IUHET 205
October 1991

NULL EXPERIMENTS FOR NEUTRINO MASSES

Alan Chodos,[a] V. Alan Kostelecký,[b] Robertus Potting[b] and Evalyn Gates[a]

[a]Center for Theoretical Physics
Yale University,
New Haven, CT 06511, U.S.A.

[b]Physics Department
Indiana University
Bloomington, IN 47405, U.S.A.

Taken at face value, experimental central values of the masses of the electron and muon neutrinos suggest that they are faster-than-light particles. We summarize the current experimental situation and propose a class of null experiments to settle this issue. We also comment on some theoretical aspects.

Published in Mod. Phys. Lett. A **7***, 467 (1992)*

1. Introduction. Some years ago, it was suggested [1] that at least one species of neutrino might be a tachyon, i.e., a faster-than-light particle. Although several theoretical issues concerning tachyons remain to be resolved, the main thrust of this suggestion was that the question is primarily experimental. In the intervening years, additional experimental data relevant to this issue have become available. In this paper, we wish to (i) summarize the experimental situation, (ii) propose a class of null experiment that could directly test the tachyonic-neutrino hypothesis, and (iii) offer some comments on the theoretical situation regarding tachyonic neutrinos.

2. Summary of Recent Data. Three experiments, all performed at SIN, have reported improved bounds on the muon-neutrino mass squared $m^2_{\nu_\mu}$ during the past decade. The first studied pion decay $\pi \to \mu\nu$ in flight, making careful measurements of the muon energy and momentum [2]. A negative central value for $m^2_{\nu_\mu}$ was obtained. The second experiment studied rest-frame decay, using pions stopped in plastic scintillator [3]. It also yielded a negative central value for $m^2_{\nu_\mu}$. The most recent [4] determined m_π with greater accuracy via pionic X-ray transitions in ^{24}Mg. This experiment provides the current best published value,

$$m^2_{\nu_\mu} = -0.097 \pm 0.072 \ MeV^2/c^4 \quad . \tag{1}$$

It continues to favor a tachyon, although the effect remains barely more than one standard deviation and hence is hardly conclusive.

In the same time period, four groups have been studying the electron neutrino. All experiments obtain information on $m^2_{\nu_e}$ by measuring the shape of the endpoint spectrum from tritium beta decay. One performed at SIN uses tritium implanted into carbon [5]. It yields a negative central value for $m^2_{\nu_e}$. Another at INS in Japan uses tritium-labeled arachidic acid ($C_{20}H_{40}O_2$) as a Cd salt [6]. The best fit obtained depends on the spectrum of final states used for the theoretical analysis. Four choices are presented, of which two yield negative central values for $m^2_{\nu_e}$. A third experiment at ITEP uses tritium in valine [7]. It reports a positive definite value for $m^2_{\nu_e}$ that disagrees with the bounds set by the other three groups. Finally, over the past five years a set of increasingly sensitive experiments using tritium gas has been performed at Los Alamos [8], in which the value of the electron-neutrino mass squared has consistently been measured as tachyonic. The latest reported value is

$$m_{\nu_e}^2 = -147 \pm 55 \pm 58 \; eV^2/c^4 \quad . \tag{2}$$

In principle, another source of information on the electron-neutrino mass is the set of IMB, Kamiokande and LSD events detected in conjunction with the supernova SN 1987A. Detailed analyses, including references to the several dozen papers on the subject that appeared in 1987, may be found in [9, 10]. In particular, ref. [9] allows for the possibility of tachyonic neutrinos. The likelihood functions generated for $m_{\nu_e}^2$ typically have significant portions of their span in the sector with $m_{\nu_e}^2 < 0$, and some supernova models favor negative central values for $m_{\nu_e}^2$. However, the data available and the current understanding of the neutrino thermal production spectrum in supernovae are insufficient to permit a definite statement.

3. Null Experiments for Tachyonic Neutrinos. The evidence presented above cannot be viewed as conclusive, but to the unbiased mind it is certainly suggestive. To obtain definitive information, one needs to identify an experimental signature that is unambiguously due to tachyonic neutrinos. We therefore propose here a type of null experiment that exploits a unique feature of tachyon Lorentz transformations. The point is that the kinematics of tachyons are such that processes forbidden in one frame may be allowed when boosted. The reasons for this are summarized in section 4 below; here, we present details for some candidate null experiments. In what follows, we set $c = 1$.

