 131 kB pdf Technical Report at <http://bit.ly/M7rsVS>.

Selden, A. 2002. “Two research traditions separated by a common subject: Mathematics and mathematics education,” online as a 111 kB pdf at <http://bit.ly/16BluDt>.

Seldon, A. & J. Seldon. 2013. *Research Sampler*: Bibliography; online at <http://bit.ly/19bibL2>. [The links given at <http://bit.ly/19bibL2> for the four topics below were all *dead* on19 Dec 2013. The old links below will have to serve as substitutes.
a. Bibliography on Constructivism, Nov. 1996: <http://bit.ly/1cznm2w>,
b. Preservice Teachers' Conceptions (from Research Sampler 3) <http://bit.ly/1hZUFU0>,
c. What Does it Take to be an Expert Problem Solver? (30 Aug 1997) <http://bit.ly/1jfXxxn>,
d. Why Mathematics Education Research Papers are Accepted or Rejected (Nov 1998) <http://bit.ly/1fkZao2>.

Selden, A., E. Dubinsky, G. Harel, & F. Hitt, eds. 2003. *Research in Collegiate Mathematics Education V*. AMS & CBMS, publisher’s information at <http://bit.ly/KOquKC>. Amazon.com information at <http://amzn.to/LKBKYy>, note the searchable “Look Inside” feature. [Also titled as *CBMS Issues in Mathematics Education*, vol. 12].

Selden, A., F. Hitt, G. Harel, & S. Hauk, eds. 2006. *Research in Collegiate Mathematics Education VI*. AMS & CBMS, publisher’s information at <http://bit.ly/Ne91NS>. Amazon.com information at <http://amzn.to/OWjl0W>, note the searchable “Look Inside” feature. [Also titled as *CBMS Issues in Mathematics Education*, vol. 13].

Silverberg, J. 1999. “Does Calculus Reform Work?” in pp. 245-248 in Gold et al. (1999), online at <http://bit.ly/1e6ShtL>.

Silver, E.A. & P.G. G. Herbst. 2007. “Theory in Mathematics Education Scholarship,” in *Second Handbook of Research on Mathematics Teaching and Learning* [Lester (2007)] at <http://bit.ly/NgiHfk>.

Small, D.B. & J.M. Hosack. 1990. *Calculus, An Integrated Approach*. McGraw-Hill. Amazon.com information at <http://amzn.to/L8xpCv>. According to Small (1999) this is the “first reformed calculus text.”

Small, D.B. 1999. “Core Mathematics for Engineers, Mathematicians and Scientists.” *Int. J. Engineering Ed.* **15**(6): 432-436; online as a 74 kB pdf at <http://bit.ly/Ldl8xO>.

Smith, D.A. 1994. “Trends in Calculus Reform,” in Solow (1994, pp. 3 -13); also online at <http://bit.ly/YJuRNF>.

Smith, D.A. 2002. “Essays on Education,” online at <http://bit.ly/Ix33vd>.

Smith, D.A. & L.C. Moore. 2010. *Calculus: Modeling and Application*, 2nd ed., <http://calculuscourse.maa.org/>. [Recommended by Stroyan (2011) as speaking to students’ interests.]

Solow, A.E. ed. 1994. *Preparing for a new calculus: Conference proceedings*. MAA. Amazon.com information at <http://amzn.to/MuGwwL>.

Solow, A.E. ed. 1996. Resources for Calculus, vol. 1: *Learning by Discovery: A Lab Manual for Calculus* (Classroom Resource Materials) (Volume 1). MAA. Amazon.com information at <http://amzn.to/KQdCmA>, note the searchable “Look Inside” feature.

Sofronas, K.S. & T.C. DeFranco. 2010. "An Examination of the Knowledge Base for Teaching Among Mathematics Faculty Teaching Calculus in Higher Education," in *Research in Collegiate Mathematics Education VII* [Hitt et al. (2010)] at <http://bit.ly/KOn6zx>].

Stacey, K., H. Chick, & M. Kendal. 2004. *The Future of the Teaching and Learning of Algebra: The 12th ICMI Study* (New ICMI Study Series). Springer, publisher's information <http://bit.ly/LkAPnR>, note the "Read Online" provision - the "Front matter" is free as a 610 kB pdf at <http://bit.ly/OkhP7N>. Amazon.com information at <http://amzn.to/LvusJG>, note the searchable "Look Inside" feature.

