

SDI LAB #0.1. FRAMES OF REFERENCE, POSITION, AND VECTORS

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I. INTRODUCTION

This first Socratic Dialogue Inducing (SDI) lab is intended as a heads- and hands-on learning experience in applying *vectors* to describe a walk by you between *initial* and *final positions* with respect to a laboratory *frame of reference*. Using *well-defined measurement operations*, you will determine your initial and final position coordinates and position *vectors*, and then use the position vectors to calculate your vector *displacement* from your initial to your final position. This early start in understanding *frames of reference*, *position*, and *vectors* should assist your physics learning, since these ideas are essential underpinnings of kinematics and mechanics. As emphasized by David Hestenes, the statement that every particle has a definite position with respect to a given reference frame is part of a tacit but very fundamental "Zeroth Law" (N0) of Newtonian mechanics.

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A. OBJECTIVES – To learn (or review)

1. the ground rules for Socratic Dialogue Inducing (SDI) Labs;
2. the algebra and trigonometry needed for elementary use of vectors;
3. the meaning of *operational definition*, *frame of reference*, and *position*;
4. how to represent a vector by means of (a) an arrow, (b) components, (c) magnitude and direction;
5. how to determine the magnitude and direction of a vector both graphically and analytically;
6. how to use vectors to specify position and determine the displacement from one position to another;
7. how to add and subtract vectors both graphically and analytically;
8. the meaning of a vector equation;
9. how position and displacement vectors are affected by the translation of a coordinate system.

B. HOW TO PREPARE FOR THIS LAB

Please study the "Ground Rules for SDI Lab" below. Rules 2, 7, 11, and 12 are especially important for this lab. You will learn more if you also study the sections of the text *Physics*, 4th ed. by Douglas Giancoli (hereafter called "Giancoli") indicated below (or corresponding material in whatever text you may be using).

1. In Chap. 1, "Introduction": Sec. 1.4, Measurement and Uncertainty; Sec. 1.5, Units, Standards, and the SI System (note "operational definitions" on p. 11 and Question 3 on p. 15).
2. In Chap. 2, "Describing Motion...." : Sec. 2.3, Reference Frames and Coordinate Systems; Sec. 2.4, Average Velocity and Displacement; Sec. 2.6, Vectors and Scalars.
3. In Chap. 3, "Kinematics in Two and Three Dimensions; Vectors: Sec. 3.1. Addition of Vectors– Graphical Methods; Sec. 3.2, Subtraction of Vectors, and Multiplication of a Vector by a Scalar; Sec. 3.3, Analytic Method for Adding Vectors: Components.

C. GROUND RULES FOR SDI LABS[†]

What I cannot create I do not understand.

Richard Feynman

1. **The primary goal of SDI labs is to help you attain a good understanding of the basic concepts of Newtonian mechanics through *creative engagement with simple mechanics experiments*.** SDI labs should also improve your ability to analyze and solve mechanics problems, acquaint you with various modes of productive scientific thinking, and introduce you to some effective strategies of theoretical and experimental physics.

2. You are encouraged to *work collaboratively* with other students. Nevertheless, the diagrams and commentary in your lab report should be your own work and not simply copied from the work of others. **TRY TO FIGURE THINGS OUT FOR YOURSELVES.** However, if you reach an impasse please don't hesitate to call in an instructor who will try to put you on the right track. Simply raise your hand or else flip the sign above your table to "HELP."

3. In order to increase your understanding of physics concepts it is essential that you take an **ACTIVE** part in the labs and actually perform ALL the experiments yourself. Only in this way will you increase your understanding of physics. **YOU MUST TAKE RESPONSIBILITY FOR YOUR OWN LEARNING.**

4. Because in physics one labeled sketch or graph is often worth a teraword ($1.0 \text{ teraword} \equiv 1.0 \times 10^{12} \text{ words}$), an important activity in SDI labs is the drawing of *force-motion-vector diagrams* showing a BODY at rest or in motion at an instant in time (a "snapshot sketch") with color-coded force (red), velocity (green), and acceleration (orange) vectors. The quadrille ruling in the lab manuals will often allow you to save valuable time by making freehand drawings without the use of rulers or straight edges. We **INSIST** that the **V**ectors be shown with their **T**ails **O**n the **B**ody (**VTOB**), so that you (and we) can always relate the vectors to the proper body, as, e.g.,

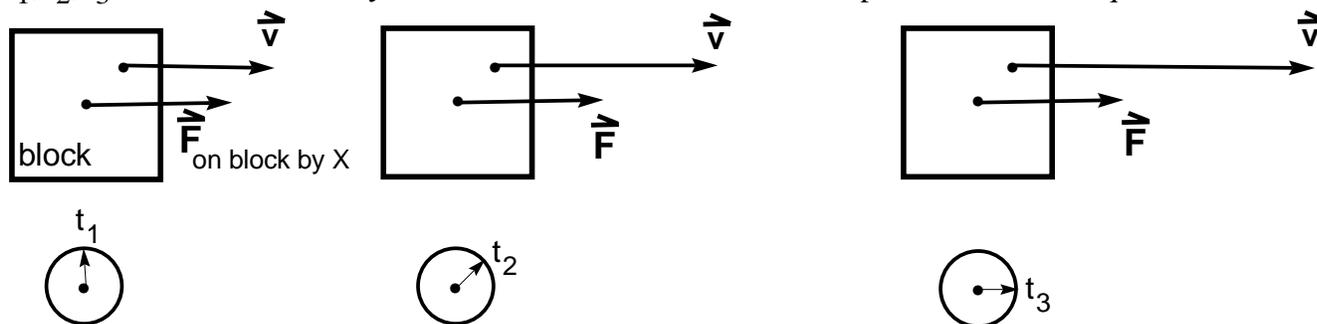


Here "X" is some other body which is interacting with the block. We insist that Force vectors always be Subscripted as **F_{On A by B}** (**FSAB**) where A and B are always two interacting particles or bodies.

Except in end-on views show vector tails as dots (•). For end-on views of vectors, we shall adopt the standard convention that: ⊙ means perpendicular to and *out* of the paper, and ⊗ means perpendicular to and *into* the paper.

[†]*Rules and models destroy genius and art.* [William Hazlitt (1778 - 1830)] Nevertheless, long experience with SDI Labs has shown that instructors can do a better job of coaching and help you to learn more physics if follow these rules and models, even though some of them may seem rather arbitrary and confining at first sight.

5. In some cases you will draw a series of "snapshot sketches" at sequential *instants of time*, e.g., t_1, t_2, t_3 . In such cases *always* show clocks near each sketch to emphasize the time sequence:



Here a block is acted upon by a force \vec{F} on block by X where X is some other body. The snapshots are shown at *equal* intervals of time as indicated by the hands of the clocks. That the block is speeding up is shown by (a) the increase in the length of the \vec{v} vector in the successive snapshots, (b) the fact that the displacement during the second time interval ($t_3 - t_2$) is larger than the displacement during the first time interval ($t_2 - t_1$). To save time, the subscripts "on block by X" for the force vector \vec{F} are shown only in the first snapshot.

6. In order to help carefully distinguish the various vectors used to describe motion, we **INSIST** that you use the standard course color code (vectors are indicated by **boldface** letters with arrows above them in this manual):

RED: force \vec{F} , torque $\vec{\tau}$;

BLUE: displacement \vec{D} ;

GREEN: velocity \vec{v} , angular velocity $\vec{\omega}$, precessional velocity $\vec{\Omega}$;

ORANGE: acceleration \vec{a} , angular acceleration $\vec{\alpha}$;

PURPLE: linear momentum \vec{p} , angular momentum \vec{L} ;

YELLOW: bodies to which Newton's First or Second Laws are applied (but please place *black-pencil outlines* around all bodies colored yellow).

All other bodies in your sketches (e.g., people, sticks, tables, floors, Earth) should be drawn with ordinary black pencil. This color code is virtually the same as that used in the course text *Physics*, 4th ed., by D.C. Giancoli (see p. xiii), with the exception of the body color YELLOW. [NOTE: If you are using a text which employs some other systematic color code, then please use that code and change color designations accordingly in this manual.]

7. Please use pencils (**erasable**) in making sketches and answering questions, since you may wish to revise your work as the lab progresses and your ideas change (if your ideas are not changing you are probably not learning very much!).

