SDI LAB #0.2. INTRODUCTION TO KINEMATICS

NAME ______________________________ ____________________ ______ ______ ______
Last (Print Clearly) First (Print Clearly) ID Number

LAB SECTION _______________________LAB TABLE POSITION___________________________

At present it is the purpose of our Author merely to investigate and to demonstrate some of the properties of accelerated motion (whatever the cause of this acceleration may be).
Galileo, *Dialogues Concerning Two New Sciences* (1638)

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The force probe, sonic motion detector, computer tools, and software were developed at the *Center for Science and Mathematics Teaching* at Tufts University. Except for Section V, this lab is based (under a cross licensing agreement) on parts of the first two "Tools for Scientific Thinking" labs (© 1987–90 CSMT Tufts University): "Introduction to Motion" and "Introduction to Motion – Changing Motion."

I. INTRODUCTION

In SDI Lab #0.1, *Frames of Reference, Position, and Vectors*, you measured your lab position and constructed a general basic *operational definition* of the word *position* in terms of the measurement of xyz coordinates in a meterstick-marked orthogonal coordinate system in a *reference frame.* Here you will consider your **position vs *time.*** [That a trajectory as a function of time t actually does exist is the second part of the "Zeroth Law" (N0) of Newtonian mechanics mentioned in the introduction to SDI Lab #0.1.] Time "t" can be operationally defined as a *clock reading*, where (following Fred Reif) a clock is defined as:

***Clock**: A system repeatedly returning to the same state.* ....(D1)

A time interval Δt can then be operationally defined as the difference $\Delta t = t_f - t_i$ between two clock readings.

In this lab, your position along a line is measured by a detector/computer/software system in an indirect manner which is nonetheless consistent with the basic operational definition of position developed in SDI #0.1. The computer/software measures the *time interval* for the round-trip flight (echo) of sound pulses between the motion detector (a sound transmitter/receiver) and your body. Knowing the speed of the sound-wave pulse, the computer can then calculate your *instantaneous position* at the time the pulse was reflected. Such "sonar" (*so*und *na*vigation *r*anging) measurement of position is used by submarines, bats, and some cameras. As you walk, the computer calculates your position with respect to a one-dimensional (1D) coordinate system (call it an *x-axis*) with origin O at the transmitter-receiver. The computer continuously displays this position as a function of time (see below) on the computer screen.

In these experiments the time is also measured by a computer. The computer contains a clock and prints the position measurements on the screen at closely spaced *clock readings.* Thus the computer displays a "real-time" graph of your position vs time! The computer can also convert the position vs time data to display velocity and acceleration as functions of time using procedures consistent with Eqs. (2) and (4) of this manual. Thus these computer plots are all consistent with the operational definitions of *instantaneous velocity* and *instantaneous acceleration* which have been discussed in the lecture.

A. OBJECTIVES – To understand:

1. the use of the sonar position-measuring device and the *MacMotion*/*PC-Motion* computer program,
2. position vs time graphs,
3. velocity vs time graphs,
4. acceleration vs time graphs,
5. interrelationship of the above three types of graphs,
6. relationship of the above graphs to the *operational definitions* of velocity and acceleration,
7. (Sec. 5) "1" - "6" above with emphasis on the *MacMotion*/*PC-Motion* "tangent and integral tools," and force-motion-vector diagrams.

B. HOW TO PREPARE FOR THIS LAB

1. Study this manual **BEFORE** coming to the lab.
2. Review"Ground Rules for SDI Labs," in SDI#0.1, Sec. C.
3. Review Chapter 2, "Describing Motion: Kinematics in One Dimension," in the course text *Physics,* 4th ed. by Douglas Giancoli (hereafter called "Giancoli"), especially Sec. 2 - 11 "Graphical Analysis of Linear Motion" (or corresponding material in whatever text you may be using).
4. Study the material on vectors in Chapter 3, "Kinematics in Two or Three Dimensions; Vectors" in Giancoli (or corresponding material in whatever text you may be using).

II. MAKING POSITION vs TIME GRAPHS OF YOUR MOTION

A. WALKING TOWARD AND AWAY FROM THE MOTION DETECTOR

Open the *MacMotion*/*PC-Motion* program by double clicking on the *MacMotion*/*PC-Motion* icon. Note that you can make the panel larger and easier to read by clicking on the "zoom button" in the upper right-hand corner of the panel. When you’re ready to start graphing distance, click once on the "Start button" in the bottom left corner of the computer screen. If the sound-pulse transmitter-receiver (let’s call it a motion detector or simply "detector" for short) is ready to go it will make soft clicking sounds.

Note that the detector measures the position of the closest object directly in front of it. Also, the detector correctly measures only positions further than 0.5 meters from itself, so in graphing your own motion stay at least 0.5 meters away from the detector. Since you will start and stop your motion at or beyond the edge of the table, an easy way to insure that you are always at least 0.5 meters from the detector is to tape a meter stick on the table so that it is flush with and perpendicular to the edge of the table. Then place the detector on the table at the 0.5 m position. If you want to repeat a trace then click on the Start Button and the previous curve will be erased.

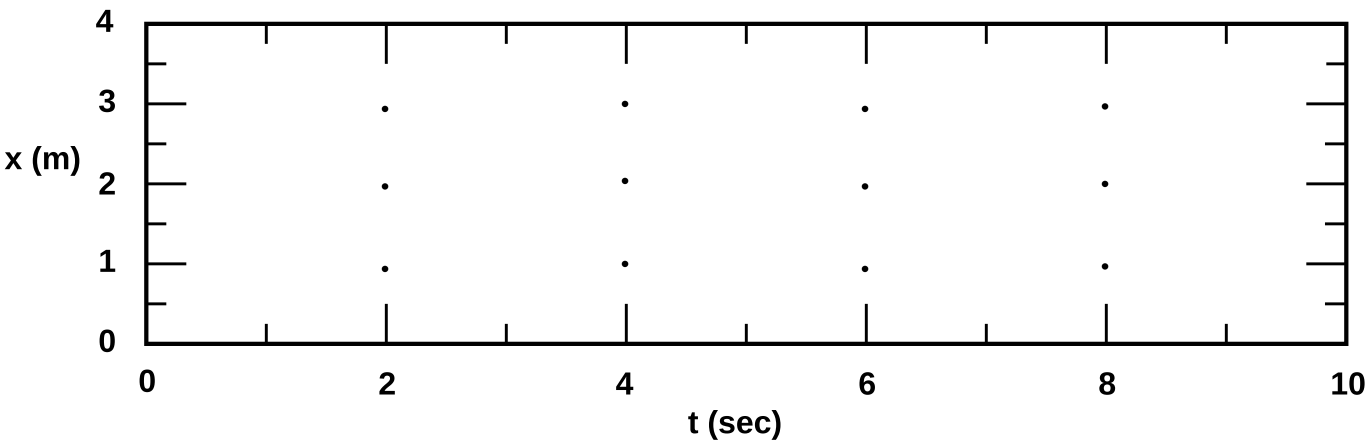
Throughout this lab you will obtain smoother and more accurate position, velocity, and acceleration curves if you (a) walk with smooth, short, shuffling steps without swinging your arms, (b) *always face the detector* (even if this means walking backward) so that you can continuously monitor your motion, (c) hold an approximately 12 x 18-inch box top (in Sec. V a 3/4 x 20 x 24-inch wooden drawing board) in front of you as you walk.

Please make the four position vs time graphs in parts 1-4 on the next two pages. Use an ordinary black-lead pencil to draw all the x vs t curves, in keeping with the SDI color code. (Blue should not be used because position x is *not* the same as displacement.) **It will be much easier to answer the questions on p. 6 if you *keep the scales exactly the same* for parts #1 - #4 below.**

1. Starting about 0.5 meter from the detector, make a position vs time (x vs t) graph by walking ***away from*** the detector ***slowly*** *and steadily* (i.e., at a low constant speed). In the space below, sketch the curve which appears on the screen. Before drawing in the curve it will be helpful to first indicate by marks "O" the points where important changes of the motion took place. (Show the irregular wiggles and bumps in the curve only in a qualitative way.)