Star, J.R. & J.P. Smith. 2006. "An Image of Calculus Reform: Students' Experiences of Harvard Calculus," in *Research in Collegiate Mathematics Education VI* [Seldon et al. (2006)] at <http://bit.ly/Ne91NS>.

Steen, L.A., ed. 1987, *Calculus for a New Century: A Pump, Not a Filter, A National Colloquium*, 28-29 October; online as a 5.2 MB pdf at <http://bit.ly/1c8cvlr>. Amazon.com information at <http://amzn.to/MBORv9>. The abstract reads:

> "This document, intended as a resource for calculus reform, contains 75 separate contributions, comprising a very diverse set of opinions about the shape of calculus for a new century. The authors agree on the forces that are reshaping calculus, but disagree on how to respond to these forces. They agree that the current course is not satisfactory, yet disagree about new content emphases. They agree that the neglect of teaching must be repaired, but do not agree on the most promising avenues for improvement. The document contains:
>
> (1) a record of presentations prepared for a colloquium;
> (2) a collage of reactions to the colloquium by a variety of individuals representing diverse calculus constituencies;
> (3) summaries of 16 discussion groups that elaborate on particular themes of importance to reform efforts;
> (4) a series of background papers providing context for the calculus colloquium;
> (5) a selection of final examinations from Calculus I, II, and III from universities, colleges, and two-year colleges around the country;
> (6) a collection of reprints of documents related to calculus; and
> (7) a list of colloquium participants."

Steen, L.A., ed. 1989. *Reshaping College Mathematics*. MAA Notes #13. Amazon.com information at <http://amzn.to/LnNJ0m>.

Steen, L.A. 1990. "Numeracy: A contribution on quantitative literacy to a special issue of *Daedalus* **119** (2): 211-231; online at <http://bit.ly/1cKY2Ky>.

Steen, L.A. ed. 1992. *Heeding the Call for Change: Suggestions for Curricular Action*, MAA; online as a 22.4 MB pdf at <http://bit.ly/1di5drN>.

Steen, L.A. 1994. "Twenty questions about research on undergraduate mathematics education," in *Research in Collegiate Mathematics Education I* in [Dubinsky et al. (1994)] at <http://bit.ly/NWJisn>.

Steen, L.A. 1995. **"**Preparing Teachers of Mathematics: Asking the Right Questions" A discussion document prepared for a February 1995 workshop on the preparation of science and mathematics teachers sponsored by the National Science Foundation, online at <http://bit.ly/1b3ZFT6>.

Steen, L. A. ed. 1997. *Why Numbers Count: Quantitative Literacy for Tomorrow's America*. The College Board, Amazon.com information at <http://amzn.to/MUu5tO>.

Steen. L.A. 1999. "Assessing Assessment," in *Assessment Practices in Undergraduate Mathematics* [Gold et al. (1999)]; also online at <http://bit.ly/18Pmj0O>.

Steen, L.A., ed. 2000, "Revolution by Stealth: Redefining University Mathematics," pp. 303-312 in *Teaching and Learning of Mathematics at the University Level* [Holton et al (2001) at <http://bit.ly/KPPbWq>; also online at <http://bit.ly/Mgv6IX>. Regarding undergraduate mathematics Steen wrote [bracketed by lines "SSSS. . . .":

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Demands for relevance and accountability are no strangers to undergraduate mathematics. Indeed, postsecondary mathematics can be viewed as higher education in microcosm. Growth in course enrollments has been enormous, paralleling the unprecedented penetration of mathematical methods into new areas of application. These new areas–ranging from biology to finance, from agriculture to neuroscience–have changed profoundly the profile of mathematical practice. Yet for the most part these changes are invisible in the undergraduate mathematics curriculum, which still marches to the drumbeat of topics first developed in the eighteenth and nineteenth centuries.