8. In order to draw force-motion-vector diagrams, please bring the following supplies to every lab:
 - a. Set of 12 Faber-Castell "Col-erase" color pencils, Cat. # 20516, available at most bookstores. Ordinary color pencils *cannot* be used because they are not erasable.
 - b. Plastic see-through ruler (even though most drawings can be made freehand).
 - c. Eraser.
9. A plentiful supply of scratch paper will be provided at each lab table for color-coded sketching by you and your instructors.
10. In most labs signs displaying helpful physics information will be placed above each lab table. It may help you to refer to these signs as your work proceeds.
11. The lab manual questions are designed to help you **THINK** about the experiments and how they relate to Newton's laws. You will often be asked to *predict* the outcome of an experiment and then *perform* that experiment. A curly bracket {.....} indicates that you should **ENCIRCLE** **O** a response within the bracket and then, we **INSIST**, briefly **EXPLAIN** or **JUSTIFY** your answers in the space provided on these sheets. The letters { Y, N, U, NOT } stand for {Yes, No, Uncertain, None Of These}.
12. **Lab reports must be handed in at the end of each lab period.** These reports will be *annotated* (NOT graded) by your lab instructor and returned to you at the start of the next lab period. A "**D**" near a section if your lab report means that you should **D**iscuss that section with your lab instructor.
13. *Your grade in SDI labs will be determined by your scores on SDI-lab exams* (samples from previous years are available). These exams will be given as parts of the periodic evening 2-hr course exams as indicated in the *Assignment Sheet*. The cumulative SDI-lab-exam scores will count for about 30% of your final course grade (for details see the *Information Sheet*). Thus, even in the off chance that you are only interested in the final grade, your goal in SDI labs should be to **UNDERSTAND** the material rather than merely to complete a lab report.
14. **ATTENDANCE AT ALL SDI LABS IS REQUIRED.** If some catastrophe causes you to miss an SDI lab, *apply to the lab coordinator immediately for a transfer slip to another lab. There will be NO lab make-ups!*
15. The SDI lab manuals sometimes include more material [SDI #0.1 (\approx 2 hr), SDI #0.2 (\approx 4 hr), SDI #1 (\approx 6 hr), SDI #2 (\approx 6 hr), SDI #3 (\approx 6 hr), SDI #4 (\approx 4 hr), SDI #5 (\approx 6 hr.), SDI #6 (\approx 3 hr.)] than can be adequately covered in the allotted time . At the start of each lab period your instructor will indicate the sections which you should attempt to complete. However, you should work at your own pace. *It is more important to understand the material you work on rather than to "cover" all the prescribed sections.* If you find yourself somewhat ahead of your lab partners why not try to explain some physics to them (explainers often learn more than listeners).
16. Both time and space are at a premium in SDI labs. Please (1) help students who follow you devote time to physics rather than clean up by placing lab material back in order after you complete an experiment, (2) allow room for the experiments by hanging your coats and jackets on the coat hooks – please don't put them on lab tables or stools.

17. In order to become better acquainted with students and thereby offer more personalized instruction, we shall take snapshots of each lab-table group near the start of the semester.

18. SDI labs have been under development for several years and have proven to be quite effective in enhancing students' conceptual understanding of Newtonian mechanics. Nevertheless, *we are always trying to improve them and are very interested in any comments, suggestions, or criticisms that you might care to offer.* You will have a formal opportunity to critique SDI labs in the mid-semester evaluation.

II. MATHEMATICS REVIEW

Vectors describe quantities which possess both magnitude and direction, e.g., displacement, force, velocity, acceleration. In contrast, *scalars* possess only magnitude, e.g., time, temperature, density, volume. Proper treatment of the magnitude and direction of vectors in this lab requires some familiarity with the trigonometric properties of right triangles (i.e., triangles which contain a 90° angle), as contained in the Pythagorean Theorem and the *definitions* given below. (Note that the symbol " \equiv " in Eqs. (2, 3) means "equal to *by definition*.")

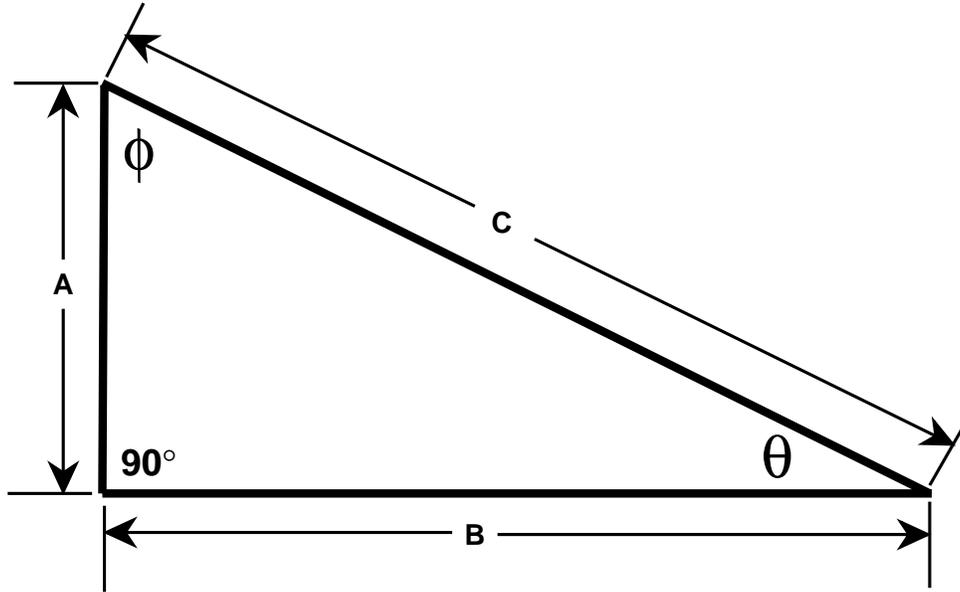


Fig. 1. A right triangle.

Pythagorean Theorem: $A^2 + B^2 = C^2$, $C = (A^2 + B^2)^{1/2}$(1a, b)

Trigonometric Definitions: $\sin\theta \equiv A / C$, $\cos\theta \equiv B / C$, $\tan\theta \equiv A / B = (\sin\theta) / (\cos\theta)$;.....(2a, b, c)

$\sin\phi \equiv B / C$, $\cos\phi \equiv A / C$, $\tan\phi \equiv B / A = (\sin\phi) / (\cos\phi)$ (3a, b, c)

Geometry: $\theta + \phi + 90^\circ = 180^\circ$ (4)

III. YOUR INITIAL POSITION

A. SOME DEFINITIONS*

The *position* of a point is a specification of the relationship between this point and some other objects. These objects are said to constitute a *reference frame*, defined as

Reference frame: *A chosen set of objects relative to which one specifies the positions of points of interest.*(D1)

("D1" stands for "definition #1).

In the present experiment the "chosen set of objects" which serve as a reference frame will be the lab room. This is a common choice of reference frame and is often called a "lab reference frame."

The conventional method of specifying a point relative to some reference frame is to choose a convenient *coordinate system* in that frame. We define a coordinate system as

Coordinate system: *A particular point (called the "origin") and a set of mutually perpendicular reference directions (often called "x, y, z" or "i, j, k") specified relative to some reference frame.*(D2)

B. MEASURE YOUR INITIAL POSITION

In this experiment , the *position* of any point in the lab reference frame can be unambiguously specified in terms of its x-, y-, and z-coordinates for any *arbitrarily selected* coordinate system in that frame. In this experiment we choose a three-dimensional (3D) coordinate system represented by plastic pipe and hung from the ceiling. The plastic pipe was marked at 0.1 meter intervals using a **meterstick**. *The meterstick serves as a (secondary) length standard.* Henceforth, we shall use the term "your lab position" to mean the xyz-coordinates of your eyeball. For present purposes, your eyeball is small enough relative to the dimensions of the lab to be considered a *particle*, defined as

Particle: *An object whose position can be adequately specified by a single point.*(D3)

Measure "your lab position" along the x-axis by carrying out these measurement *operations*:

- (1) look directly at the x-axis *so that your line of sight makes a 90° angle with the x-axis*, and then
- (2) read off your x-position coordinate from the scale on the x-axis, as shown schematically in Fig. 2a.