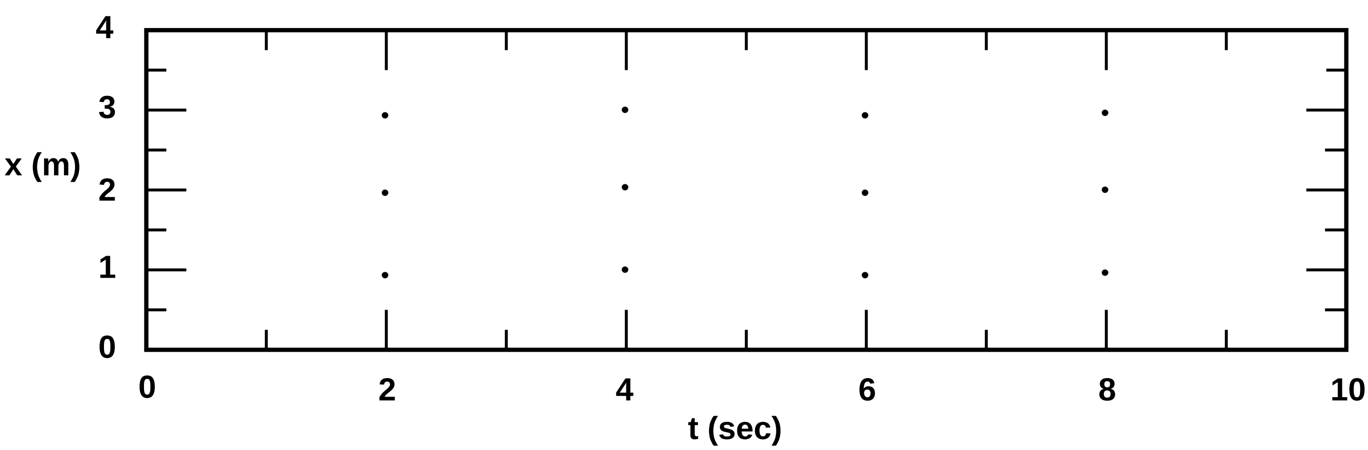
Note that we have changed the labeling of the axes from that which appears on the computer screen, replacing "distance (m)" with the more precise "x (m)" and "time (sec)" with "t (sec)." Here "x" is the *position* of the object being tracked. The detector/computer/software has been arbitrarily configured to regard the motion of the object as along an x-axis with origin at the detector and positive direction *away* from the front of the detector.

#1. SLOW MOTION *AWAY FROM* THE DETECTOR



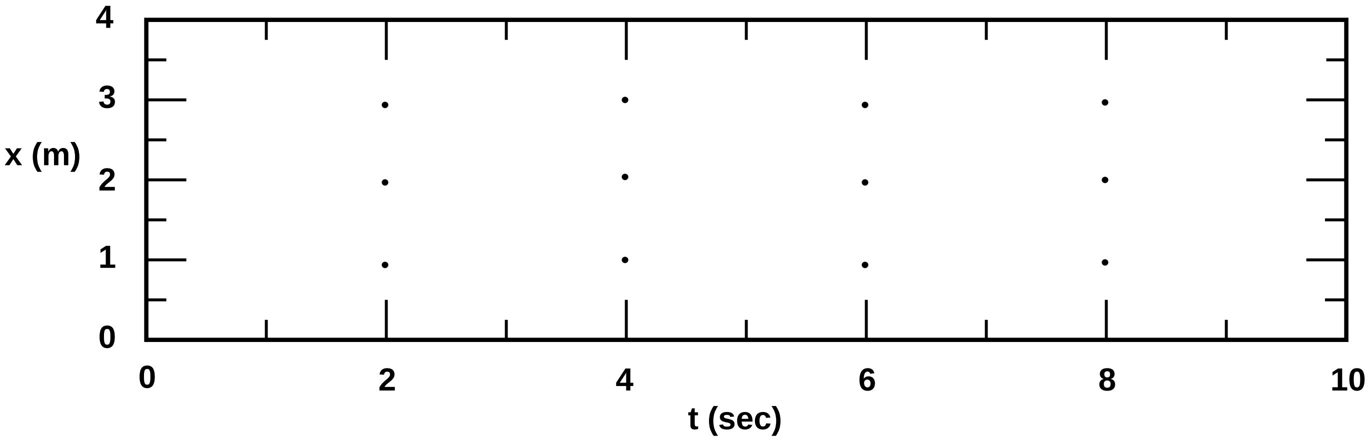
2. Repeat the procedure in "1" above, walking ***away from*** the detector, but now walk somewhat ***faster*** than before (but don't sprint). Sketch the curve in the graph below.

#2. FAST MOTION *AWAY FROM* THE DETECTOR



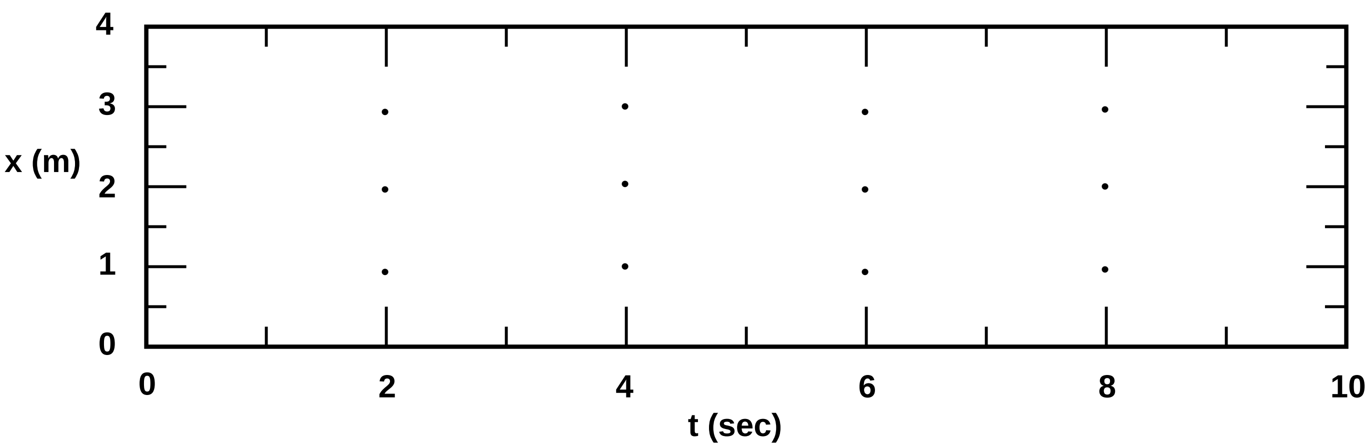
3. Starting several meters from the detector, make a position vs time graph by walking ***towards*** the detector ***slowly*** *and steadily.* Sketch the graph in the space below.

#3. SLOW MOTION *TOWARDS* THE DETECTOR



4. Repeat the procedure in "3" above, walking ***towards*** the detector, but now walk somewhat ***faster*** than before (but don't sprint). Sketch the curve in the graph below.

#4. FAST MOTION *TOWARDS* THE DETECTOR



5. *Qualitatively* describe the similarity and/or difference between
GRAPH #1. SLOW MOTION *AWAY FROM* DETECTOR and
GRAPH #2. FAST MOTION *AWAY FROM* DETECTOR

a. In regard to the sign of x(t) [here and elsewhere in this manual "x(t)" is a shorthand way of writing "x as a function of t" or "x versus t"]:

b. In regard to the sign and magnitude of the *slope* of x(t):

6. *Qualitatively* describe the similarity and/or difference between
GRAPH #1. SLOW MOTION *AWAY FROM* DETECTOR, and
GRAPH #3. SLOW MOTION *TOWARDS* THE DETECTOR.

