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All the recent direct experiments measuring the electron- and muon-neutrino masses have yielded negative central values for the squared mass, suggesting these particles might have spacelike energy-momentum dispersion. I summarize the situation and discuss some possibilities for alternative experiments designed to test this unconventional result.

1. Introduction

It is a pleasure for me to join in honoring Lou Witten on the occasion of his retirement. I wish him much happiness in the years ahead.

Some years ago, the possibility was raised that the squared mass of one or more of the neutrinos might be negative* [1]. Although the neutrino physics implied by the resulting spacelike energy-momentum dispersion is unconventional, direct theoretical arguments are at present insufficient to refute it [2]. In this context, theory can act only as a guide; the true nature of the neutrino is ultimately an experimental issue.

Two possibilities exist for experimental verification or disproof of the notion of spacelike dispersion. The direct approach consists of measuring a nonzero positive or negative value for a squared neutrino mass. The null method involves the observation of some effect unique to theories of specified timelike or spacelike dispersion. In this work, I summarize the current situation for both direct and null experiments and suggest a new class of null experiment using cosmic rays.

2. Direct Experiments

In the past two years, five distinct groups have reported new or improved values for the squared mass of the electron neutrino obtained by measuring the endpoint spectrum from tritium beta decay. The results are as follows.

* Particles with negative squared mass are sometimes called 'tachyons.' As this term is often taken to imply attributes other than or in addition to negative squared mass, I will avoid it here.

- Tokyo [3]: $m^2_{\nu_e} = -65 \pm 85 \pm 65\ \mathrm{eV}^2/\mathrm{c}^4$
- Los Alamos [4]: $m^2_{\nu_e} = -147 \pm 68 \pm 41\ \mathrm{eV}^2/\mathrm{c}^4$
- Zürich [5]: $m^2_{\nu_e} = -24 \pm 48 \pm 61\ \mathrm{eV}^2/\mathrm{c}^4$
- Livermore [6]: $m^2_{\nu_e} = -72 \pm 41 \pm 30\ \mathrm{eV}^2/\mathrm{c}^4$
- Mainz [7]: $m^2_{\nu_e} = -39 \pm 34 \pm 15\ \mathrm{eV}^2/\mathrm{c}^4$

Combining statistical and systematic errors in quadrature, I find the average of these five experiments to be

$$m^2_{\nu_e} = -69 \pm 33\ \mathrm{eV}^2/\mathrm{c}^4 \quad . \tag{1}$$

Measurements of the muon-neutrino mass over the past decade have used pion decay at PSI. The results are as follows.

- Pion decay in flight [8]: $m^2_{\nu_\mu} = -0.14 \pm 0.20\ \mathrm{MeV}^2/\mathrm{c}^4$
- Rest-frame decay [9]: $m^2_{\nu_\mu} = -0.163 \pm 0.080\ \mathrm{MeV}^2/\mathrm{c}^4$
- Improved pion mass [10]: $m^2_{\nu_\mu} = -0.097 \pm 0.072\ \mathrm{MeV}^2/\mathrm{c}^4$

In the past year, an improved measurement of the muon momentum in pion decay has been made [11]. Using the prior world average for the pion mass [12] and the current best fit to the muon mass [13], this gives

$$m^2_{\nu_\mu} = -0.154 \pm 0.045\ \mathrm{MeV}^2/\mathrm{c}^4 \quad . \tag{2}$$

All these direct experiments are difficult, and the two- or three-standard-deviation effects observed cannot be taken as conclusive. Nonetheless, they make more tantalizing the possibility of spacelike neutrino dispersions and motivate a consideration of possible null experiments.

3. Null Experiments

The kinematics of particles with timelike and spacelike dispersions differ in several ways. Discussions of the differences and some efforts to construct consistent field theories of spacelike particles can be found in refs. [2, 14-17] and references therein. In particular, unconventional decays may be kinematically allowed when a spacelike particle is involved. For example, if the muon neutrino is spacelike a sufficiently energetic muon may decay into a pion and a muon antineutrino.