It is, therefore, not at all surprising that the three themes identified at the UNESCO conference are presaged in the Discussion Document for this ICMI Study: the rapid growth in the number of students at the tertiary level; unprecedented changes in secondary school curricula, in teaching methods, and in technology; and increasing demand for public accountability [Holton et al. (2001)]. Worldwide demands for radical transformation of higher education bear on mathematics as much as on any other discipline. Postsecondary students study mathematics for many different reasons. Some pursue clear professional goals in careers such as engineering or business where advanced mathematical thinking is directly useful. Some enroll in specialized mathematics courses that are required in programs that prepare skilled workers such as nurses, automobile mechanics, or electronics technicians. Some study mathematics in order to teach mathematics to children, while others, far more numerous, study mathematics for much the same reason that students study literature or history–for critical thinking, for culture, and for intellectual breadth. Still others enroll in postsecondary courses designed to help older students master parts of secondary mathematics (especially algebra) that they never studied, never learned, or just forgot. (This latter group is especially numerous in countries such as the United States that provide relatively open access to tertiary education [Phipps (1998)].

Postsecondary students study mathematics for many different reasons. Some pursue clear professional goals in careers such as engineering or business where advanced mathematical thinking is directly useful. Some enroll in specialized mathematics courses that are required in programs that prepare skilled workers such as nurses, automobile mechanics, or electronics technicians. Some study mathematics in order to teach mathematics to children, while others, far more numerous, study mathematics for much the same reason that students study literature or history–for critical thinking, for culture, and for intellectual breadth. Still others enroll in postsecondary courses designed to help older students master parts of secondary mathematics (especially algebra) that they never studied, never learned, or just forgot. (This latter group is especially numerous in countries such as the United States that provide relatively open access to tertiary education [Phipps (1998)].

In today's world, the majority of students who enroll in postsecondary education study some type of mathematics. Tomorrow, virtually all will. In the information age, mathematical competence is as essential for self-fulfillment as literacy has been in earlier eras. Both employment and citizenship now require that adults be comfortable with central mathematical notions such as numbers and symbols, graphs and geometry, formulas and equations, measurement and estimation, risks and data. More important, literate adults must be prepared to recognize and interpret mathematics embedded in different contexts, to think mathematically as naturally as they think in their native language [Steen (1997)]. Since not all of this learning can possibly be accomplished in secondary education, much of it will take place in postsecondary contexts–either in traditional institutions of higher education (such as universities, four- and two-year colleges, polytechnics, or technical institutes) or, increasingly, in non-traditional settings such as the internet, corporate training centers, weekend short-courses, and for-profit universities.

This profusion of postsecondary mathematics programs at the end of the twentieth century contrasts sharply with the very limited forms of university mathematics education at the beginning of this century. The variety of forms, purposes, durations, degrees, and delivery systems of postsecondary mathematics reflects the changing character of society, of careers and of student needs. Proliferation of choices is without doubt the most significant change that has taken place in tertiary mathematics education in the last one hundred years.

Steen, L.A., 2004. *Achieving Quantitative Literacy*," Amazon.com information at <xxx>. Publisher's information at <http://bit.ly/L1fjnO>:

"A century after the United States crossed the threshold into universal secondary education, we are crossing a quite different threshold into universal postsecondary education. Consequently, society now expects of higher education what in the early 20th century it asked of secondary schools, namely, to prepare students for civic and economic life. In contrast to that earlier time, however, our age is dominated by computers and data, not factory assembly lines. These changes in society have created an urgent demand for multifaceted literacy far more sophisticated than what previously served as the foundation of today's curriculum. This greater demand for higher order competencies is nowhere more apparent than in the area of quantitative literacy. Although no less important for all citizens than fluency in reading and writing, quantitative literacy too often continues to be the province of the few. Indeed, for too long our educational system has produced a scientific and mathematical elite while failing to nurture the literate citizenry required for robust democracy. As a result, the gap between expert and citizen has widened dangerously, most notably when numbers and data are brought to bear in deciding public and private issues- and one can scarcely think of an issue in contemporary life where this is not the case."

Steen, L.A., 2005, 2006. "Asking the Right Questions. *Supporting Assessment in Undergraduate Mathematics*," Mathematical Association of America, 2006, pp. 11-18. Also in *Focus*, 25:8 (November 2005) 43-50. The Focus version is online as a 221 kB pdf at <http://bit.ly/1hpesvN>.

Steen, L.A., ed. 2006b. "Supporting Assessment in Undergraduate Mathematics," online as a 2.1 MB pdf at <http://bit.ly/IxCE04>.