Our first example concerns muon neutrinos. Consider the "decay" $\mu \to \pi\nu$. This process is kinematically forbidden if the muon is at rest because it requires the neutrino energy to be negative. Since the neutrino is assumed to have spacelike four-momentum, however, we can perform a boost on the system that renders positive the neutrino energy. This eliminates the kinematic obstacle to the reaction taking place. A beam of muons of sufficient energy E would therefore decay into $\pi\nu$.

The threshold energy E_{th} is readily found by determining the boost required to make the neutrino energy vanish. The associated gamma factor is $\gamma_{th} = p_\nu/|m_\nu|$, so we require

$$E \geq E_{th} = \frac{m_\mu p_\nu}{|m_\nu|} \quad . \tag{3}$$

Here, p_ν is the neutrino momentum in the muon rest frame,

$$p_\nu^2 = \lambda^2 + |m_\nu|^2 \quad , \tag{4}$$

where λ is the negative of the neutrino energy in the muon rest frame,

$$\lambda = \frac{m_\pi^2 - m_\mu^2 + |m_\nu|^2}{2m_\mu} \quad . \tag{5}$$

For small $|m_\nu|$, $E_{th} \propto 1/|m_\nu|$. Using the central value given by eqn. (1), we find $E_{th} \simeq 14$ GeV. For a 1 TeV muon beam the threshold is attained for tachyonic-neutrino masses with modulus of order 4 keV. A 20 TeV beam could access $|m_{\nu_\mu}| \simeq 200\ eV$.

In the absence of a complete theory of tachyonic neutrinos, we can attempt an approximation to the rate for this process by using the standard weak-interaction matrix elements with massless neutrinos. This neglects correction effects of order $|m_\nu|/m_\mu$. Suppose the muon beam has energy E in the laboratory frame. First, we determine the rate ω_* for $\mu \to \pi\nu$ in the rest frame of the muon. The only neutrino momenta that are kinematically relevant in this frame are those that correspond to a positive neutrino energy in the laboratory frame. These form a cone centered about the direction of the boost, with opening angle θ such that

$$\cos\theta = \frac{\lambda}{p_\nu} \frac{E}{\sqrt{E^2 - m_\mu^2}} \quad . \tag{6}$$

Note that since $\cos\theta \leq 1$ we recover the lower bound E_{th} on E given by eqn. (3). Finally, the rate $\omega(E)$ in the laboratory frame is found by dividing with the appropriate γ factor, $\omega(E) = m_\mu \omega_*(E)/E$.

The result is

$$\omega(E) = \frac{1}{2\tau_\pi} \left(\frac{m_\pi}{m_\mu}\right) \frac{1}{1 - m_\mu^2/m_\pi^2} \left[\frac{p_\nu}{E} - \frac{\lambda}{\sqrt{E^2 - m_\mu^2}}\right] \quad , \tag{7}$$

where τ_π is the lifetime of the charged pion. For small $|m_\nu|$, the maximum of $\omega(E)$ occurs at $E_{max} = \sqrt{3}\ E_{th}$. We also find

$$\omega(E_{max}) = \frac{2}{3\sqrt{3}} \frac{1}{\tau_\pi} \left(\frac{|m_\nu|}{m_\pi}\right)^3 \frac{1}{(1 - m_\mu^2/m_\pi^2)^3} \quad . \tag{8}$$

Note that $\omega(E_{max}) \propto |m_\nu|^3$.

In Fig. 1, $\omega(E)$ is plotted for the case that the neutrino mass is given by eq. (1). We see that $E_{max} \simeq 24\ GeV$ and $\omega(E_{max}) \simeq 2\ s^{-1}$. The branching ratio

$$\frac{\Gamma_{\mu\to\pi\nu}}{\Gamma_{\mu\to all}} \simeq 10^{-4} \tag{9}$$

is likely to be within reach of a precision experiment. This calculation suggests that nonobservation of the decay $\mu \to \pi\nu_\mu$ at this level could set a new *lower* bound on $m^2_{\nu_\mu} < 0$.

More generally, one can seek pairs X, Y such that $X \to Y + \nu$ is a decay allowed by conservation laws but forbidden by kinematics. To minimize the boost needed for the decay to proceed, we must minimize the energy difference blocking the decay in the rest frame of X. Note that the threshold analysis above holds even if Y is composite because in a favorable situation at the threshold the components of Y are formed without kinetic energy.