To see why this is possible, consider a two-body decay $X \to AB$, where the particles have masses m_X, m_A, m_B. Consider first the normal case where

X, A, B all have timelike energy-momentum dispersion. Lorentz invariance then constrains each particle of mass m, energy E, and momentum $\vec{p}$ to lie on a two-sheeted hyperboloid $E^2 - \vec{p}^2 = m^2$. *A priori,* both positive and negative energies can appear. Difficulties with negative energy are avoided by reinterpreting a negative-energy particle as a positive-energy antiparticle moving backwards in time. This represents an additional kinematical restriction. One consequence of the projection to the positive-energy sheet is that the decay $X \to AB$ will not proceed if $m_X < m_A$. Note that if instead both sheets were admitted, the decay would be kinematically allowed (e.g., in dimensionless units in the rest frame of X, one might have $m_X = 2$, $m_A = 5$, $m_B = 1$ giving $E_A = 7$, $E_B = -5$).

Suppose instead that particle B has spacelike energy-momentum dispersion. Lorentz invariance now constrains it to lie on the single-sheeted hyperboloid $E_B^2 - \vec{p}_B^2 = -m_B^2$. As before, to avoid problems with negative energies, a particle with $E_B < 0$ must be reinterpreted as a positive-energy antiparticle moving backwards in time. However, the separation of $E > 0$ and $E < 0$ is no longer Lorentz covariant. This induces a preferred frame, which can be taken as the frame at rest with respect to the microwave background, say.* This has an interesting consequence for any two-body decay $X \to AB$ that is kinematically forbidden in the rest frame of X because $E_B < 0$. The decay can proceed if X is boosted by $\beta_X > E_B/|\vec{p}_B|$ (note $\beta_X < 1$) because this transforms $E_B < 0$ into $E'_B > 0$.

The reader is referred to the literature for information on the difficult and unresolved issue of developing a complete and consistent theory incorporating spacelike neutrino dispersion along with the known electroweak neutrino physics. Some recent progress towards this end is reported in ref. [2]. Here, I concentrate on the possibility of using the distinct kinematical properties of spacelike neutrinos to develop null experiments constraining allowed ranges for negative squared masses.

4. The Electron Neutrino

Consider first the process $p \to n e^+ \nu_e$ for ν_e with spacelike dispersion. The threshold proton energy E_{th} above which this decay proceeds is

$$\begin{aligned} E_{th} &= \frac{1}{2|m_{\nu_e}|} \left[\left(m_p^2 - (m_n + m_e)^2 - |m_{\nu_e}|^2 \right)^2 + 4 m_p^2 |m_{\nu_e}|^2 \right]^{1/2} \\ &\simeq \frac{1.7 \times 10^3}{|m_{\nu_e}/\text{MeV}|} \ \text{ MeV} \quad . \end{aligned} \tag{3}$$

For example, the central value given in Eq. (1) yields $E_{th} \simeq 200$ TeV. This is beyond the range of available or proposed accelerators.

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* Since the proper motion of the earth in this frame is nonrelativistic, the preferred frame can also be taken as the laboratory frame for present purposes.

One possibility for using this process in a null experiment is to study cosmic rays of energies greater than about 10^4 GeV, say. Conventional physics suggests that at lower energies these are primarily protons, with some α-particles and heavier nuclei but only a small admixture of neutrons [18]. At the higher energies of interest here, there is some controversy about the composition, but the neutron component is still expected to be small. The search for a substantial neutron component appearing above some threshold energy would therefore provide a null test for spacelike neutrino dispersion.

A consequence of the existence of the process $p \to ne^+\nu_e$ is that a nucleon of sufficiently high energy $E > E_{th}$ could oscillate from proton to neutron and back. Through particle emission, energy would be lost with each oscillation until the nucleon energy falls below E_{th}. Provided the oscillation time is small compared to the residence time in the galaxy, this effect would lead to an apparent deficit of cosmic ray protons above E_{th}. Amusingly, such an effect is experimentally observed: there is a 'knee' in the cosmic-ray spectrum at an energy of $E_{knee} \simeq 5 \times 10^6$ GeV. Using Eq. (3), this would correspond to a value $|m_{\nu_e}| \simeq 0.3$ eV. However, there is a plausible hypothesis using conventional physics that can explain the knee: particles of energy greater than E_{knee} are expected to escape the Galaxy more easily because their gyroradius exceeds the effective galactic containment size. Moreover, even if the neutrino does have spacelike dispersion, this or other conventional effects could well mask any impact on the spectrum. Using cosmic rays to make a convincing case for spacelike dispersion therefore seems to require explicit measurement of a neutron component above a threshold energy E_{th}.