Case studies editors: Bonnie Gold, Laurie Hopkins, Dick Jardine, & William A. Marion. Bernard L. Madison, Project Director; William E. Haver, Workshops Director; Peter Ewell, Project Evaluator; Thomas Rishel, Principal Investigator (2001–02); Michael Pearson, Principal Investigator (2002–05)

Steen, L.A., ed. 2007. " On Being a Mathematical Citizen: The Natural NExT Step," the ninth James R.C. Leitzel Lecture, the Mathematical Association of America, San Jose, California, 4 August; online at <http://bit.ly/KJGI9i>. Steen wrote, concerning "College Outcomes":

"I begin with something close to all our hearts: undergraduate education. Specifically, how should we measure its value? The increasing cost of higher education, and its increasing importance, has generated ever increasing calls for greater public accountability. A few years ago assessment guru Peter Ewell and I wrote a brief survey of this new environment for *Focus* with the alliterative title 'The Four A's: Accountability, Accreditation, Assessment, and Articulation' [Ewell & Steen (2003)]. One result of this public concern is the growing influence of (and related controversy about) college ranking systems such as the *U.S. News and World Report*. Faculty and administrators often argue that the work of higher education is too complex and too varied to be accurately judged by simple output measures. Nonetheless, we live in a world in which simple measures thrive, whether or not they measure anything important, or anything at all. One could spend a full semester plumbing the depths of the challenge posed by assessment of higher education. Here I want to touch on just three particulars to illustrate my argument about the value of mathematical thinking. One concerns measures of quantity (graduation rates), another measures of quality (general education), and a third measures of readiness (alignment)."

Steen, L.A., 2013. Selected Publications, online at <http://bit.ly/1eJ5GEK> and <http://bit.ly/1dDD0fq>.

Stein, S. 1997. "Preparation of Future Teachers," *Notices of the AMS* **44** (3): 311-312; online at <http://bit.ly/LF4Snx> / "Letters to the Editor", scroll down, where "/" means "click on."

Straffin, P. ed. 1996. Resources for Calculus, vol. 3: *Applications of Calculus*. Amazon.com information at <http://amzn.to/MXY3gO>.

Strauss, V. 2013. "The future of high school math education," *Washington Post*, 06 Dec; online at <http://wapo.st/19gaF1q>. Strauss wrote [bracketed by lines "SSSSS. . . . "; *emphasis* in the original except where indicated]:

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A few weeks ago a group of senior mathematicians, teachers, statisticians, and curriculum developers met in Boston to discuss the future of high school mathematics, revisiting issues addressed by a 2008 conference organized by the Center for Mathematics Education at the University of Maryland. This time, the Common Core State Standards was front and center of the discussion. Participants in the Boston meeting, sponsored by the non-profit Consortium for Mathematics and Its Applications <http://www.comap.com/>, formulated a set of recommendations for progressive action in the field and drafted an essay to explain their ideas.. . . . . Here's their essay:

Results from the most recent Program for International Student Assessment showed once again that U.S. high school students are in the middle of the pack when it comes to science, mathematics, and literacy achievement. The findings quickly elicited an outburst of public hand wringing, criticism of U.S. schools and their teachers, and calls to emulate the curriculum and teaching practices of high achieving countries. Then, just as predictably, there were a variety of explanations why we cannot import the policies and practices of other quite different countries (e.g., South Korea, Taiwan, Finland, and Singapore). *Instead, schools were urged to redouble efforts along lines that have been largely ineffective for the past decade and are not common in any high performing country—a regimen of extensive standardized testing with mostly punitive consequences for schools and teachers that fail to make adequate yearly progress*. . . . .[[My *italics*.]]. . . .. Public attention to the challenge of international competition has already begun to fade and we will hear little about the meaning of the PISA results until the next "wakeup call" arrives. ***What might happen if we tried something different this time around?***

Countries that have made real progress in their performance on international assessments share several characteristics. First and foremost is broad agreement on the goals of education and sustained commitment to change over time. In the United States there has been steady, if modest, improvement in student mathematics performance at the elementary and middle school levels on the National Assessment of Educational Progress (NAEP) and some improvement in results on college entrance examination tests (SAT and ACT) over the past two decades—a period when efforts have been guided by the National Council of Teachers of Mathematics (NCTM) standards for curriculum, evaluation, teaching, and assessment.