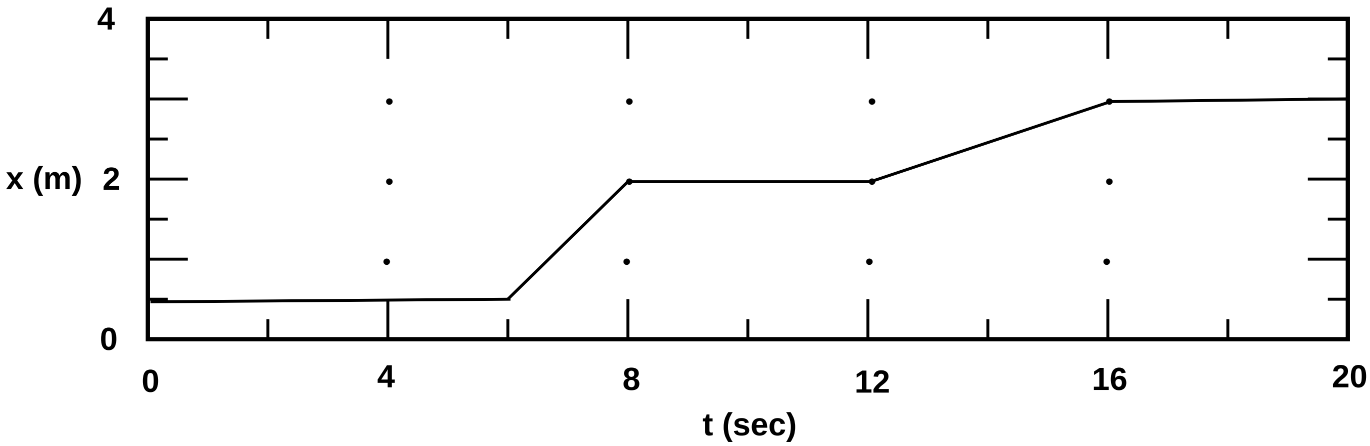
a. In regard to the sign of x(t) [here and elsewhere in this manual "x(t)" is a shorthand way of writing "x as a function of t" or "x versus t"]:

b. In regard to the sign and magnitude of the *slope* of x(t):

B. MATCHING POSITION vs TIME GRAPHS

1. Computer-stored Graph.

To display the computer-stored position vs time graph on the screen: (a) Pull down the File Menu and select **Open**. Double click on **Distance Match**. The "distance graph" below will appear on the screen.



This graph is stored in the computer as **Data B.** New data from the motion detector are always stored as **Data A**, and can therefore be collected without erasing the **Distance.Match** graph. (You can clear any data remaining from previous experiments in **Data A** by selecting **Clear Data A** from the Data Menu.)

Move so as to match the position vs time graph above. Work as a team and keep trying until you achieve a good match. Each person should take a turn. Use a ***black*** pencil to draw in the group's best match in the above graph. (Show the irregular wiggles and bumps in the curve only in a qualitative way.) *Label on the graph the type of walking associated with distinctive parts of the curve*, e.g., if you walked vertically upward towards the ceiling during the first segment (0 – 6 sec) of the curve then write "walking vertically upward"on the graph with an arrow pointing to the first segment. Be sure to indicate the differences in the walking which produced the differences in the slopes of the two positive-slope curves.

III. MAKING VELOCITY vs TIME GRAPHS OF YOUR MOTION

In the previous section the computer displayed your position vs time as you walked away from or towards the detector. The computer is also programmed to show *how fast* you are moving as a function of time. How fast you move is taken here to mean your *instantaneous speed* (as measured for a car by its speedometer). It is the rate of change of position with respect to time. In contrast *instantaneous velocity* is a **vector** quantity. The magnitude of the vector velocity $\vec{\mathbf{v}}$ is the speed and the direction of $\vec{\mathbf{v}}$ indicates the direction of the motion.

For one-dimensional motion along the x-axis it is convenient describe motion in terms of the x-component of the vector velocity. The x-component is positive when in the direction of the positive x-axis and negative when in the direction of the negative x-axis. The *MacMotion*/*PC-Motion* program uses the word "velocity" in place of "x-component of the vector velocity" and for simplicity we shall do likewise in this manual. It is important to keep in mind that (a) the "sign of the velocity" in this manual means the "sign of the x-component of $\vec{\mathbf{v}}$," (b) the vector velocity $\vec{\mathbf{v}}$ has no sign.

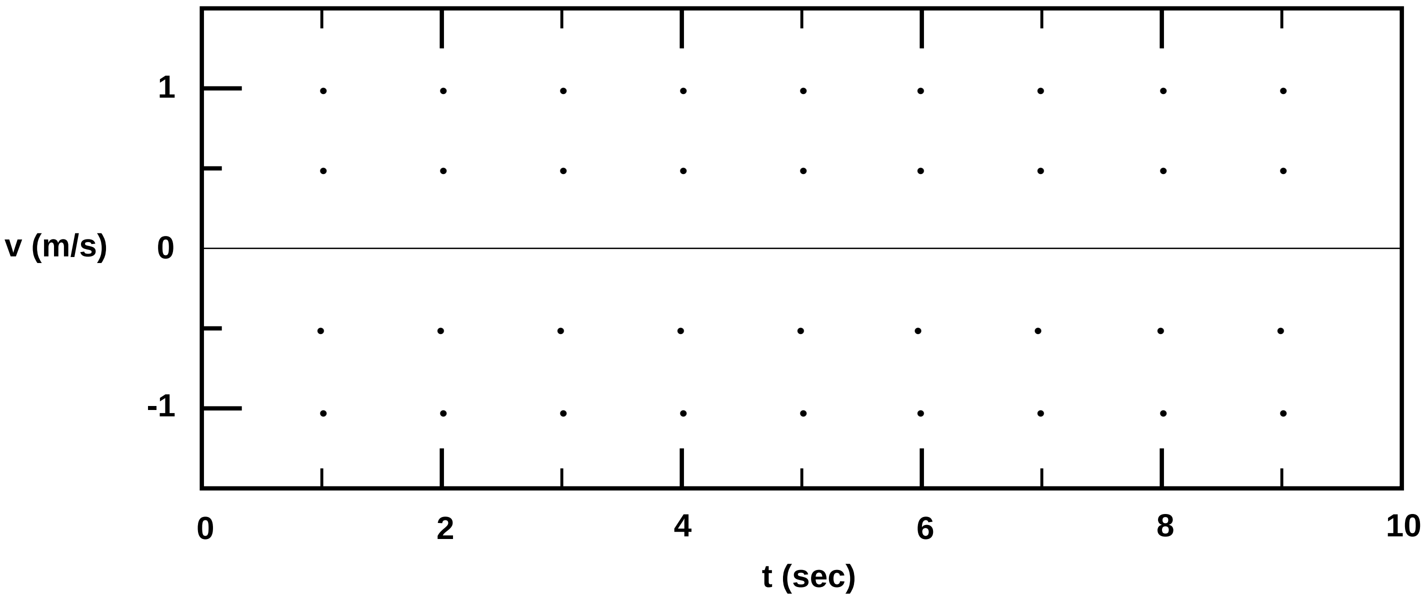
Please make the four velocity vs time graphs in parts 1-4 on the next two pages. Use a ***green*** pencil to draw all the curves, in keeping with the SDI color code that green indicates velocity. **It will be much easier to answer the questions on p. 10 if you *keep the scales exactly the same* for parts #1 - #4 below.**

A. WALKING TOWARD AND AWAY FROM THE MOTION DETECTOR

Prepare the computer to measure velocity. Double click anywhere on the "distance graph" to display the "dialog box." Then select **Velocity** for the ordinate and set the range from **–1** to **1** m/sec. Also change the **Time** scale to read **0** to **10** sec.