For the muon neutrino, the reaction $\mu \to \pi\nu$ discusssed above is the best experimental candidate we have found. For the electron neutrino an interesting case [11] is the reaction $p \to ne^+\nu_e$. The phase space factor is relatively small and proton beams of high energy are readily available. The threshold proton energy for this decay to proceed is

$$E_{th} \simeq \frac{1.7 \times 10^3\ MeV^2}{|m_{\nu_e}|} \quad . \tag{10}$$

The central value given by eqn. (2) provides a threshold $E_{th} \simeq 140\ TeV$. Current p beams are sensitive to tachyonic-neutrino masses $|m_{\nu_e}| \simeq 1\ keV$, while the proposed 20 TeV beams of the SSC will be sensitive to $|m_{\nu_e}| \simeq 100\ eV$.

Similar considerations apply to the case of ordinary nuclear beta decay,

$$N_1 \to N_2 e^- \bar{\nu}_e \quad . \tag{11}$$

A particularly clean test is provided by nuclei N_1 that are stable in the rest frame, since it then suffices to boost N_1 and seek evidence of N_2 in the beam. There are several such candidate nucleus pairs, including ^{82}Se/^{82}Br, ^{123}Sb/^{123}Te, and ^{163}Dy/^{163}Ho. The latter is particularly favorable. The nucleus $^{163}_{66}$Dy is stable, with a natural abundance of about 25%, while $^{163}_{67}$Ho has a half life of about 4600 years. The reaction ^{163}Dy$\to^{163}$Ho $e^-\bar{\nu}_e$ is excluded by a small phase-space factor in the rest frame. A beam of ^{163}Dy would be able to decay if its energy E exceeds the threshold value

$$E_{th} \simeq \frac{4.2 \times 10^2 \ MeV^2}{|m_{\nu_e}|} \quad . \tag{12}$$

Current plans for RHIC call for the capacity to boost $^{197}_{79}$Au to about 100 GeV/nucleon, corresponding to a beam energy of about 20 TeV. This provides sensitivity to electron-neutrino masses approaching the central value in eqn. (2).

One can also consider candidate nuclei N_1 that are unstable in the rest frame. For example, both members of the pair ^{243}Am/^{243}Cm are α-unstable with half lives of about 7000 and 29 years, respectively. The threshold energy in this case is larger than the ^{163}Dy/^{163}Ho case by about a factor of three. Another possibility is to identify candidate nuclei N_1 that are unstable to beta decay in the rest frame but that have long half lives due to restricted phase space. Boosting such nuclei by amounts comparable to the maximum decay energy of the products could substantially enhance the decay rate in the laboratory frame. A good candidate for this purpose is the beta decay of ^{187}Re to ^{187}Os, which has a half life of about 4.35×10^{10} years and a decay energy comparable to the ^{163}Dy/^{163}Ho splitting.

4. Theoretical Issues. We begin this section with a brief exposition of how even the most primitive notion of Lorentz invariance can be compatible with the possibility that a process forbidden in one frame can occur in another. Our treatment is a recasting of material that has appeared in the earlier literature on tachyons [12-15].

In quantum field theory, one generally associates positive frequencies with annihilation operators, which we denote generically by a, and negative frequencies with creation operators. This assignment fixes the vacuum state $|0\rangle$, defined (at least for a free theory) as usual by the condition $a(k)|0\rangle = 0$. For tachyonic fields, the same association is made. The distinguishing feature of this case is that tachyons have spacelike four-momenta k_μ, which means that k_μ with $k_0 > 0$ can always be Lorentz transformed to k'_μ with $k'_0 < 0$. It follows that a covariant separation between annihilation and creation operators cannot be made.

The quantization procedure for tachyons therefore necessarily entails the presence of a preferred frame. Processes in the vacuum $|0\rangle$ can be described satisfactorily via positive-energy particles in the preferred frame. However, in certain other frames we would then be led to introduce negative-energy particles. A procedure is needed to avoid this. For example, one might choose [12] to reinterpret the negative-energy states as positive-energy antiparticles going backwards in time, thereby transforming in to out states or vice versa [16]. Another possibility is to insist on a description with positive-energy oscillators in any frame [13], which involves the interchange of creation and annihilation operators. In any case, the vacuum in the preferred frame is seen by a boosted observer to be occupied by precisely those tachyons whose energies would have been negative in the unboosted frame but which have been promoted to physical positive-energy tachyons by virtue of the boost. A careful analysis of this situation reveals that passive Lorentz invariance (where the particles are boosted) is not preserved, whereas active Lorentz invariance (where the observer is boosted) remains, at least with respect to the physics of the ordinary particles participating in a reaction.