Over the past three years, 46 of the 50 U. S. states have been engaged in an effort to implement Common Core State Standards (CCSS) for mathematics and literacy. With respect to mathematics, those standards, prepared under the aegis of the National Governors' Association with generous private financial support, are in many ways an extension of key ideas in the earlier NCTM standards. Despite understandable controversy about particulars of the CCSS and the processes by which they were developed and states were induced to adopt them, *the Common Core standards provide a useful framework for further efforts*. . . . .[[My *italics*.]]. . . . Partisan political pressures (from both left and right) are already leading some state governors to reconsider their participation in this national compact to improve education—before even the first assessments of progress are reported. *But we believe that education policy makers and mathematics educators should resist the common wish for a quick fix and stay the course, modifying goals and efforts as results suggest such actions.*

What should students, teachers, parents, and policy-makers look for in the emerging reform of high school mathematics? From our perspective - as mathematicians, teachers, statisticians, teacher educators, and curriculum developers with extensive experience in school mathematics innovation—there are at least four key elements of the Common Core program that provide a basis for productive change in U. S. high school mathematics:

Comprehensive and Integrated Curriculum. . . . . . *A broad and integrated vision of high school mathematics would serve our students better than the narrow and compartmentalized structure of traditional programs...*

Mathematical Habits of Mind. . . . . . *Developing important mathematical habits of mind should become a central goal of high school instruction, especially the process of mathematical modeling that is required to solve significant real-world problems.*

Balanced Attention to Technique, Understanding, and Applications. . . . . . . *Improved performance on international assessments like PISA are likely to result from moves toward curricula and teaching methods that balance and integrate mathematical techniques, understanding, and applications.*

Information Technologies. . . . . *Improved performance on international assessments like PISA are likely.Personal computers, tablets, smartphones, and other computing devices will almost certainly transform school mathematics in fundamental ways. Intelligent response to that challenge will require creative research and development efforts and the courage to make significant changes in traditional practices.*

If the content and teaching of high school mathematics are transformed in the directions we recommend, schools and teachers will also need new tools for assessing student learning. One of the clearest findings of educational research is the truism that what gets tested gets taught. PISA is not a perfect or complete measure of high school student achievement. Neither are the TIMMS international assessments, the NAEP tests, the SAT and ACT college entrance exams, college placement exams, or, quite likely, the coming assessments attached to the Common Core State Standards.

Some would respond to the inadequacy of current assessment tools by sharply curtailing high stakes standardized testing; others would actually increase the testing and raise the consequences for students and schools. It is almost certainly true that the best course lies somewhere between those extremes. We need new and better tools for assessing student learning. . . . .[[e.g., *Concept Inventories* <http://en.wikipedia.org/wiki/Concept_inventory> used in formative pre/post testing as by Epstein (2013)]] . . . . . , and we need to employ those assessments in constructive ways to help teachers improve instruction and to inform educational policy decisions.

Finally, we need to change the tenor of public discourse about mathematics education. If we are to reach the shared goal of preparing young people for productive and satisfying lives, we need to work together to develop progressive goals for school mathematics and high quality instructional resources. Most important of all, *we need to dial down the acrimonious policy arguments and relentless criticism of schools and teachers. Teaching is one of the most important and demanding tasks for adults in our society, and teachers deserve our encouragement and support as they work to provide the best possible life preparation for their students*. . . . . .[[My i*talics*.]]. . . .

- Jim Fey, Sol Garfunkel, Diane Briars, Andy Isaacs, Henry Pollak, Eric Robinson, Richard Scheaffer, Alan Schoenfeld, Cathy Seeley, Dan Teague, Zalman Usiskin

Regarding the CCSS, compare the above with (a) "The Common Core State Standards" [Bressoud (2010i); (b) "*Next Generation Science Standards*: Good or Bad for Science Education?" [Hake (2013a)]; (c) "Mathematics and Education" (2013)]; (d) "Engaging students in mathematics" [McCallum (2013) in *REFERENCES*]; (e) "Why I Cannot Support the Common Core Standards" [Ravitch (2013a); and (f) "Study supports move toward common math standards" [Schmidt (2012b)].