[Simple "eyeballing" as shown in Fig. 2 will provide sufficient accuracy ($\Delta x \approx \pm 0.2$ m) for present purposes.]

*Here we follow F. Reif, *Understanding Basic Mechanics* (Wiley, 1995)

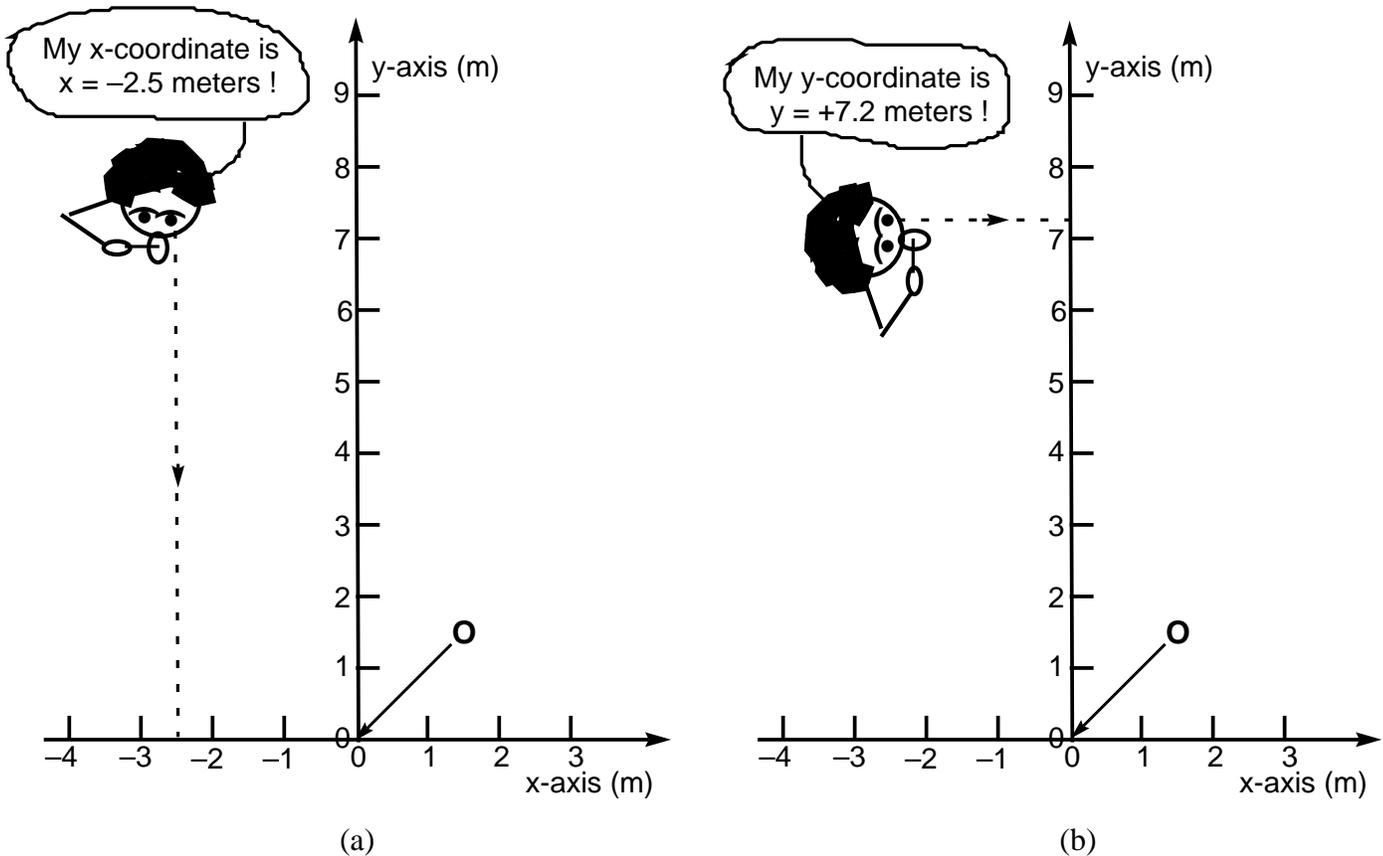


Fig. 2. A student *measures* her/his lab position as $x = -2.5$ meters and $y = +7.2$ meters.

- Similarly, measure your lab position along the y-axis by carrying out these measurement *operations*:
- (1) look directly at the y-axis *so that your line of sight makes a 90° angle with the y-axis*, and then
 - (2) read off your y-position coordinate from the scale on the y-axis, as shown schematically in Fig. 2b.

- Similarly, measure your lab position along the z-axis by carrying out these measurement *operations*:
- (1) look directly at the z-axis *so that your line of sight makes a 90° angle with the z-axis*, and then
 - (2) read off your z-position coordinate from the scale on the y-axis.

Please record your measured initial lab position below (you need not specify the uncertainty, but don't forget the UNITS!):

$x_i =$ _____, $y_i =$ _____, $z_i =$ _____(5)

B. OPERATIONAL DEFINITION OF "POSITION"

To properly understand mechanics, it is necessary to appreciate the meaning and significance of operational definitions[†], defined as

Operational definition: a description which *specifies the experimental significance of a word or concept in terms of well-defined measurement methods*. (D4)

In the above section you determined your lab position in terms of well-defined measurement methods. Thus, a statement of those *concrete operations* would serve to "*operationally define*"[†] the term "lab position" in the restricted sense used here. Can you abstract the essential elements from this particular case to give a **general basic operational definition** of the term "position" for any point "p" in three-dimensional (3D) space? {Y, N, U, NOT} [HINT #1: Your sketch and explanation below should show a point "p" in an xyz-coordinate reference frame and indicate the operations for *marking* the coordinate scales and then *measuring* the position coordinates. HINT #2: If you are uncertain about how to represent a 3D coordinate system on a 2D sheet of paper, then please confer with an instructor.]

[†] See, e.g., G. Holton and S.E. Brush, *Introduction to Concepts and Theories in Physical Science* (Princeton Univ. Press, 2nd ed., 1985) p. 176-181.

IV. YOUR INITIAL POSITION VECTOR \vec{r}_i

A. GRAPHICAL REPRESENTATION OF INITIAL POSITION VECTOR

In addition to the specification of your coordinates (x_i, y_i, z_i) , your initial lab position can also be described by a "**position vector**," \vec{r}_i which has its "tail" at the origin of the coordinate system and its "head" at your initial lab position. [The vector "tail" and "head" correspond, respectively, to the feathered end and tip of the arrowhead of an archer's arrow. In this manual a **vector** is indicated by a boldface letter with an arrow above it (as \vec{r}_i), while the *magnitude of the vector* is indicated by non-boldface (as r_i) or else with vertical bars around the vector symbol (as $|\vec{r}_i|$).] We define a vector as

Vector: A quantity specified jointly by a magnitude (i.e., a positive number) and a direction relative to some reference frame.(D5)

To represent \vec{r}_i to scale as an arrow one must first represent to scale the lab xy-coordinate system as a graph such as that shown in Fig. 3. (In future labs you will be expected to properly draw your own graphs.) On Fig. 3, plot your initial (x_i, y_i, z_i) lab position (Eq. 5) as a data point "•" with a small circle "O" around it (to prevent its being "lost"). (For simplicity ignore your z position since it will remain nearly constant and therefore not affect the displacements of concern in this lab.) Label the point " x_i, y_i ". Then draw (using an ordinary **black** lead pencil in keeping with the course color code) your initial lab-position vector \vec{r}_i as indicated above and label it. Place an arrow "Δ" over the symbol r_i (as in Fig. 8) to indicate that it's a vector.