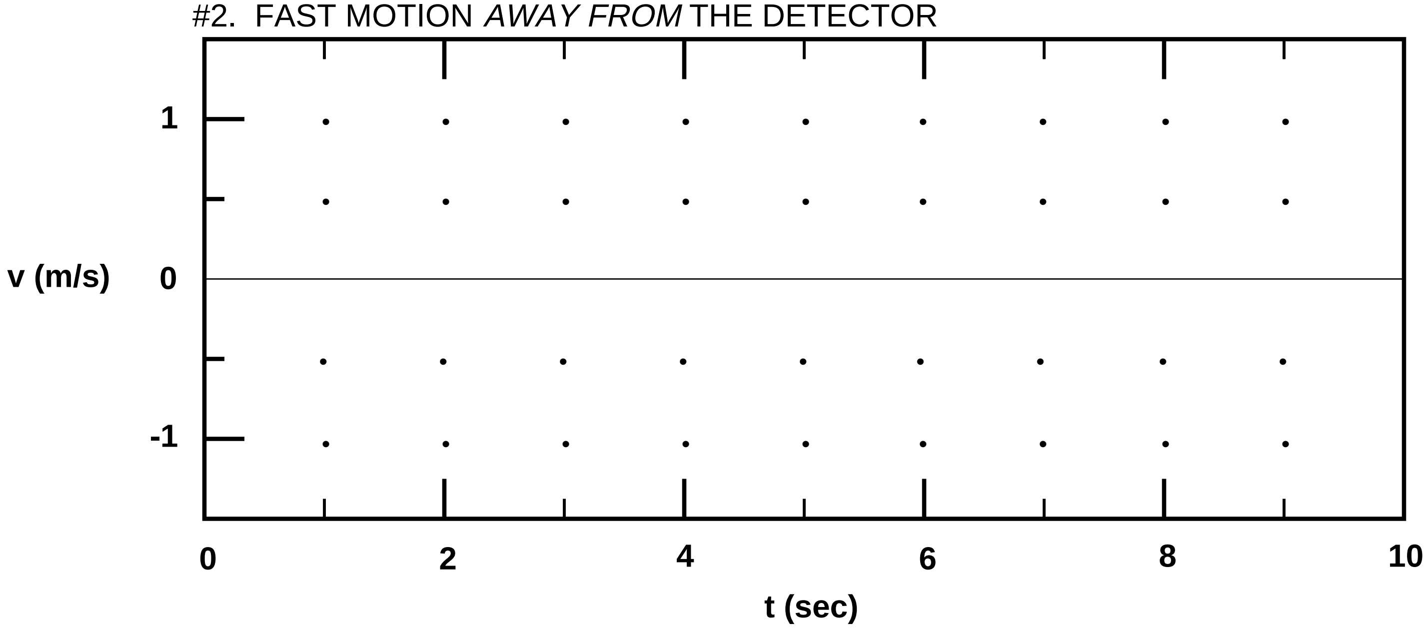
1. Starting about 0.5 meter from the detector, make a velocity vs time (v vs t) graph by walking ***away from*** the detector ***slowly*** *and steadily* (i.e., at a low constant velocity). When you have obtained a satisfactory plot, sketch the curve which appears on the screen in the space below. (Show the irregular wiggles and bumps in the curve only in a qualitative way.)

#1. SLOW MOTION *AWAY FROM* THE DETECTOR



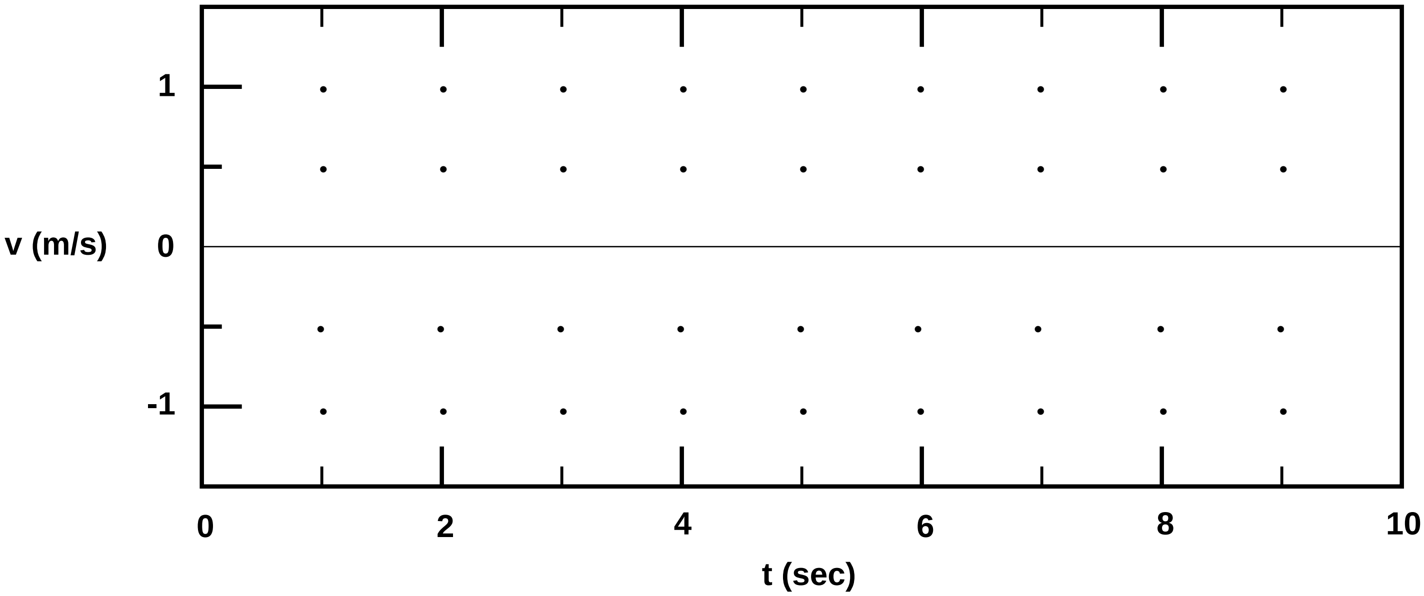
2. Repeat the procedures in "1" above, walking ***away from*** the detector, but now walk somewhat ***faster***. Sketch the graph below.

#2. FAST MOTION *AWAY FROM* THE DETECTOR



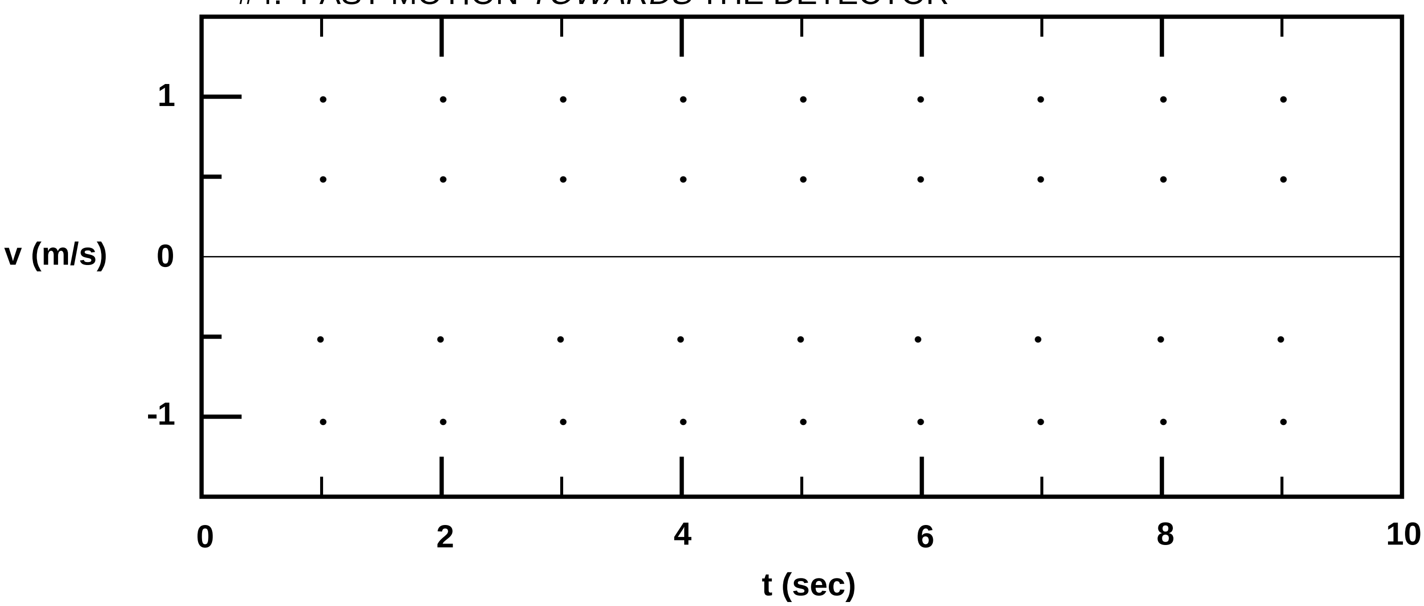
3. Starting several meters from the detector, make a velocity vs time graph by walking ***towards*** the detector ***slowly*** *and steadily*. Sketch the graph in the space below.