Thurston, W.P. 1990. "Mathematical Education" *Notices of the AMS* **37**: 844– 850; online as 123 kB pdf at <http://bit.ly/M0Nypt>. Thurston <http://en.wikipedia.org/wiki/William_Thurston> wrote:

"In my parents' generation (during the 1940s), the standard first college mathematics course was college algebra. Soon afterward, the standard first college course was calculus, until the early 1960s, when calculus became standard for the best high school mathematics students. By now first year calculus has largely migrated to high school in affluent school districts, so that most of the better mathematics and science students at our best universities have already taken calculus before they arrive. At Princeton, for instance, two-thirds of entering students placed out of at least one semester of calculus last year. *The acceleration of the curriculum has had its cost: there has been an accompanying trend to prune away side topics*." [My *italics*.]

Tucker, A.C. & J.R.C. Leitzel. 1995. "Assessing Calculus Reform Efforts: A Report to the Community," 1995. MAA. Amazon.com information at <http://amzn.to/JiUTqR>.

Tucker, A.C., ed. 1999. *Models that work: Case studies in effective undergraduate mathematics programs*, MAA Notes no. 38; online as a 61 kB pdf at <http://bit.ly/JFcoBZ>.

Tucker, T.W., ed. 1990. *Priming the Calculus Pump: Innovations and Resources*, MAA Notes #17. Amazon.com information at <http://amzn.to/KG9u8M>. Tucker wrote:

"In the four years since the Tulane conference on calculus reform issued a call for a 'lean and lively calculus', countless articles have been written and contributed paper sessions, talks and panels have been presented on the topic. Individuals and institutions have responded with energy and imagination in rethinking from top to bottom what should go into a calculus course. This creative energy has resulted in real change in how calculus is taught at several institutions across the country. Unfortunately, these activities have not received the publicity they deserve. That is the purpose of this book - to provide the mathematical community with detailed examples of calculus reform at work. The ten featured projects in this book, together with abstracts of more than sixty other projects and a collection of reference materials and resources, are designed to give individuals and departments a concrete idea of what they can do, as well as information on how to do it and what resources are available. MAA, pp. 175-198, 1990.

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Gives the author, title, reference (sans hot links), and abstract of the 55+ references (20+ on calculus) listed in 2002.

Vinner, S. 1989. “The avoidance of visual considerations in calculus students,” *Focus on Learning Problems in Mathematics* **11**: 149-156. An ERIC abstract is online at <http://1.usa.gov/1bq1CVF>.

Vinner, S. & T. Dreyfus. 1989. “Images and Definitions for the Concept of a Function,” *Journal for Research in Mathematics Education* **20**(4): 356-366. An ERIC abstract is online at <http://1.usa.gov/1bq1CVF>.

Voss, J.F., D.N. Perkins. & J.W. Segal, eds. 1991. *Informal Reasoning and Education*. Routledge. Amazon.com information at <http://amzn.to/KURJIr>.

White, R. 1987. “Calculus of reality,” in *Calculus for a New Century: A Pump, Not a Filter* [Steen (1987)] at <http://bit.ly/1c8cvlr> (5.2 MB).

White, P. & M. Mitchelmore. 1996. “Conceptual Knowledge in Introductory Calculus,” *Journal for Research in Mathematics Education* **27**(1): 79-95. The first page is online at <http://bit.ly/18yOUcP>.

Walshaw, M. 2004. “A powerful theory of active engagement.” *For the Learning of Mathematics*, **24**(3): 4-10. The first page is online at <http://bit.ly/1khSFWC>.

Wikipedia. 2013a. "Calculus," online at <http://bit.ly/1cbtuDg>.

Wikipedia. 2013b. "Continuous Function," online at <http://bit.ly/LoAz5S>.

Wilson, R. 1997. “A Decade of Teaching Reform Calculus Has Been a Disaster, Critics Charge — Mathematicians divide over a curricular movement that some say has cheated students. *The Chronicle of Higher Education* 43, Feb. 7. p. A12-A13; an abstract is online at <http://bit.ly/LKu79k>:
“Ten years ago a movement to reform the teaching of calculus began in the nation's colleges and universities. Today, many mathematics professors are questioning whether the reform was a good idea. Critics charge that reformed calculus courses and textbooks have been watered down and do not give students enough background in solving complicated mathematics problems. The proponents of reform believe the new approach helps students develop a deeper understanding of the concepts and uses of calculus, in part by shifting the burden of lengthy calculations to computers. This article discusses the controversy over calculus that has erupted in the mathematical community. The controversy appears to be dividing the profession into those who favor reform and those who are against it.”