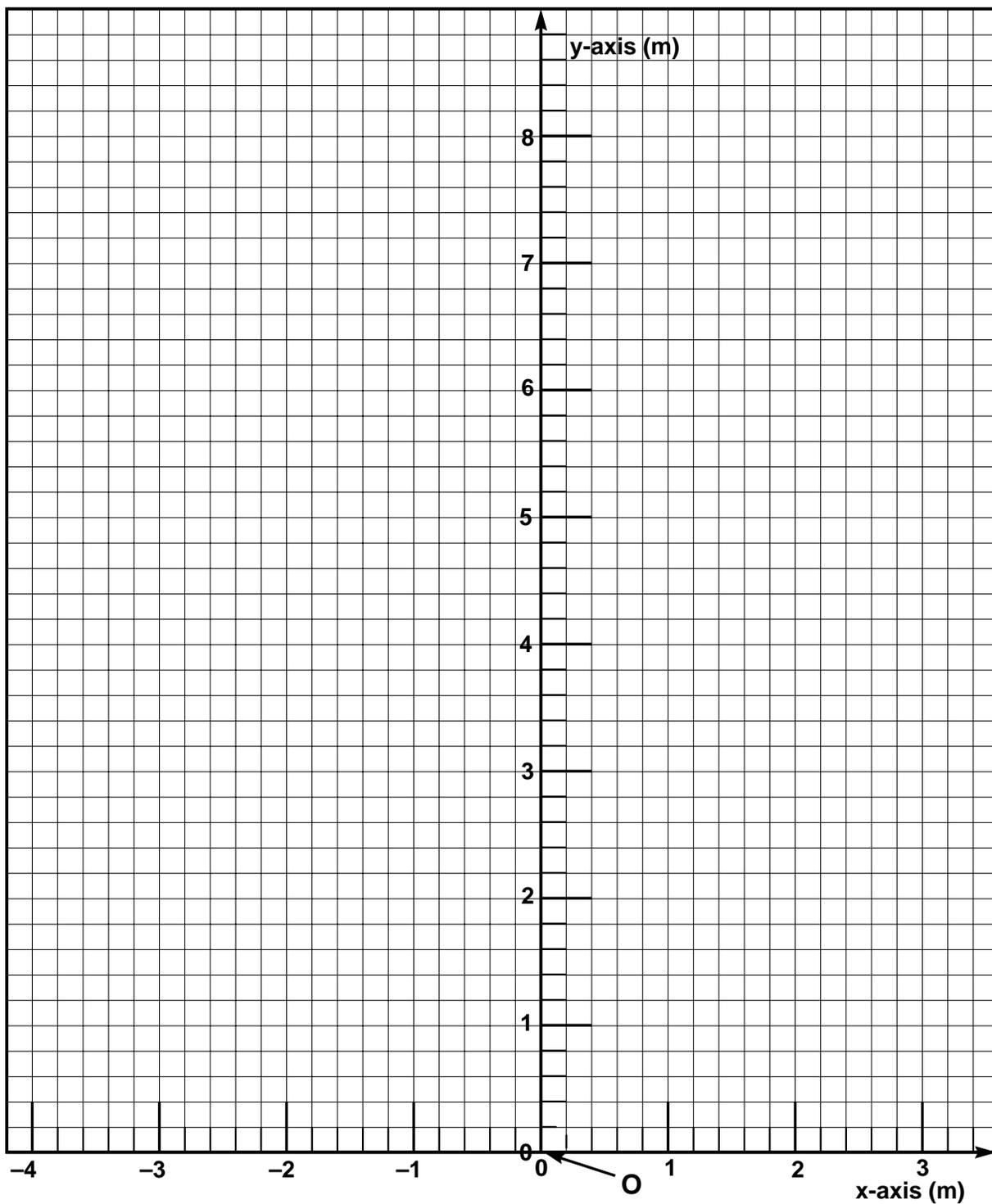


Fig. 3. A graph of the lab xy-coordinate system. The point "O" is the origin. Show and label your initial lab position (x_i, y_i) ; position vector \vec{r}_i as discussed on the preceding page ; and the components x_i and y_i as discussed on the next page.

B. REPRESENTATION OF INITIAL POSITION VECTOR IN TERMS OF ITS COMPONENTS

Aside from its *graphical* representation as a vector arrow in Fig. 3, the vector \vec{r}_i can be represented in several equivalent ways. It is often useful to represent a vector in terms of its scalar *components* in some convenient coordinate system. The coordinate system that's presently convenient is, of course, the one mounted above your head and represented to scale in Fig. 3. Since \vec{r}_i has its tail at the origin, we can find its components by means of the same operations we used to find the lab position coordinates, except that now we can more conveniently perform these operations with pencil and straightedge on the Fig. 3 graph. Note that a scalar component of a vector is a number (sometimes with dimensions) which is characterized by a magnitude and a *sign* (+ or -) in contrast to a vector which is characterized by a magnitude and a *direction*.

To find the x-coordinate of \vec{r}_i , "drop" (i.e., draw in) a line (preferably dashed -----) from the head of \vec{r}_i *perpendicular to the x-axis*. The point where this line intersects the x-axis defines the x component of \vec{r}_i . Similarly, to find the y-coordinate of \vec{r}_i drop a dashed line from the head of \vec{r}_i perpendicular to the y-axis. The point where this line intersects the y-axis defines the y component of \vec{r}_i .

In Fig. 3, find, draw, and then *label* the x and y components of \vec{r}_i and indicate their numerical values below:

$$x_i = \text{_____}, \quad y_i = \text{_____}. \quad \dots\dots\dots(6)$$

Would you expect your results for the components (Eq. 6) to be the same as your results for the position coordinates (Eq. 5)? {Y, N, U, NOT}

A very convenient way to represent the position vector \vec{r}_i is in terms of its components and the *unit vectors* $\hat{i}, \hat{j}, \hat{k}$. (Note that unit vectors are specified by letters with *carets* over them.) A unit vector may be defined as

Unit vector: A vector whose magnitude is 1 (one). $\dots\dots\dots(D6)$

Conventionally the unit vectors $\hat{i}, \hat{j}, \hat{k}$ are taken along the x, y, z axes. Then \vec{r}_i can be written as

$$\vec{r}_i = \hat{i} x_i + \hat{j} y_i + \hat{k} z_i \dots\dots\dots(7)$$

The form of Eq. (7) is often very convenient for vector algebraic manipulation.

C. REPRESENTATION OF INITIAL POSITION VECTOR IN TERMS OF ITS MAGNITUDE AND DIRECTION

On Fig. 4 (next page), draw in \vec{r}_i and its components x_i and y_i as you did before in Fig. 3. Is the geometry of your drawing consistent with the relationship

$$|\vec{r}_i| \equiv r_i = [(x_i)^2 + (y_i)^2]^{1/2} ? \dots\dots\dots(8)$$

{Y, N, U, NOT} [Hint: Compare your drawing in Fig. 3 with Fig. 1.]

In order to obtain the direction of \vec{r}_i , consider (for example) the angle θ which \vec{r}_i makes with the x axis (either the + or - axis, whichever - for convenience - results in a θ which is acute). This angle can be determined, consistent with Eq. 2c:

$$\tan\theta = (|y|/|x|) ; \dots\dots\dots(9)$$

so that

$$\theta = \tan^{-1}(|y|/|x|) . \dots\dots\dots(10)$$

(Recall that $\tan^{-1}a$, called the "arc tangent a" or "inverse a" or "inv a" means "the angle whose tangent is a.") Eq. (8) and (10) serve, then, to specify \vec{r}_i in terms of its magnitude and direction.

[In passing we note that it is often the case that the magnitude and direction of a vector such as \vec{r}_i is known and one needs to find the components of \vec{r}_i . For example, if θ is known, then from Eqs. (2a, b):

$$x_i = r_i \cos\theta, \quad y_i = r_i \sin\theta. \dots\dots\dots(11a, b)$$

Eqs. such as (11a, b) are used so often in mechanics that their application should become second nature.]