#3. SLOW MOTION *TOWARDS* THE DETECTOR



4. Repeat the procedures in "3" above, walking ***towards*** the detector, but now walk somewhat ***faster***. Sketch the graph below.

#4. FAST MOTION *TOWARDS* THE DETECTOR



5. *Qualitatively* describe below the similarity and difference between
GRAPH #1. SLOW MOTION *AWAY FROM* DETECTOR, and
GRAPH #2. FAST MOTION *AWAY FROM* DETECTOR.
a. In regard to the sign and magnitude of v(t): [Hint: Recall that (a) the "sign of v" in this manual means the "sign of the x-component of $\vec{\mathbf{v}}$," (b) the vector velocity $\vec{\mathbf{v}}$ has no sign.]

b. In regard to the sign and magnitude of the *slope* of v(t):

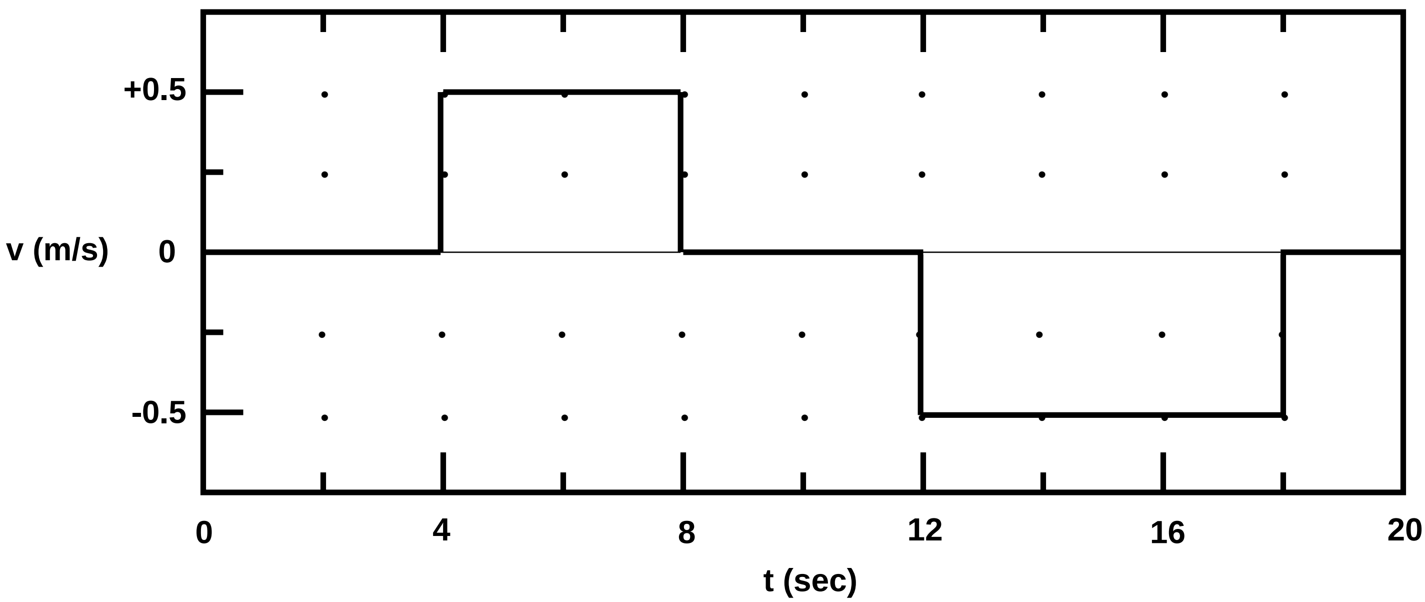
6. *Qualitatively* describe the similarity and difference between
GRAPH #1. SLOW MOTION *AWAY FROM* DETECTOR, and
GRAPH #3. SLOW MOTION *TOWARDS* THE DETECTOR.
a. In regard to the sign and magnitude of v(t):

b. In regard to the sign and magnitude of the *slope* of v(t):

B. MATCHING VELOCITY vs TIME GRAPHS

1. Computer-stored Graph.

To display the computer-stored velocity vs time graph on the screen: (a) Pull down the File Menu and select **Open**. Then double click on **Velocity.Match** (you may need to scroll to see this). The "velocity graph" below will appear on the screen.



2. Move so as to match the velocity vs time graph above. Work as a team and keep trying until you achieve a good match. Each person should take a turn. Use a ***green*** pencil to draw in the group's best match in the above graph. Indicate the starting point x_o= _______m . *Label on the graph the type of walking associated with distinctive parts of the curve*, e.g., if you walked vertically upward towards the ceiling during the first segment (0 – 4 sec) of the curve then write "walking vertically upward" on the graph with an arrow pointing to the first segment.

3. Can you match the curve if you start from a different position x_o'? {Y, N, U, NOT} Try it and record your results with a *dashed* ***green*** line "---------" in the above graph. Indicate the new starting point x_o' = _______m .

4. Can you calculate the *displacement* [$\Delta x = x (t = 20sec) - x (t = 0 sec)$] of a person who moves in accord with the v(t) curve shown in the above figure over the interval t = 0 to t = 20 sec ?
{ Y, N, U, NOT}†

5. Can you calculate the total *area* under the positive and negative parts of the v(t) curve over the interval t = 0 to t = 20 sec? ["Area under" means the area between v(t) and the v = 0 axis, counting area the v = 0 axis as positive, and area below that axis as negative.]
{ Y, N, U, NOT}

6. Is there any general relationship between displacement Δx and the area under the v(t) curve?
{ Y, N, U, NOT}

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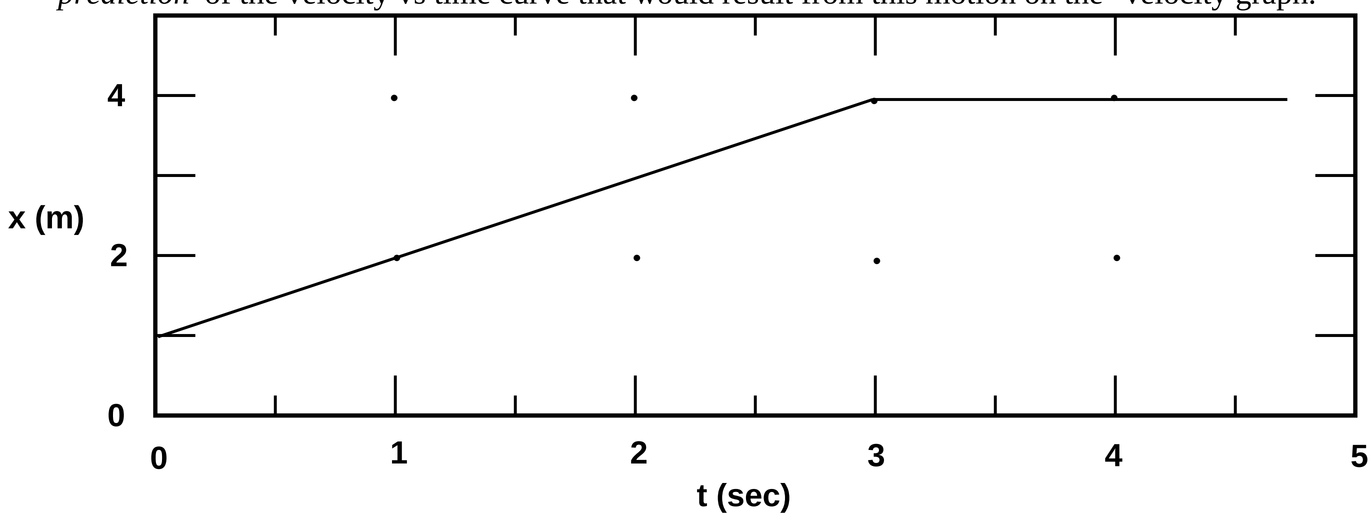
† Recall the SDI lab ground rule that a curly bracket {.......} indicates that you should **ENCIRCLE** O a response within the bracket and then, we **INSIST**, briefly **EXPLAIN** or **JUSTIFY** your answers in the space provided on these sheets. The letters { Y, N, U, NOT} stand for {Yes, No, Uncertain, None Of These}.