For our purposes, the main consequence of this state of affairs is that an observer boosted along with the muon sees not just the muon at rest but also a background spectrum of neutrinos [17]. The process appearing as $\mu \to \pi + \nu$ to the observer in the lab is seen by the observer in the muon's rest frame as $\mu + \overline{\nu} \to \pi$, where the $\overline{\nu}$ is a member of this background. Thus, observers moving with respect to one another agree on whether a given process takes place, but they may disagree on whether there was a neutrino in the final state or an antineutrino in the initial state.

In constructing a quantum field theory of tachyons, one suspects in advance that certain cherished principles may need to be modified or even abandoned. For example, in a direct approach the three-momentum of the tachyon field satisfies $\vec{p}^{\,2} \geq |m|^2$, which suggests the violation of local commutativity [18]. It is, therefore, of some use to have a set of principles that one intends to maintain in the presence of tachyons, and which can thus provide a guide to the construction of a well-defined theory. We believe that the following three should be among those principles.

(A) *The principle of Lorentz invariance*, at least in the modified sense described above. A theory in which Lorentz invariance has been totally abandoned (for example, a Galilean-invariant theory) allows for tachyons but is not of physical interest.

(B) *The reinterpretation principle.* This is just the idea that negative-energy tachyons propagating backward in time are to be reinterpreted as positive-energy antitachyons moving forward in time. This is standard for ordinary particles and leads immediately to the correlation of annihilation (creation) with positive (negative) frequency. Tachyons are unusual only in that the assignment is observer dependent.

(C) *The tachyon correspondence principle.* This states that as $m \to 0$ the tachyon theory tends to the corresponding theory for massless particles. For present purposes, this principle has two important consequences. First, since neutrinos are very nearly massless, it means tachyons cannot be excluded except by actually measuring a positive definite m^2. There are no "zeroth-order" effects in $|m_\nu|$ that could give away the neutrino's tachyonic nature. Second, it implies that tachyonic neutrinos must carry half-integer spin; otherwise, a theory of fermionic integer-spin particles appears in the limit $m \to 0$, contradicting the principle.

Finally, we turn to possible field-theoretic formulations of spin-1/2 tachyons. Most of the previous literature on tachyons has been devoted to the spin-0 case. The reason usually given is that for spacelike momenta the little group is noncompact, so that the only unitary representations other than the one-dimensional one are infinite dimensional. The implication is that any tachyon of spin other than zero must be described by an infinite-component field. The usual proof of this, however, requires the one-particle states to transform irreducibly under the Lorentz group. As we have seen, the zero-tachyon state is not Lorentz invariant, and *a fortiori* the one-particle states do not transform irreducibly either. This is not to say that a finite-component theory of tachyons with nonzero spin is guaranteed to work, but it is a possibility that cannot be dismissed out of hand.

In reference [1] the lagrangian

$$\mathcal{L} = \overline{\psi}(i\gamma^5\gamma^\mu\partial_\mu - m)\psi \tag{13}$$

was proposed for free tachyonic neutrinos, leading to the wave equation

$$(i\gamma^5\gamma^\mu\partial_\mu - m)\psi = 0 \quad . \tag{14}$$

This is the simplest of a class of finite-component wave equations derivable from hermitian lagrangians that are Lorentz invariant and possess tachyonic solutions. As explained in ref. [1], however, a straightforward quantization of this system requires the introduction of negative-norm states. In fact, for each $p^2 = -m^2$ there are two plane-wave solutions to eq. (14) of opposite helicity, and to preserve the proper Lorentz-transformation properties of the fields one of these helicities must be associated with a negative norm. Consideration must therefore be given to the physical interpretation of this theory.