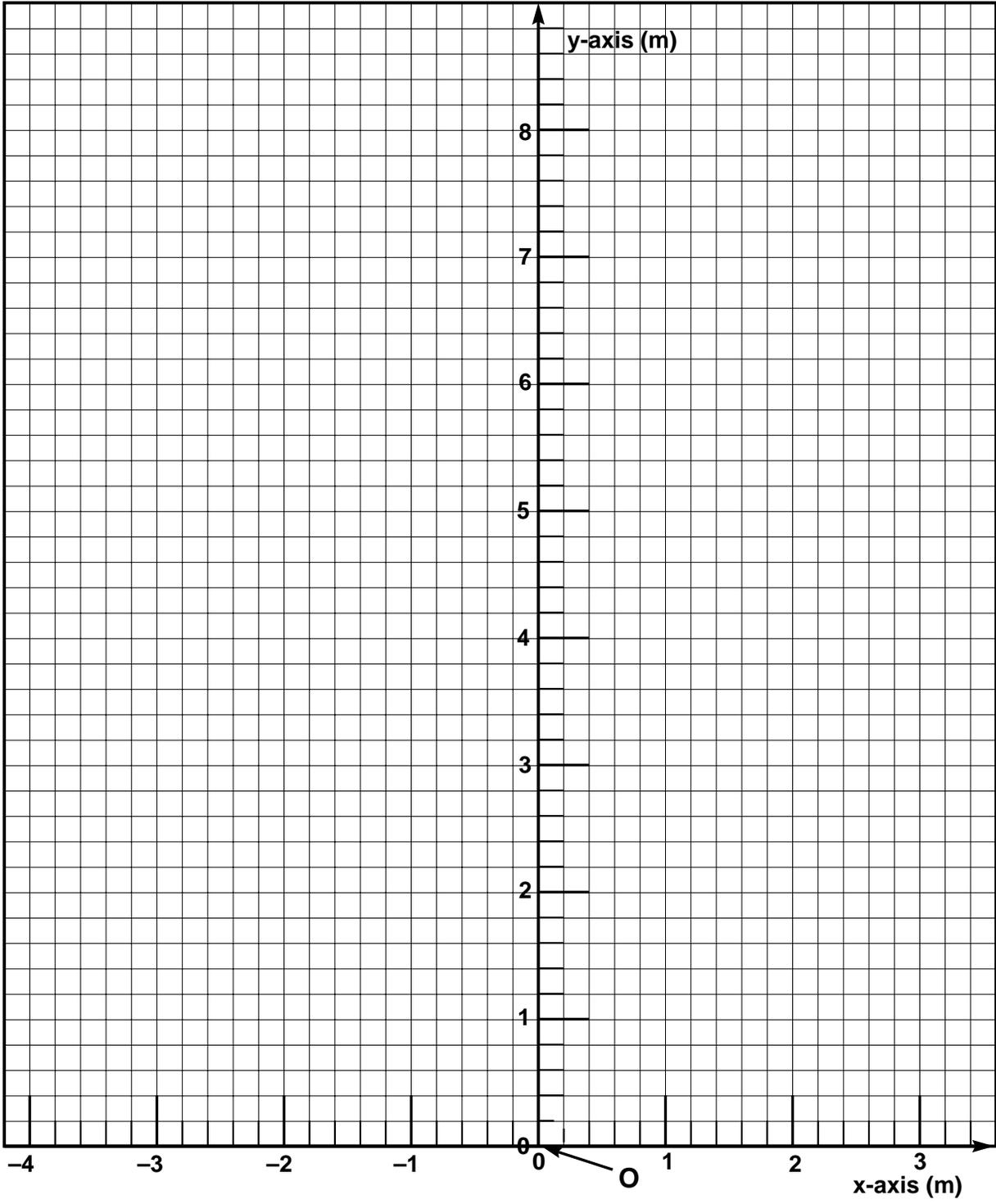


Fig. 4. A graph of the lab xy-coordinate system. Draw in and label \vec{r}_i and its components x_i, y_i as in Fig. 3.

1. Calculate r_i and θ .

In Fig. 4, clearly *label* the angle θ (between \vec{r}_i and the x-axis) that you want to use to specify the direction of \vec{r}_i [why not make life easy and choose an acute (less than 90°) angle?]. Use the components x_i and y_i (Eq. 6) to calculate r_i and θ in accord with Eqs. (8, 10) in the space below.

Here and henceforth in lab reports and homework assignment, *always* (a) draw a clearly labeled sketch or graph (here, Fig. 4), (b) write down the formula being used *in symbolic form*, (c) explicitly show the substitution of numerical values, (d) show the answer with correct UNITS, proper significant figures, and an arrow " \leftarrow " to indicate its location.

Please enter your answers here:

$$r_i(\text{calculated}) = \text{_____}, \theta(\text{calculated}) = \text{_____}. \dots\dots\dots(12)$$

2. Graphically determine r_i and θ .

In Fig. 4, use a compass(or a ruler) and a protractor to determine r_i and θ . Enter these values here:

$$r_i(\text{graphical}) = \text{_____}, \theta(\text{graphical}) = \text{_____}. \dots\dots\dots(13)$$

[Hint: An easy way to determine $r_i(\text{graphical})$ is to use a compass to draw an arc, centered at the origin O, from the tip of \vec{r}_i to the x or y axis. Then read off $r_i(\text{graphical})$ from the scale along the axis.]

3. In the space below calculate the % relative difference between the above calculated and graphical values of r_i and θ . {HINT: For θ : (% rel. diff.) = $[(\theta_{\text{graph}} - \theta_{\text{calc}}) \times 100] / \theta_{\text{calc}}$. }

Are these % relative differences reasonable? {Y, N, U, NOT} [Hint: as a very approximate rule of thumb, reasonably careful graphical work on reasonably *large-scale graphs* usually gives values accurate to $\approx \pm 5\%$.]

V. YOUR FINAL POSITION

A. WALK TO (AND THEN MEASURE) YOUR FINAL POSITION

Congratulations! You’ve done some nice work in this first lab and it’s time for a break! Please take a *meandering* walk from your initial position (x_i, y_i) *out of the lab* and return shortly to a some *final* position (x_f, y_f) *far* from your initial position such that: (1) x_f has a sign *opposite* that of your initial coordinate x_i , (2) the value of $x_f(y_f)$ is *at least 2 meters greater or less* than $x_i(y_i)$. Determine your final coordinates using the method of Sec. IIIA (Fig. 2) above and record them below:

$x_f = \text{_____}, y_f = \text{_____}. \dots\dots\dots(14)$

VI. YOUR FINAL POSITION VECTOR \vec{r}_f

A. GRAPHICAL REPRESENTATION OF FINAL POSITION VECTOR

On Fig. 5 (next page) plot and label your final position coordinates x_f and y_f . Draw in and label your final position vector \vec{r}_f .

B. REPRESENTATION OF FINAL POSITION VECTOR IN TERMS OF ITS COMPONENTS

Determine the components of \vec{r}_f by dropping perpendiculars to the x and y axes as you did in Sec. IVB (Fig. 3). Label the components in Fig. 5 and record them below:

$x_f = \text{_____}, y_f = \text{_____}. \dots\dots\dots(15)$

C. REPRESENTATION OF FINAL POSITION VECTOR IN TERMS OF ITS MAGNITUDE AND DIRECTION

1. Calculate r_f and θ .

In Fig. 5, clearly *label* the angle θ (between \vec{r}_f and the x-axis) that you want to use to specify the direction of \vec{r}_f . Use the components x_f and y_f (Eq. 15) to calculate (in the space below) r_f and θ in accord with Eqs. (8, 10).

Enter your answers here:

$r_f(\text{calculated}) = \text{_____}, \theta(\text{calculated}) = \text{_____}. \dots\dots\dots(16)$