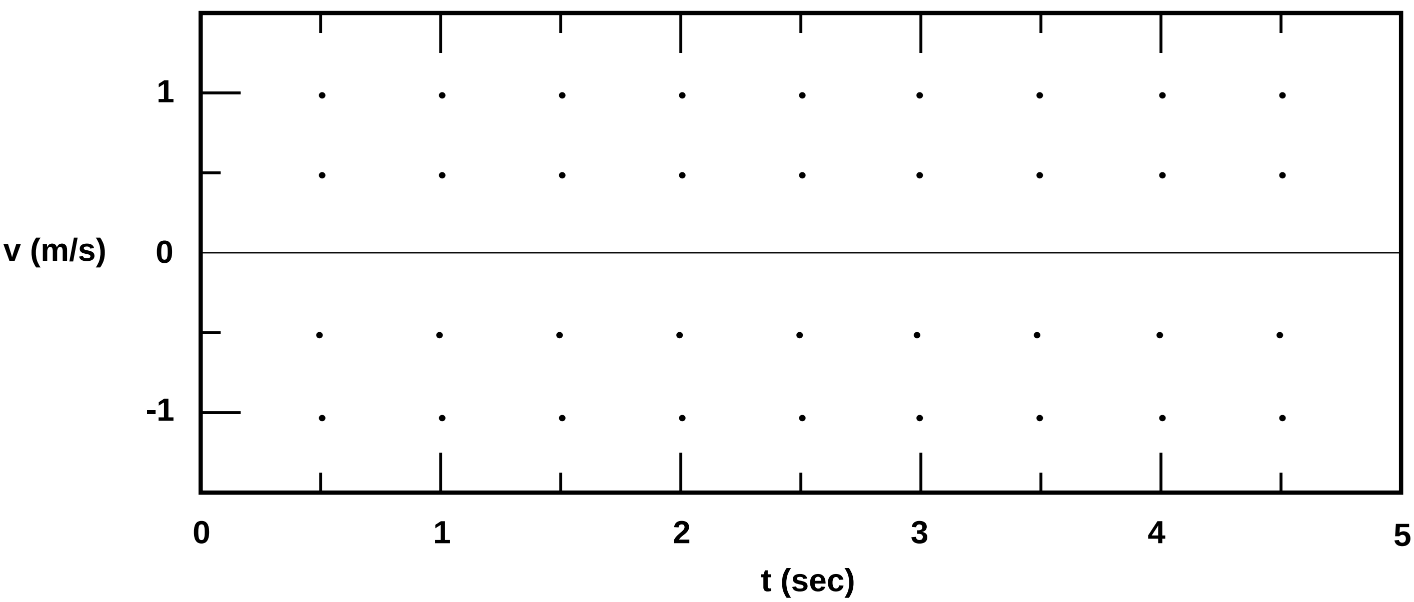
IV. INTERRELATIONSHIP OF POSITION vs TIME AND VELOCITY vs TIME CURVES

Prepare the computer to graph *both* position and velocity. Pull down the display menu and select **Two Graphs**. Double click on the top graph and set it up to display **Distance** from **0** to **4** m for a time of **5** sec. Then set up the bottom graph to display **Velocity** form **–1.5** to **1.5** m/s for **5** sec. Clear any previous data.

A. PREDICT A VELOCITY vs TIME CURVE FROM A POSITION vs TIME CURVE

1. Study the "distance graph" shown below. Using a *dashed* ***green*** line "**-----------**", sketch your *prediction* of the velocity vs time curve that would result from this motion on the "velocity graph."





2. Single click anywhere on the computer screen's "distance graph" so that the computer will first show a real-time graph of your position vs time and then fill in the velocity vs time graph afterwards. Each person in your group should attempt to match the "distance graph" shown above. When you have made a good match sketch the position (***black***) vs time and the velocity (***green***) vs time curves using *solid* lines on the above graphs.

Is your *prediction* of the velocity vs time curve in reasonable agreement with the experimental velocity vs time curve shown above? {Y, N, U, NOT}

3. How would the position vs time [x vs t or x(t)] curve be different if you had moved faster?

4. How would the velocity vs time [v vs t or v(t)] curve be different if you had moved faster?

B. COMPARISON OF x vs t and v vs t CURVES WITH DEFINITION OF VELOCITY

The *definition* of instantaneous velocity for one dimension is

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0}(\Delta\vec{\mathbf{x}}/\Delta t) = \lim_{\Delta t \to 0} [(\vec{\mathbf{x}}_{\mathbf{f}} - \vec{\mathbf{x}}_{\mathbf{i}}) /\Delta t], \qquad (1)$$

where $\vec{\mathbf{x}}_{\mathbf{f}}$ and $\vec{\mathbf{x}}_{\mathbf{i}}$ are, respectively, the final and initial *position vectors* [recall SDI Lab #0.1, "Vectors, Position, and Frames of Reference"] at the end and beginning of the time increment Δt. Eq. (1) constitutes an *operational* definition of $\vec{\mathbf{v}}$ if methods for the *measurement* of $\vec{\mathbf{x}}_{\mathbf{f}}$, $\vec{\mathbf{x}}_{\mathbf{i}}$ and the operations involved in the limiting process can be specified. As previously indicated, one-dimensional motion along the x-axis can be described in terms of the x-component of the vector velocity and following the *MacMotion*/*PC-Motion* convention, we simply call this "v". Thus (1) can be written

$$v \equiv \lim_{\Delta t \to 0}(\Delta x/\Delta t) = \lim_{\Delta t \to 0} [(x_f - x_i) /\Delta t], \qquad (2)$$

where x_f and x_i are the *position coordinates* at the end and beginning of the time increment Δt. Recall that the motion detector/computer/software is configured so that the origin O is at the detector and +x is directed away from the front of the detector.

1. Are your x vs t and v vs t curves on the preceding page qualitatively consistent with the Eq. (2) definition? {Y, N, U, NOT}

C. PREDICTING THE SIGN OF THE VELOCITY

1. Can you predict the *sign* of your velocity if you know (a) the direction of motion, and (b) that the detector/computer regards the positive x direction as the direction away from the front of the detector? {Y, N, U, NOT} [HINT: Consider Eq. (2).]