Windham, D.M. 2008. "Faculty Perceptions of a Calculus Reform Experiment at a Research University: A Historical Qualitative Analysis"; Dissertation, Department of Middle and Secondary Education, Florida State University; online as a 1.4 MB pdf at <http://bit.ly/PlJtE8>.

Windham, D.M. 2011. *Faculty perceptions of a calculus reform experiment at a research university: A historical qualitative analysis*. ProQuest. Amazon.com information at <http://amzn.to/Rn5Hmn>. The book description reads [my *italics*]:

"From 1999 through 2004, the mathematics department at Research University experimented with using a reform text, Hughes-Hallett et al.'s Calculus, to teach the undergraduate calculus sequence. A historical qualitative analysis was undertaken involving three linked case studies to determine, from the perspective of the professors in the classroom, the success of the experiment in reform. Three professors, one a self-identified reform advocate, one an arch-traditionalist who vehemently opposed reform, and one who professed himself to be in between, gave insight into the results of the switch and the departmental atmosphere that *led to the return in 2004 to a more traditional calculus instruction*. The results of these case studies include a picture of a department in transition, trying to better serve its students but having difficulty adjusting to the changes in instruction coincident with reform. Each of the participants admitted using the textbook as little more than a delivery vehicle for homework assignments; *none of the three participants changed their lecture style or teaching methods to respond to the demands of the reform movement*. Calculus reform's founders and those who have inherited the movement and brought it into the 21st century advocate technological exploration, real world applications, group projects, and conceptual understanding.

Each one of the participants admitted to applying some of these in their teaching style, but each in turn rejected other tenets of the reform movement as unusable, or unwieldy. As the department did not change any other aspect of calculus instruction at the university other than the text used, this experiment could have been dismissed as naive, insincere, or half-hearted. But in fact, the department may have benefited indirectly from the move by even the more traditional text they embraced post-reform, as all participants acknowledged that even traditional texts now contain elements of reform themselves. However, the case studies analyzed in this research would indicate that *any reform effort conducted in a research university should expect to meet some resistance of the type exposed at this university*. Anyone attempting to reform the teaching of calculus at their college can benefit from reading the perceptions of these professors and addressing them, either with seminars and *research that can convince faculty that a change is needed*, or at the very least by adjusting curricular structure and pacing so the reforms have a chance to succeed. Also, *educational researchers could benefit greatly from a nationwide qualitative/quantitative research focus on the acceptance of calculus reform at mainstream colleges and universities that do not have a vested interest in proving the reforms a success to maintain funding levels*. Finally, those educational researchers interested in the perceptions of college math professors at research institutions could further analyze how those professors' perceptions could impede or enhance efforts at reform, and how those perceptions differ from those predominant at teaching-focused institutions."

Woods, D.R. 2011. "Re: Interactive Engagement Typically Lowers Student Evaluations of Teaching?" STLHE-L post of 20 Mar 2011 16:23:36 -0400; online on the OPEN! STLHE-L archives at <http://bit.ly/gpVdGx>. Woods wrote:

"Any new intervention to improve learning usually, in my experience, creates a drop in SETs. Students dislike change and usually respond via the SET. However, forewarned means that action can to taken to prevent the lower SET and indeed to result in higher SETs. At least that's my experience."

Similar pro-SET opinions have expressed by Felder (1992).

Zandieh, M.J. 2000. "A theoretical framework for analyzing student understanding of the concept of derivative," in *Research in Collegiate Mathematics Education IV*. [Dubinsky et al. (2000)] at <http://bit.ly/MNQbwI>.

Zorn, P. , D.M. Bressoud , & M. Pearson. MAA response to the President's Council of Advisors on Science and Technology (PCAST) report, *Engage to Excel: Producing one million additional college graduates with degrees in Science, Technology, Engineering, and Mathematics*, online as a 963 kB pdf at <http://bit.ly/1cSq3xQ>.