Measurements suggest that protons dominate the cosmic-ray flux up to energies of order 5×10^4 GeV. This would provide an upper bound of $|m_{\nu_e}| \lesssim 35$ eV for spacelike dispersion. From Eq. (3), the higher the energy to which one can experimentally show the absence of a significant neutron component in cosmic rays, the tighter the bound on a possible negative value of $m^2_{\nu_e}$. The physics of cosmic-ray acceleration and propagation is likely to be affected by spacelike kinematics. Nonetheless, a crude estimate for the least upper bound on $|m_{\nu_e}|$ available in principle can be obtained as follows. For a neutron energy $E_n \simeq 10^n$ GeV with $n \gtrsim 4$, say, a neutron decay length is $l_n \simeq ct_n = \gamma c\tau_n \simeq 10^{n-5}$ pc. The scale size for containment within the galactic disk is about a kiloparsec, so disregarding energy losses due to ionization, spallation, etc. in the interstellar medium a neutron will escape the Galaxy before decay if $n \gtrsim 8$. Let the laboratory-frame time scale for the process $p \to ne^+\nu_e$ be t_p. This quantity depends on the details of the theory as well as on kinematics. The residence time t_{res} for protons in the Galaxy is believed to be roughly of order 10^5 yr at the energies relevant here, and if $t_p \gtrsim t_{res}$ no oscillation will occur. The least upper bound for $|m_{\nu_e}|$ occurs for the highest nucleon energy compatible with sufficient oscillations to generate a significant neutron flux, so the above considerations suggest that the least upper bound occurs for $E_n \simeq 10^8$ GeV provided $t_p \lesssim t_{res}$. Nonobservation of neutrons at this energy could therefore provide in principle a best possible bound of $|m_{\nu_e}| \lesssim 10^{-2}$ eV for spacelike dispersion.

A side effect of the oscillations is the production of neutrinos. The kinematics of the process $p \to ne^+\nu_e$ near threshold are such that the emitted neutrino carries away relatively little energy, so these neutrinos might be hard to detect. However, in neutron decay the antineutrino can carry away much of the energy. If the oscillation time scales t_p, t_n are small enough, this process could result in an increased flux of high-energy antineutrinos. Observation of this flux might provide another means of testing the hypothesis of spacelike dispersion.*

Another possible class of null experiments involving the electron neutrino makes use of nuclear beta decay [2]. Ideally, one seeks a nucleus, stable in its rest frame, for which beta decay is forbidden only by a small phase-space factor. A relatively small boost would then permit the decay to proceed. An example is the reaction ${}^{163}_{66}\mathrm{Dy} \to {}^{163}_{67}\mathrm{Ho}\; e^-\bar{\nu}_e$, which for spacelike dispersion could occur if the Dy were boosted to

$$E > E_{th} \simeq \frac{4.2 \times 10^2}{|m_{\nu_e}/\mathrm{MeV}|} \quad \mathrm{MeV} \quad . \tag{4}$$

The central value in Eq. (1) gives a threshold energy per nucleon that is comparable to those anticipated at RHIC and in the CERN heavy-ion program. Some elaborations on this type of experiment are presented in ref. [2].

5. The Muon Neutrino

Null experiments can also be established for the muon neutrino. Consider in particular the process $\mu \to \pi\nu_\mu$. The threshold muon energy E_{th} above which this reaction proceeds is $E_{th} = m_\mu p_\nu / |m_{\nu_\mu}|$, where the neutrino momentum p_ν in the muon rest frame is given by

$$p_\nu^2 = \frac{(m_\mu^2 - m_\pi^2 - |m_{\nu_\mu}|^2)^2 + 4m_\mu^2 |m_{\nu_\mu}|^2}{4m_\mu^2} \quad . \tag{5}$$

For example, the central value in Eq. (2) gives $E_{th} \simeq 11$ GeV. One possible null experiment is therefore to examine a muon beam of energy $E > E_{th}$ for pionic content.

A possible method for obtaining a muon beam is as follows (see, for example, ref. [20]). A target is placed in a primary beam (of protons, say) from an accelerator. A momentum selection for pions is made, and a long decay channel

* This suggestion was made by Buford Price (email communication) in the context of the Antarctic-ice neutrino detector [19].

(of order a kilometer) permits a few percent of these to decay into muons. At the end of the channel the pions (and other hadrons) are stopped in an absorber, and further momentum selection for the penetrating muons is made.