2. Can you state a general rule? {Y, N, U, NOT} [HINT: Consider Eq. (2).]

V. GRAPHING DISPLACEMENT, VELOCITY, AND ACCELERATION CURVES

A very fine motion, yes indeed.
A very fine motion, yes indeed!
A very fine motion, yes indeed!!
Rise............up...........sister rise! Early American Folk Song

A. EXECUTE, DISPLAY, AND DESCRIBE "A VERY FINE MOTION"

1. Open the *MacMotion*/*PC-Motion* program by double clicking on the *MacMotion*/*PC-Motion* icon. Set the x axis to read 0 to 4 m and the time axis to read from 0 to 10 sec. To save time, *each group should appoint its smoothest walker to produce the fine motion x(t) graph below in Fig. 2.*

2. Starting *at rest* anywhere in front of the detector, execute forward and backward motion in which you (the group's smoothest walker) (a) remain at rest for ≈ 1/2 sec after the detector starts beeping in order to produce an initial constant x baseline trace near t = 0 (see Fig. 1), (b) change your direction of motion at least twice, (c) *smoothly and continuously* vary your speed, (c) produce a ***very*** **sMOOOOOOOOTH** x(t) graph. A possible example is indicated on the next page but, as shown, is probably not smooth enough for a good computer analysis by the methods of this experiment. To obtain a sufficiently smooth curve we strongly recommend (as before) that you (1) walk with smooth, short, shuffling steps without swinging your arms, (2) *always face the detector* (even if this means walking backward) so that you can continuously monitor your motion, (3) hold an approximately 3/4 x 20 x 24-inch wooden drawing board (one is at each table) in front of you as you walk [a box top will probably not yield smooth enough a(t) curves].

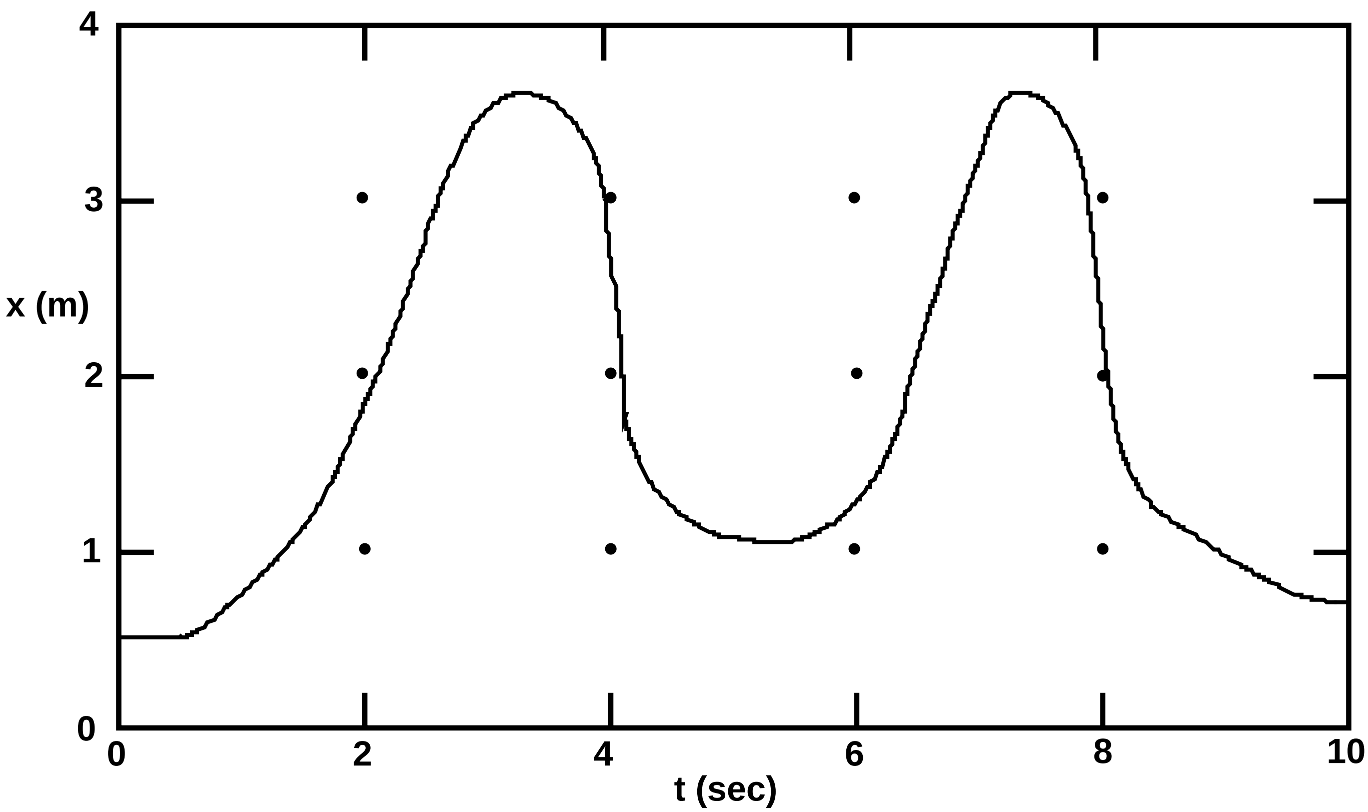


Fig. 1. An example of a motion with three changes in direction at about 3.5, 5.5, and 7.5 seconds. As indicated above, try for an x(t) curve which is considerably *smoother* than this example.

Keep practicing your motion until you obtain a *very smooth* x (t) curve. Protect your data by pulling down the Data Menu and clicking on Data A → Data B. Print a copy for each member of the group (you will need to first press "command p" - this means "hold down the button with the picture of the apple on it and depress the p key"). Trim copies to size with the scissors at your table and tape them on the next page.

Fig. 2. The position vs time curve for the very fine motion of the group's smoothest walker: (insert name)____

B. COMPUTER ANALYSIS OF MOTION USING THE *TANGENT TOOL*

1. Pull down the *Display Menu* and select **Four Graphs** for the data of Fig. 2. Arrange the placement of the graphs so that:

(a) on the left side of the screen

(1) the upper graph shows the *"distance" vs time curve*,

(2) the lower graph shows the *velocity vs time curve*;

(b) on the right side of the screen

(1) the upper graph shows the *velocity vs time curve*,

(2) the lower graph shows the *acceleration vs time curve;*

(c) the time scales of all three graphs are the same and "vertically" aligned for each pair of graphs.

2. Pull down the *Analyze Menu*, select the data (A or B) that you wish to analyze and select "*tangent*" to bring up the ingenious *TANGENT TOOL.* Move a tangent line along the x(t) curve as it appears on the computer screen. The number after "*Tangent =*" at the upper left of the graph is actually the *slope of the tangent* at the "distance"– time point indicated at the bottom left of the graph. The *slope of the tangent* at a point on a curve is *by definition* the *slope of the curve* at that point.

3. Move the tangent line to t = 2 sec, print out copies of the four-graph picture (you will need to first hit "command p" - this means "hold down the button with the picture of the apple on it and depress the p key") for each member of the group. Trim the figures to size with the scissors at your table and tape them above the Fig. 3 caption on the next page. Draw vertical pencil lines at t = 2 sec through the x(t) and v(t) graphs at the right-hand side and the v(t) and a(t) graphs on the left-hand side.

4. Move the tangent line to t = 8 sec, print out copies of the four-graph picture (you will need to first hit "command p" - this means "hold down the button with the picture of the apple on it and depress the p key") for each member of the group. Trim the figures to size with the scissors at your table and tape them above the Fig. 4 caption on the next page. Draw vertical pencil lines at t = 8 sec through the x(t) and v(t) graphs at the right-hand side and the v(t) and a(t) graphs on the left-hand side.

5. For future reference, move the tangent tool along the x(t) curve and transfer the computer's calculations shown at the bottom of the screen to Table 1 below:

Table 1. Computer determined values of kinematic parameters during the fine motion (indicate + or – and specify 3 significant figures).

TIME	POSITION (x)	VELOCITY	ACCELERATION
[sec]	[m]	[m/s]	{m/[(s)(s)]}
2.00			
4.00			
6.00			
8.00			

Fig. 3. Printout of the computer's 4 graphs of x(t) above v(t) on the left, and v(t) above a(t) on the right. Tangents to the curves are shown for t = 2 sec.

Fig. 4. Printout of the computer's 4 graphs of x(t) above v(t) on the left, and v(t) above a(t) on the right. Tangents to the curves are shown for t = 8 sec.

6. Considering the velocity and acceleration curves of Figs. 3 and 4, are there any times at which the velocity is zero *and the acceleration is **not** zero*? {Y, N, U, NOT}

a. If "No", do you think that such a situation is PA (Physically Absurd) ? {Y, N, U, NOT}.

b. If "Yes", please specify these times by drawing *dashed* vertical lines through the v(t) and a(t) graphs on the right hand side of Fig. 3. Do you think errors in the computer curves occurred at these times? {Y, N, U, NOT}.