One possibility is to exploit the observation of Dirac [19] and Feynman [20] that the presence of negative norms is not *a priori* fatal, provided they are experimentally unobservable. In fact, this is precisely the situation for the standard electroweak model with the usual neutrino kinetic term replaced by eqn. (13). The point is that direct detection of the negative-norm states is hard (if possible at all) because the only observable neutrino interactions are via weak left-handed currents. As noted in ref. [1], the presence of the left-chirality projection $(1-\gamma_5)/2$ is just what is needed to ensure zeroth-order decoupling of the negative-norm states. To lowest order, the only function of the right-handed negative-norm state is to generate a tachyonic dispersion relation. Note in particular that the presence in the standard electroweak model of only the left-handed interactions is "explained" in this scenario by the requirement that negative norms be unobservable. Note also that this means the limit of vanishing mass is smooth, in accord with principle (C) above.

The presence of negative norms is reminiscent of the situation in the covariant quantization of gauge field theories. This suggests another possibility: the identification of an appropriate constraint that defines a physical subspace free of negative-norm states. The natural choice is the helicity projection operator. However, direct use of this is unsatisfactory because a Lorentz transformation mixes states of different helicity. The existence of a constraint that entirely eliminates the negative-norm states while maintaining unitarity and allowing for the Lorentz-transformation properties of the vacuum has yet to be established.

A related alternative is to define the set of physical states by projecting instead with the chiral operator $(1-\gamma_5)/2$. This is a Lorentz-invariant choice. For energies E large compared to the neutrino mass $|m_\nu|$, it approximates the helicity projection. Physical neutrino states then contain a negative-norm component that is suppressed by a power of the ratio $|m_\nu|/E$. This means states of net negative norm would be hard to create experimentally. Moreover, any violations of unitarity that might be induced by the projection would also be suppressed. For energies at the weak scale $E \simeq M_W$, the suppression factor is $\simeq 10^{-10}$. At the least, in the absence of a completely satisfactory theory of finite-component tachyonic neutrinos, this approach is likely to yield a phenomenologically viable model.

Notwithstanding the arguments given above, one could also consider the use of an infinite-component field to describe the neutrino. Unitary representations of the spacelike little group $SO(2,1)$ may be placed into one of two classes determined by the eigenvalue of the squared Pauli-Lubanski vector. If one uses the discrete series, the representations are characterized by the "spin," which is a half-integer $s \geq -1/2$, and the sign of the helicity. The helicity takes all integer values from $\pm(s+1)$ to $\pm\infty$. As $|m_\nu| \to 0$ one indeed recovers [21] the usual type of massless spin-1/2 theory as required by principle (C). A zero-mass neutrino of helicity 1/2 can thus be viewed as the zero-mass limit of a tachyonic neutrino with spin $s = -1/2$ and helicity 1/2.

An interesting aspect of this approach is that as the mass of a tachyonic neutrino goes to zero increasingly large boosts are required to connect different helicities, so for all practical purposes different helicities decouple. This helicity-trapping phenomenon suggests that the description of an infinite-component tachyonic neutrino in terms of an effective lagrangian for a single helicity (such as the lagrangian (13), for example) is likely to be a good approximation. Moreover, a given discrete series naturally contains only one sign of the helicity, which provides a connection to the "explanation" described above of the presence in the electroweak model of the left-chiral projection [22].

It appears feasible to construct a satisfactory model of an infinite-component neutrino, including an electroweak coupling to the Z^0_μ. The principal theoretical difficulty here is to be able to couple such a neutrino to the $W^\pm_\mu$ and the corresponding lepton in a Lorentz-invariant manner. One needs nonzero Clebsch-Gordan coefficients in the decomposition of two finite-dimensional representations with the infinite discrete series, in a formalism that would also incorporate the Lorentz-transformation properties of the vacuum. An approach incorporating the finite temperature and chemical potential of the neutrino background may be useful.

5. Summary. Our present conclusion is that a completely satisfactory quantum-field-theoretic formulation of a massive tachyonic neutrino has not yet been shown to exist, although we have presented candidates for a phenomenologically viable model. We wish to reiterate, however, our central points: whether the neutrino is a tachyon is fundamentally an experimental question, the experimental evidence is at present ambiguous but favors the tachyonic case, and the situation could be considerably clarified by means of the type of null experiment described above.

Acknowledgments. We thank Terry Goldman, Archie Hendry, Alex Krasnitz, Mike Nieto, and Harry Ogren for discussion. This work was supported in part by the United States Department of Energy under contracts DE-AC02-ER03075 and DE-AC02-84ER40125.