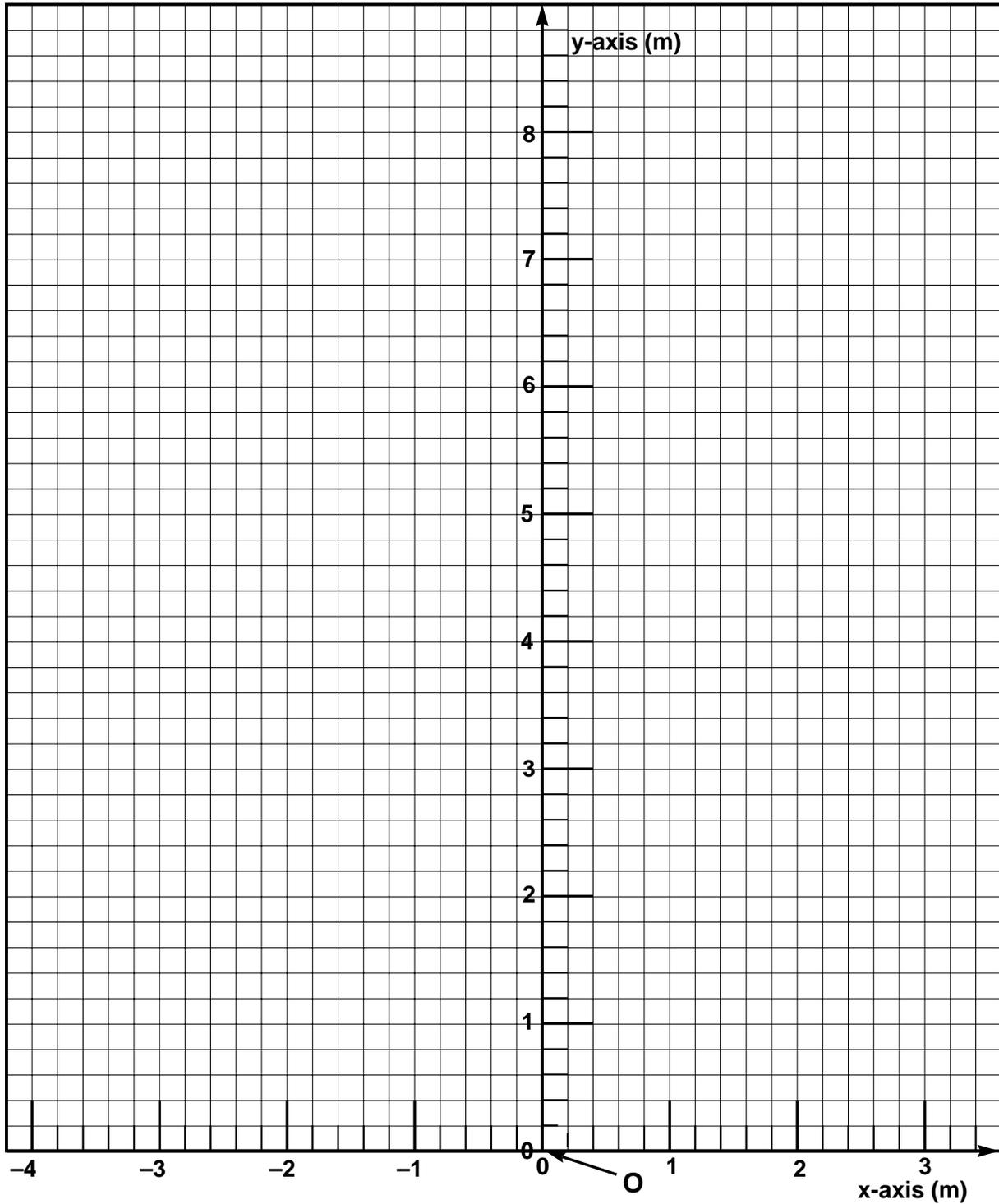


Fig. 5. A graph of the lab xy-coordinate system. Show and label your final lab position (x_f, y_f) ; position vector \vec{r}_f , and components x_f, y_f . Show and label the direction angle θ between \vec{r}_f and the x-axis.

2. Graphically determine r_f and θ .

In Fig. 5, use a compass (or a ruler) and a protractor to determine r_f and θ . Enter these values here:

$$r_f(\text{graphical}) = \underline{\hspace{2cm}}, \theta(\text{graphical}) = \underline{\hspace{2cm}}. \dots\dots\dots(17)$$

3. In the space below calculate the % relative difference between the above calculated and graphical values of r_f and θ .

Are these % relative differences reasonable? {Y, N, U, NOT}

VII. YOUR DISPLACEMENT VECTOR \vec{D}

A. DRAW YOUR DISPLACEMENT VECTOR TO SCALE ON A GRAPH

In Fig. 6, draw in and label \vec{r}_i from Fig. 3 and \vec{r}_f from Fig. 5. The *displacement* vector \vec{D} between the initial position (x_i, y_i) and the final position (x_f, y_f) is defined to be a vector with its tail at (x_i, y_i) (the head of \vec{r}_i) and its head at (x_f, y_f) (the head of \vec{r}_f). In Fig. 6, draw in and label the displacement vector \vec{D} (for an example, see Fig. 8, ignoring the $x'y'$ coordinate system). *The displacement vector \vec{D} represents the **change** in your position when you moved from your initial position to your final position.*

Is the *magnitude* of \vec{D} the same as the *length* of the path you took in moving between your initial and final positions? {Y, N, U, NOT}

Is the *direction* of \vec{D} the same as the *direction* of the path you took in moving between your initial and final positions? {Y, N, U, NOT}

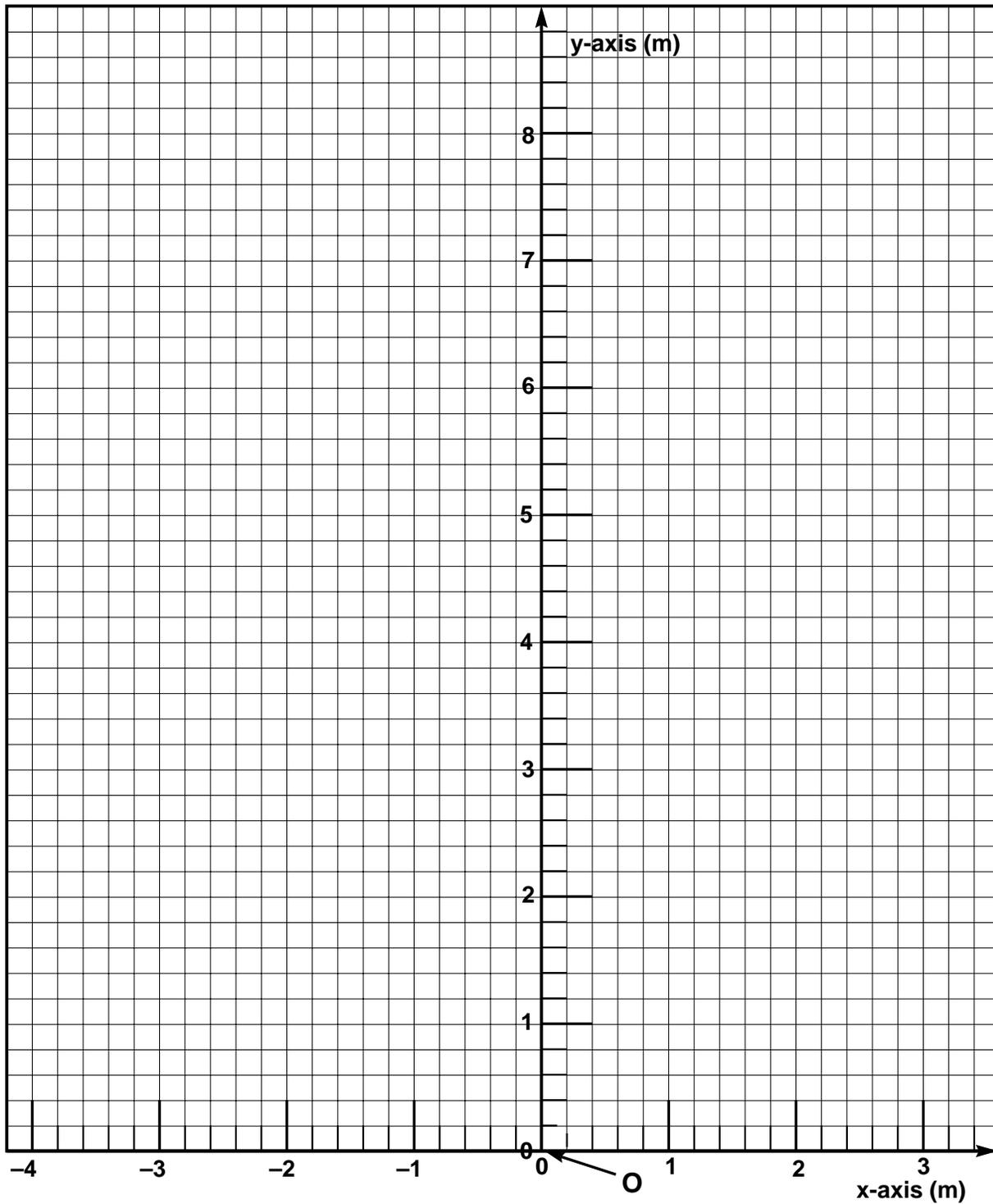


Fig. 6. A graph of the lab xy-coordinate system. Show and label your initial position vector \vec{r}_i ; your final position vector \vec{r}_f ; your displacement vector \vec{D} ; and the *graphically* determined components $D_x(\text{graphical})$, $D_y(\text{graphical})$.