7. Can you indicate how the slope of the tangent of x(t) relates to the sign and magnitude of the velocity in the graph directly below it on the computer screen? {Y, N, U, NOT}

8. Is the above comparison in accord with Eq. 2, p. 14 (and repeated below) {Y, N, U, NOT}

$$v \equiv \lim_{\Delta t \to 0}(\Delta x/\Delta t) = \lim_{\Delta t \to 0} [(x_f - x_i)/\Delta t], \quad \text{..(2)}$$

9. Recall that the *definition* of "acceleration" is

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0}(\Delta\vec{\mathbf{v}}/\Delta t) = \lim_{\Delta t \to 0} [(\vec{\mathbf{v}}_{\mathbf{f}} - \vec{\mathbf{v}}_{\mathbf{i}})/\Delta t], \quad \text{..(3)}$$

where $\vec{\mathbf{v}}_{\mathbf{f}}$ and $\vec{\mathbf{v}}_{\mathbf{i}}$ are, respectively, the *velocity vectors* at the end and beginning of the time increment Δt. Eq. (3) constitutes an *operational* definition of $\vec{\mathbf{a}}$ if methods for the *measurement* of $\vec{\mathbf{v}}_{\mathbf{f}}$, $\vec{\mathbf{v}}_{\mathbf{i}}$ and the operations involved in the limiting process can be specified. As before, for one-dimensional motion along the x-axis, vector notation is not needed and we can write

$$a \equiv \lim_{\Delta t \to 0}(\Delta v/\Delta t) = \lim_{\Delta t \to 0} [(v_f - v_i)/\Delta t], \quad \text{..(4)}$$

where v_f and v_i are the components of the vector velocity along the x-axis at the end and beginning of the time increment Δt, and a is the component of the vector acceleration along the x-axis.

10. Move the tangent over the v(t) curve as it appears on the computer screen. Notice how small irregularities in the v(t) curve are magnified in the a(t) curve. Can you indicate how the slope of the tangent of v(t) relates to the sign and magnitude of the acceleration in the graph directly below? {Y, N, U, NOT}.

11. Is the above comparison in accord with Eq. 4 {Y, N, U, NOT}

C. SNAPSHOT SKETCHES (*force-motion-vector diagrams)*

1. On the x(t) curve of Fig. 2, use an ordinary lead pencil to draw a circle of about 1/4 inch diameter to represent the person who produced that curve at his/her x position for t = 2, 4, 6, 8 seconds. *Color the person yellow.* Show the appropriate velocity (***green***) and acceleration (***orange***) vectors *with their tails on the person.* Be sure that both the $\vec{\mathbf{v}}$ vectors and the $\vec{\mathbf{a}}$ vectors are drawn roughly to scale in accord with the v and a values tabulated in Table 1. Of course, the vector lengths need not be exactly to scale since we are primarily interested in the *qualitative* time variation of x, $\vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$.

[**HINT**: Since the motion depicted is parallel to the x direction, the $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ vectors must be shown in either the positive or negative x direction. With the addition of these vectors, Fig. 2 becomes more complicated and it needs to be understood that the lengths of the $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ vectors are proportional to the magnitudes of the velocity of the walker in m/s and the acceleration of the walker in m/s^2.]

2. Can you relate the direction and magnitude of the $\vec{\mathbf{v}}$ vector at t = 2, 4, 6, and 8 sec to the x(t) curve? {Y, N, U, NOT}

3. Can you relate the direction and magnitude of the $\vec{\mathbf{a}}$ vector at t = 2, 4, 6, and 8 sec to the x(t) curve? {Y, N, U, NOT} [Hint: $\vec{\mathbf{a}}$ is equal to the time-rate of *change* of the slope of x(t). As t increases through t = 2 sec, how is the slope of x(t) changing? If the slope of of x(t) is *increasing* in time (becoming less negative or more positive) then $\vec{\mathbf{a}}$ is in the direction of positive x; if the slope is decreasing in time (becoming more negative or less positive) then $\vec{\mathbf{a}}$ is in the direction of negative x.]

4. Do you think kinematic diagrams and graphs such as Figs. 2 - 4 could be used advantageously in "Teach Yourself Dancing" manuals (ballroom, tap, square, ballet, belly, etc.) ? {Y, N, U, NOT}

D. COMPUTER ANALYSIS OF MOTION USING THE *INTEGRAL TOOL* (Optional)

1. Pull down the *Analyze Menu*, select the data (A or B) that you wish to analyze and select "*integral*" to bring up the ingenious *INTEGRAL TOOL.* Starting from an initial time t = 0, hold down the mouse button and move the heavy black vertical line along the velocity vs time curve until you reach some time $t = t_f$. At t_f release the mouse button. The number after "*Integral* =" at the upper right of the graph is just the value of the integral of the v(t) curve between t = 0 and $t = t_f$ [i.e., just the area under the v(t) curve between t = 0 and $t = t_f$].

2. For t_f = 2 sec, print out copies of the four-graph picture (you will need to first press "command p") for each member of the group, trim them to size with the scissors at your table, and tape them above the Fig. 5 captions on the next page.

3. For t_f = 8 sec, print out copies of the four-graph picture (you will need to first press "command p") for each member of the group, trim them to size with the scissors at your table, and tape them above the Fig. 6 captions on the page after the next page.

4. In order to see the relationship of the area under the v(t) curve between t = 0 and $t = t_f$ to the value of the displacement at t_f, fill in Table 2 below (indicate + or – and specify 3 significant figures where possible) :

Table 2. Computer determined values of kinematic parameters using the "integral tool" to obtain the area under the v(t) curve (indicate + or – and specify 3 significant figures).

TIME	POSITION (x)	POSITION (x)	DISPLACEMENT	AREA UNDER v(t)
t (final)	at t (final)	at t = 0	x [t(final)] - x (t=0)	for 0 < t < t (final)
[sec]	[m]	[m]	[m]	[m]
2.00				
8.00				

Fig. 5. Printout of the computer's 4 graphs of x(t) above v(t) on the left, and v(t) above a(t) on the right. Areas under the curves between t = 0 and t = 2 sec are shown.

Fig. 6. Printout of the computer's 4 graphs of x(t) above v(t) on the left, and v(t) above a(t) on the right. Areas under the curves between t = 0 and t = 8 sec are shown.

5. Can you indicate the relationship of the area under the v(t) curve between t = 0 and t = t_f to the value of the displacement at t_f? {Y, N, U, NOT}

6. Is the relationship above in accord with Eq. (2), repeated below {Y, N, U, NOT}

$$v \equiv \lim_{\Delta t \to 0}(\Delta x/\Delta t) = \lim_{\Delta t \to 0} [(x_f - x_i)/\Delta t], \quad \text{...(2)}$$

[HINT: It may help to show a sketch.]

7. In order to see the relationship of the area under the a(t) curve between t = 0 and t = t_f to the change in v between t = 0 and t = t_f fill in Table 3 below:

Table 3. Computer determined values of kinematic parameters using the "integral tool" to obtain the area under the a(t) curve (indicate + or – and specify 3 significant figures).

TIME	v	v	CHANGE IN v	AREA UNDER a(t)
t (final)	at t (final)	at t = 0	v [t(final)] - v (t=0)	for 0 < t < t (final)
[sec]	[m/s]	[m/s]	[m/s]	[m/s]
2.00				
8.00				

8. Can you indicate the relationship of the area under the a(t) curve between t = 0 and $t = t_f$ to the change in the value of the velocity between t = 0 and $t = t_f$? {Y, N, U, NOT}

9. Is the relationship above in accord with Eq. (4), repeated below {Y, N, U, NOT}

$$a \equiv \lim_{\Delta t \to 0}(\Delta v/\Delta t) = \lim_{\Delta t \to 0} [(v_f - v_i)/\Delta t], \quad \text{...(4)}$$

[HINT: It may help to show a sketch.]

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