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17. Note that the proper motion of the earth (whether about the sun or with respect to the microwave background, etc.) contributes a negligible effect. The neutrino spectrum due to such motion would be a component of the known cosmological neutrino background.

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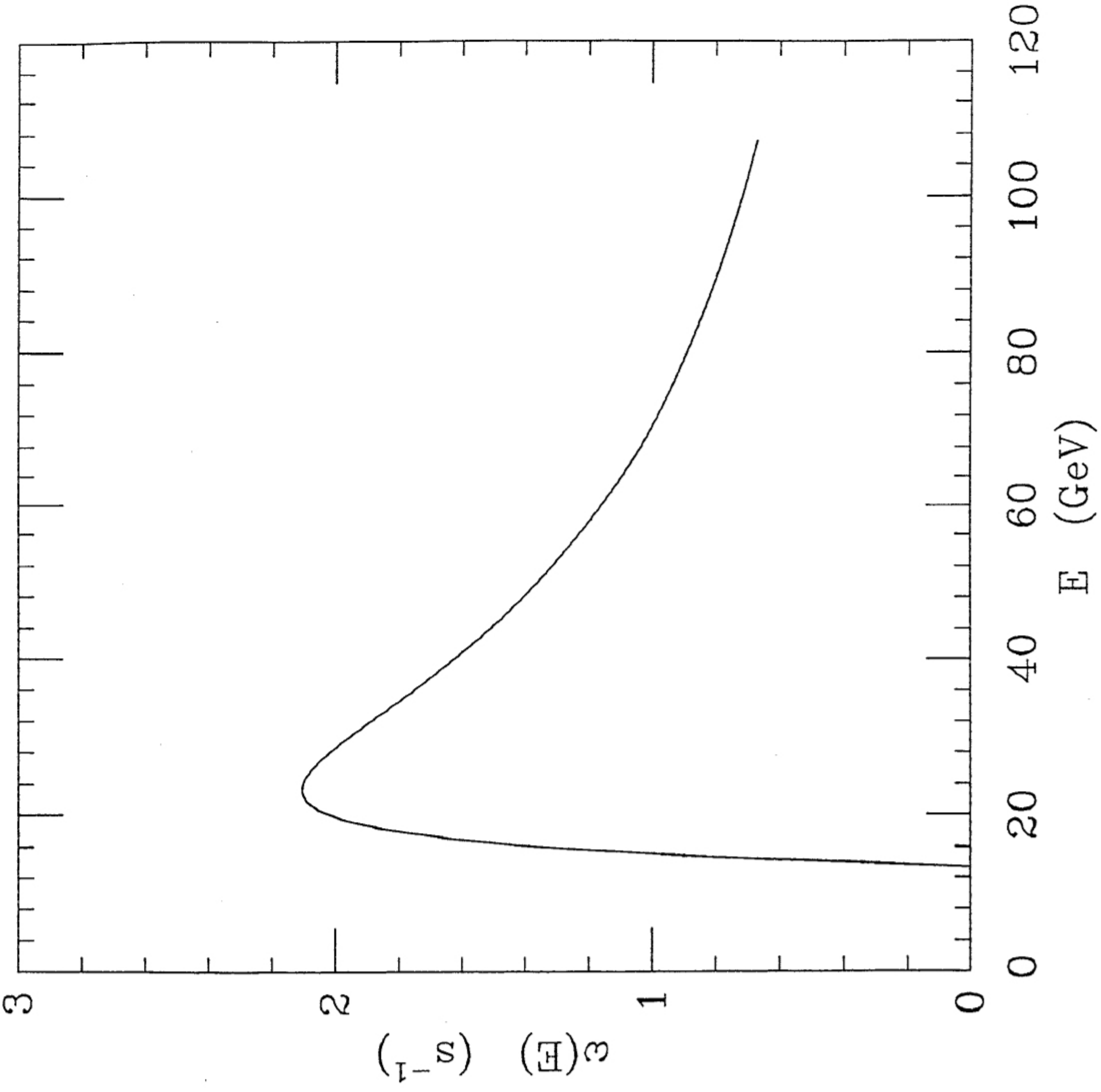

Figure 1: The rate $\omega(E)$ for the reaction $\mu \to \pi\nu$, given by eqn. (7), plotted versus the muon energy E in the laboratory frame. The central value for the muon-neutrino mass, eqn. (1), is taken.