BOWSTRING ELASTICITY

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All string-like objects elongate when their tension is increased. Put more accurately: when the pulling force on (both ends of) a string (the string ‘tension’) is increased by an amount \( \Delta F \), its original length \( \ell \) increases by \( \Delta \ell \). The relative deformation, \( \Delta \ell / \ell \), is called ‘strain’ and the force per cross-sectional area, \( \Delta F / A \), is called ‘stress’. There is a simple relation between stress and strain,

\[
\Delta F / A = Y \cdot (\Delta \ell / \ell).
\]  

(1)

The elastic modulus \( Y \) (also called Young’s modulus) is a property of the material. Rubber bands stretch easily because rubber has a very low \( Y \), which means that it takes little force to produce a given elongation.

We are interested in the elasticity of bowstrings because work is required to elongate a string. Thus, a stretched string stores energy. When we draw the bow, the string tension decreases and the string contracts and releases energy. When the bow is loosed, the corresponding energy must be returned to the string and is not available to accelerate the arrow. Thus, an elastic string makes for a less efficient bow.

Before we can make use of equation 1 above, we have to determine the cross section (area) \( A \) of our bowstring. Since bowstrings are made from a number of individual strands (with space in between), we cannot derive \( A \) from the diameter of the string. But we can weigh a piece of string and divide by its length to get the ‘linear density’ \( \mu \). Then, combining this number with the (volume) density \( \rho \) of the string material, we can calculate the cross section to be \( A = \mu / \rho \).

Let’s look at a typical string (one that has been in use on my compound bow, presumably made mostly of Dyneema, see Fig.1 and 2). My measurement of the linear string density yields \( \mu = 0.033 \) g/cm. For the density of the string material we use \( \rho_{\text{Dy}} = 0.97 \) g/cm\(^3\). Then we find \( A = \mu / \rho_{\text{Dy}} = 0.034 \) cm\(^2\) = 3.4·10\(^{-6}\) m\(^2\). For the elastic modulus we use \( Y_{\text{Dy}} = 100 \) GPa. Here, G (giga) stands for a factor of a billion, or 10\(^9\) and the metric unit Pa (pascal) is an alias for N/m\(^2\), i.e., force per area (same as pressure, 1 psi = 6895 Pa).

Now we can calculate, from eq. 1, the strain \( (\Delta \ell / \ell) \) of our bowstring for a given tension change \( \Delta F \). I pick \( \Delta F = 250 \) N (= 56.3 pounds) because this represents the change in string tension (decreasing from 300 N to 50 N) when I am drawing my bow. The result is \( (\Delta \ell / \ell) = \Delta F / (A \cdot Y) = 250 \) N/(3.4·10\(^{-6}\)m\(^2\) · 100·10\(^9\) N/m\(^2\)) = 0.074 %. In other words, a 1 m long string will change by only \( \Delta \ell = 0.74 \) mm! This is a very small effect and we must conclude that, at least for modern strings, bowstring elasticity may be ignored and plays no significant role in the performance of a bow.

This is an unexpected conclusion and calls for an independent verification. To this aim, I used my string (\( \ell = 1 \) m) to pick up a mass from the floor with the idea to measure the string elongation directly. It turned out that the string elasticity is so small that I needed a rather large mass before I saw an elongation that could be measured easily. In effect, I used a 200kg mass (440 pounds) and
an industrial crane to perform this experiment (don’t try this at home). The outcome basically confirms the calculation with an elastic modulus for my string that is somewhat smaller than the one used in the calculation \( Y = 60 \text{ GPa} \).

I now need to introduce ‘tensile strength’, \( S \), another material constant, which is defined as the stress at which the material breaks. A typical value for Dyneema is \( S = 3 \text{ GPa} \) (see fig.2). We can use this information to estimate the tension at which our string is expected to rupture: 

\[
F_{\text{break}} = S \cdot A = 3 \cdot 10^9 \text{ N/m}^2 \cdot 3.4 \cdot 10^{-6} \text{ m}^2 = 10200 \text{ N} (=2300 \text{ pounds!})
\]

This calculation assumes that the stress is uniformly distributed over all strands in the string. Since this is never quite true, the actual breaking force is somewhat lower but still much larger than the string tension actually encountered when shooting a bow.

On the web and in the literature one often finds quotes for the ‘stretch’ of string materials. In this context ‘stretch’ is defined as the strain at the breaking point, determined by the elongation of a string just before it snaps. ‘Stretch’ equals roughly \( S/Y \), however, this ignores the fact that, before the breaking point is reached, the string is stressed beyond the elastic limit and is permanently deformed. It should be obvious that this quantity of ‘stretch’ is not useful when discussing bowstring elasticity.

Among the many materials that historically have been used for bowstrings, the favorite choice has been waxed linen for hundreds of years (traditional bowyers today still are using linen). Today, strings with superior properties are available, due to the development of synthetic fibers during the second half of the 20th century. The tensile strength, the elastic modulus and the density of four materials that represent milestones in the development of modern strings are given in figs. 1 and 2.

One of the earliest synthetics in use was Dacron with strength similar to linen, but less susceptibility to moisture and temperature changes. Dacron is quite elastic (low \( Y \)) and suffers from slow deformation under stress (creep). Its successor was Kevlar with low elasticity and high strength, but unfortunately with durability problems. Today’s favorite material for bowstrings is spun, ultra-high molecular weight polyethylene (tradenames Spectra and Dyneema), sometimes blended with Vectran, a liquid-crystal polymer, to counteract a tendency for creep. An important advantage of modern string material is its low density, since lowering the string mass increases the bow efficiency.
Fig. 1. Elastic modulus (Young’s modulus) $Y$ versus density $\rho$ for four traditional bowstring materials. The synthetic materials are identified by their trade names. The data have been collected from a number of web-based sources. The modulus is not well-defined because the micro-structure of the materials depends on the manufacturing process.

Fig. 2. The tensile strength $S$ for four traditional bowstring materials. See also Fig.1 caption.