B. VECTOR ADDITION AND SUBTRACTION

Note that the three vectors you have drawn in Fig. 6 form a triangle (see also Fig. 8). They illustrate the process of *vector addition* by means of the graphical "head-to-tail method": the tail of \vec{D} is placed at the head of \vec{r}_i to yield \vec{r}_f , the sum or "resultant" of the two vectors \vec{r}_i and \vec{D} . *Note that \vec{r}_f is drawn from the tail of \vec{r}_i to the head of \vec{D} .*

That such head-to-tail vector addition is physically reasonable can be seen by imagining yourself setting out on a trip from the origin O of Fig. 6 (or 8). First you walk to point (x_i, y_i) with displacement \vec{r}_i between O and (x_i, y_i) . Then you walk from (x_i, y_i) to (x_f, y_f) yielding a displacement \vec{D} between the latter two points. It's easy to see that your final resultant displacement from the origin O would then be given as just :

$$\vec{r}_f = \vec{r}_i + \vec{D} . \dots\dots\dots(18)$$

Eq. (18) is an example of a *vector equation*. It is NOT true, in general, that the *magnitudes* of the vectors are such that $r_f = r_i + D$! Instead, for the present case in which we consider only two dimensions, Eq. (18) is equivalent to the *two* scalar equations

$$x_f = x_i + D_x, \dots\dots\dots(18'a)$$

$$y_f = y_i + D_y. \dots\dots\dots(18'b)$$

Can you write Eq. (18) in terms of the unit vectors \hat{i}, \hat{j} ? {Y, N, U, NOT} (If not, please consult with an instructor.)

Is your unit vector equation consistent with 18'a and 18'b? {Y, N, U, NOT}

Vector addition is "commutative." Thus, in the present case, Eq. (18) implies that

$$\vec{r}_f = \vec{D} + \vec{r}_i . \dots\dots\dots(19)$$

Eq. (19) can be easily verified graphically or analytically (you might wish to do this as an out-of-lab exercise).

Eq. (18) or (19) may be manipulated to yield

$$\vec{D} = \vec{r}_f - \vec{r}_i = \vec{r}_f + (-\vec{r}_i) . \dots\dots\dots(20)$$

Eq. (20) illustrates vector subtraction. It's helpful to regard this as the *addition* of the two vectors \vec{r}_f and $(-\vec{r}_i)$ as indicated on the right hand side of Eq. (20). The vector $(-\vec{r}_i)$ has the *same magnitude* as \vec{r}_i but points in the *opposite direction*.

C. COMPONENTS OF DISPLACEMENT VECTOR \vec{D}

1. By Calculation

Eq. (20), being a *vector equation* in two dimensions, can be decomposed into the *two* scalar equations, one for the x-components and one for the y-components:

$$D_x = x_f - x_i \dots\dots\dots(21a)$$

$$D_y = y_f - y_i \dots\dots\dots(21b)$$

In view of Eq. (21), it is easy to find the components of \vec{D} because we already know the components of \hat{r}_i and \hat{r}_f . Use Eqs. (6, 15, 21) to calculate the components of \vec{D} in the space below.

Please enter your answers here:

$$D_x \text{ (calculated)} = \underline{\hspace{2cm}} \text{ , } D_y \text{ (calculated)} = \underline{\hspace{2cm}} \dots\dots\dots(22)$$

2. By graphical means

In Fig. 6, drop perpendiculars from the head and tail of D to the x and y axis. The length subtended by these perpendiculars on the x(y)-axis is equal to D_x (D_y). Use a compass (or ruler) to find these lengths in Fig. 6, and enter the here:

$$D_x \text{ (graphical)} = \underline{\hspace{2cm}} \text{ , } D_y \text{ (graphical)} = \underline{\hspace{2cm}} \dots\dots\dots(23)$$

3. In the space below calculate the % relative difference between the above calculated and graphical values of D_x and D_y .

Are these % relative differences reasonable? {Y, N, U, NOT}

D. MAGNITUDE AND DIRECTION OF DISPLACEMENT VECTOR \vec{D}

1. Calculate D and θ

In Fig. 6, place a dashed line "-----" through the *tail* of \vec{D} [i.e., through (x_i, y_i)] and *parallel to the x-axis*. Clearly *label* the angle θ (between \vec{D} and this dashed line) that you want to use to specify the direction of \vec{D} . Use the calculated components of \vec{D} (Eq. 21) and equations similar to Eqs. (8, 10) to calculate D and θ :

Enter your answers here:

$$D \text{ (calculated)} = \underline{\hspace{2cm}}, \theta \text{ (calculated)} = \underline{\hspace{2cm}}. \dots\dots\dots(23)$$

2. Graphically determine D and θ .

In Fig. 6, use a compass (or a ruler) and a protractor to determine D and θ and enter these values below:

$$D \text{ (graphical)} = \underline{\hspace{2cm}}, \theta \text{ (graphical)} = \underline{\hspace{2cm}}. \dots\dots\dots(24)$$

3. In the space below calculate the % relative difference between the above calculated and graphical values of D and θ .

Are these % relative differences reasonable? {Y, N, U, NOT}

VIII. CHANGING COORDINATE SYSTEMS

In Sec. III-B, p. 7, it is stated that "the *position* of any point in the lab reference frame can be unambiguously specified in terms of its x-, y-, and z-coordinates for any *arbitrarily selected* coordinate system in that frame." The coordinate system is arbitrary in the sense that it is up to the individual to specify the coordinate system in which s(he) wishes to describe physical properties or events. It would be a bad thing for physics if its laws depended on one's private choice of coordinate system.

Suppose we were to translate our ceiling-hung xy-coordinate system parallel to itself (i.e., with no rotation) so that its origin O is displaced to O' (near the door to the corridor) as shown in Fig. 7. Call the translated system the x'y'-coordinate system, or simply the "primed system." *In the primed system* your initial lab position can be represented as (x_i', y_i') , and your final lab position can be represented as (x_f', y_f') . Likewise *in the primed system*, your initial and final lab position vectors can be represented as \vec{r}_i' and \vec{r}_f' , and your displacement can be represented as \vec{D}' .

Will any of the above primed quantities (i.e., the quantities as represented in the x'y'-coordinate system) be the *same* as the unprimed quantities (i.e., the quantities as represented in the original xy-coordinate system)? {Y, N, U, NOT} [HINT: In Fig. 7, draw your \vec{r}_i , \vec{r}_f and \vec{D} vectors with respect to the xy-coordinate system and your \vec{r}_i' , \vec{r}_f' , and \vec{D}' vectors with respect to the x'y'-coordinate system. Then think *physically* about the meaning of your displacement \vec{D} .]