Since some pion contamination of the resulting muon beam is possible, simple observation of pions downstream in a muon beam is insufficient as a null test for spacelike neutrino dispersion. Instead, the pion flux in the muon beam must be measured and compared to that expected from standard physics (estimated by Monte Carlo for the particular experimental configuration). An excess of pions would thereby provide evidence for a negative value of $m^2_{\nu_\mu}$. Note, however, that to fix $m^2_{\nu_\mu}$ (or, if no pion excess is observed, to bound it) requires knowledge of the muon decay rate at the beam energy, which involves more than kinematics.

Muon beams of several hundred GeV are available at CERN and at Fermilab. A measurement of the pion content of a muon beam with momentum $p_\mu = 280$ GeV/c has been performed by the EMC collaboration [20]. The predicted pion to muon ratio after the absorber was $\simeq 10^{-6}$, compared to an experimental upper limit of 10^{-5} measured at the experiment some 300 meters downstream. This places an upper bound on the laboratory-frame decay rate $\omega(E)$ at $E = 280$ GeV:

$$\omega(280 \text{ GeV}) \lesssim 10 \text{ Hz} \quad . \tag{6}$$

A crude estimate for $\omega(E)$, obtained under several assumptions about the underlying theory, was presented in ref. [2]. This result can be solved for $m^2_{\nu_\mu}$:

$$m^2_{\nu_\nu} \simeq -2\tau_\pi \frac{(m^2_\pi - m^2_\mu)^2}{m^3_\pi} E \, \omega(E) \quad . \tag{7}$$

Under these assumptions, the bound in Eq. (6) then gives the limit $m^2_{\nu_\mu} \gtrsim -3.7$ MeV^2/c^4. With present technology, the decay distance of the muon beam and the precision of the pion measurement could probably be increased sufficiently to result in bounds better than those in direct experiments.

Another possibility for tightening the bounds on spacelike neutrino dispersion is to use a muon storage ring (such as the $g-2$ ring at BNL) to obtain a relatively long-lasting muon beam. Conventional physics would predict an exponential decrease in pion contamination of the beam with storage time, whereas the kinematics of spacelike dispersion would result in a steady (although possibly small) flux of pions.

6. Discussion

The main thrust of this work is the observation that with currently available technology null experiments can provide information on negative squared neutrino masses. This information complements results obtained by direct experiments. It is largely model-independent: in the ideal situation it depends only on kinematics. A constraint on the squared electron-neutrino mass arises from the nonobservation of a significant neutron component in cosmic rays below energies of about 5×10^4 GeV, giving $|m_{\nu_e}| \lesssim 35$ eV for spacelike dispersion, i.e., $m_{\nu_e}^2 \gtrsim -1200$ $\mathrm{eV}^2/\mathrm{c}^4$. For the muon neutrino, with some theoretical input the EMC experiment at CERN provides a bound $m_{\nu_\mu}^2 \gtrsim -3.7$ $\mathrm{MeV}^2/\mathrm{c}^4$. These figures could probably be substantially improved with present technology.

Currently, the best values for the squared masses of the electron and muon neutrinos are negative by two to three standard deviations, Eqs. (1) and (2). Note, however, that care must be taken in interpreting the central values in these equations. The kinematics of timelike dispersion are well understood and the fitting procedure to the data is therefore well established. The correct procedure to use for negative squared mass is less clear. For the muon neutrino, the method conventionally adopted corresponds to that for the kinematics of spacelike dispersion as described above. However, for the electron neutrino the procedure normally used to obtain a fit to the Kurie plot is different from that found using spacelike kinematics (the latter result is presented in ref. [1]). This means the experimental negative central values quoted for the squared mass are not as directly meaningful as in the muon-neutrino case.* However, the effect observed for the electron neutrino is likely to remain at the level of two standard deviations in any analysis.

Another point to emphasize is that null experiments can provide useful constraints on the physically allowed range of m_ν^2. Such constraints have not been applied previously to analyses of experimental data. The quoted limits on m_ν in the literature are usually obtained by a bayesian procedure in which experimental values of quantities lying outside physically acceptable regions are disregarded (see, e.g., ref. [13] for a description of this method of analysis). This procedure is evidently not entirely satisfactory as it relies in part on theoretical prejudice (a discussion of this point is given in ref. [21]). With the null experiments described above, a large part (but not all) of the parameter space can be excluded on experimental grounds instead.

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* This has been independently emphasized by W. Stoeffl (private communication).

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