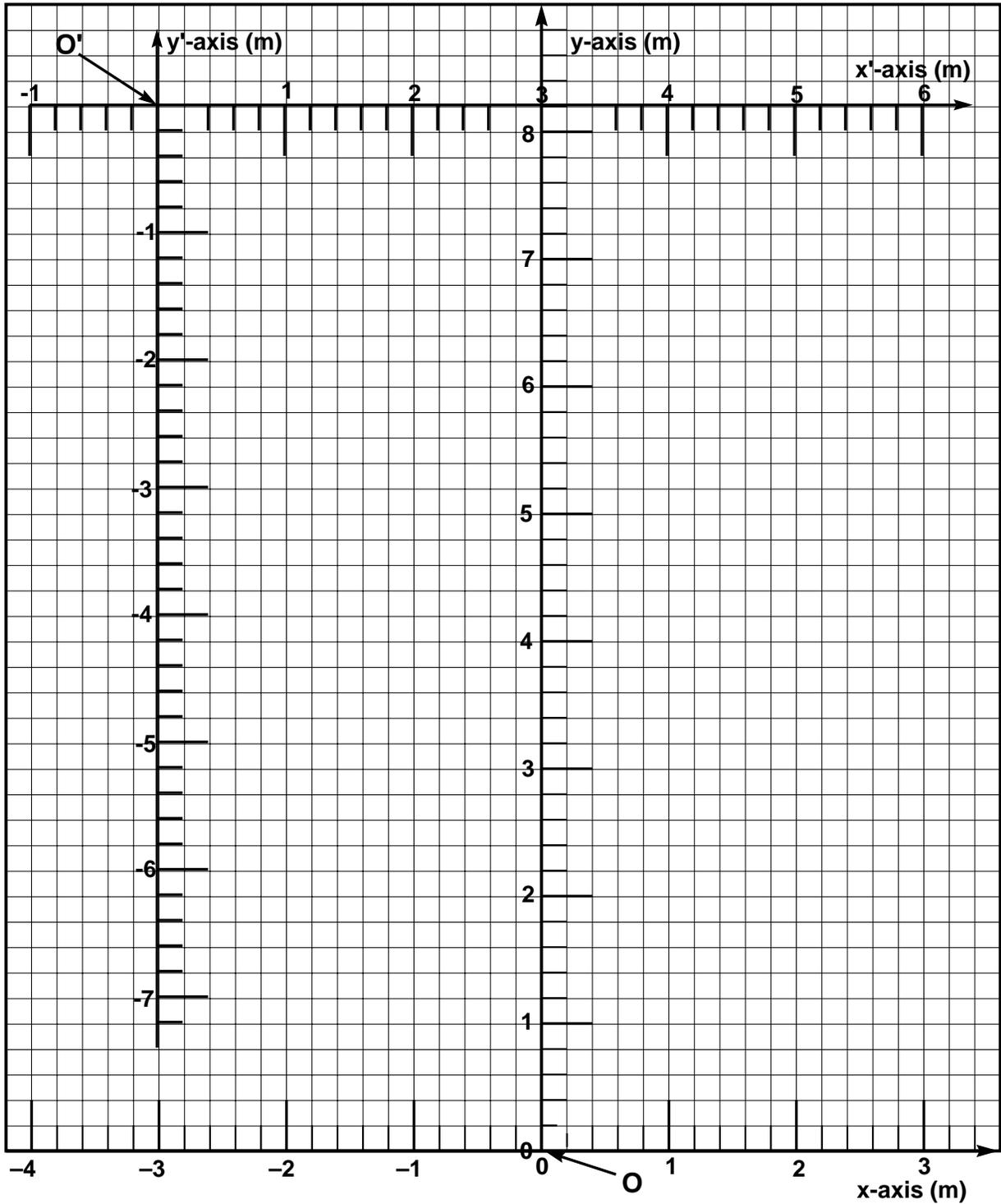


Fig. 7. A graph of the $x'y'$ -coordinate system obtained by translating the origin O of the original xy -coordinate system to the point O' .

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APPENDIX

OLP #1.[†] Famous criminal lawyer Vectoria Sharp (graduate of this course) explains Fig. 8, the vector diagram of a bank robbery to a judge and jury. The robber's initial position vector \vec{r}_i , final position vector \vec{r}_f , and displacement vector \vec{D} as recorded by a bank surveillance camera at O are drawn *with respect to the xy-coordinate system*. A key eye-witness who was at O' during the robbery has testified as to the robber's identity, initial position, final position, and displacement, giving magnitudes and directions *with respect to the x'y'-coordinate system*.

To the awe of judge and jury, Sharp discredits the witness and wins the case by showing that the witness's values are totally inconsistent those deduced by calculating the magnitudes and directions of \vec{r}_i' , \vec{r}_f' , and \vec{D}' from the \vec{r}_i , \vec{r}_f , and \vec{D} vectors as given by the surveillance camera. Sharp's calculation utilizes (a) the coordinate translation vector \vec{T} with tail at O and head at O', (b) a little vector addition.

- (a) As did Sharp, use \vec{T} and vector addition to find the magnitudes and directions of the \vec{r}_i' , \vec{r}_f' , and \vec{D}' from \vec{r}_i , \vec{r}_f , and \vec{D} as given by the camera and shown in Fig. 8. [HINT: First (a) draw the vector \vec{r}_i' in Fig. 8 so as to form a triangle with \vec{T} and \vec{r}_i , (b) write a vector equation which describes the relationship of the three vectors, (c) solve the equation for \vec{r}_i' and for its components (knowing from Fig. 8 the components of \vec{T} and \vec{r}_i .)
- (b) Check your answers by using graphical methods.

[†]Out-of-Lab Problems designated **OLP #N** (N = 1, 2, 3,) will occasionally appear in the lab manuals. Ordinarily these will be assigned as *homework problems*.

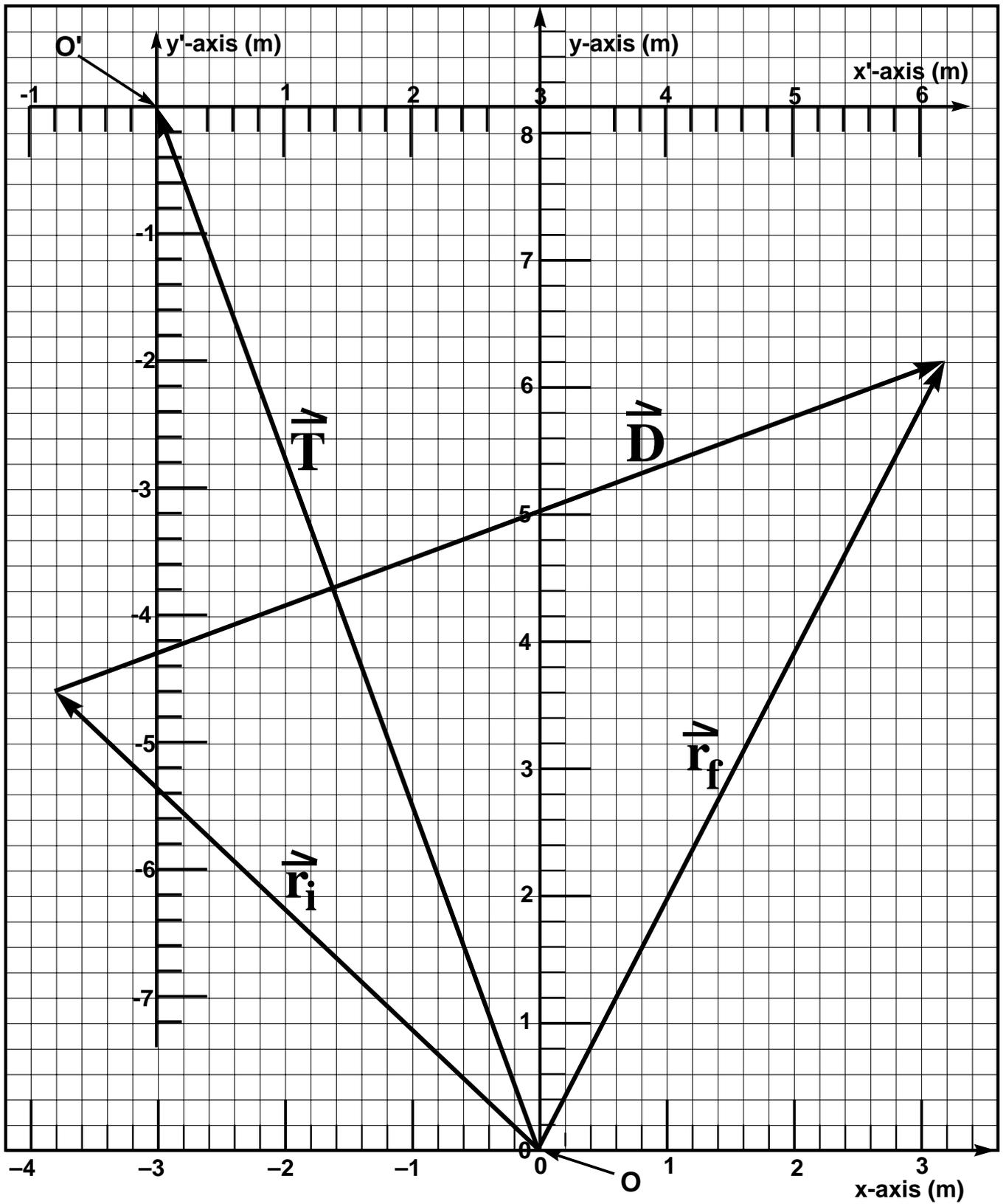


Fig. 8. The vector diagram of a bank robbery as explained in problem OLP #1.