SDI LAB # 3. CIRCULAR MOTION AND FRICTIONAL FORCES*

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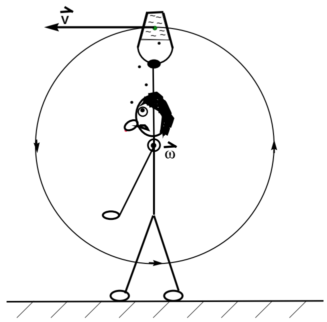
LAB SECTION _______________________LAB TABLE POSITION__________________________



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* SDI Lab #3, RRH, 4/27/98. Partially supported by NSF Grant DUE/MDR-9253965. © Richard R. Hake, 1998. (You may change, modify, copy, and distribute at will for use in your own institution, but please contact R.R. Hake before distributing your version beyond your own institution.)

I. INTRODUCTION

The experiments of this lab should help to deepen your understanding of Newton's second law, $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$, as it is applied to circular motion. Here it is important to realize that the velocity $\vec{\mathbf{v}}$ is a vector, and that *changes* in $\vec{\mathbf{v}}$ (thus the existence of an acceleration $\vec{\mathbf{a}}$) may occur due to changes in the ***direction*** of $\vec{\mathbf{v}}$ even while the ***magnitude*** of $\vec{\mathbf{v}}$ *remains constant.* A simple example is "*uniform* circular motion," i.e., motion in a circle at *constant* speed. In this lab we experience such motion and directly feel the forces associated with it. After considering frictional forces which are directed *opposite* to $\vec{\mathbf{v}}$, we investigate cases in which frictional forces are directed *along* $\vec{\mathbf{v}}$ in linear motion and along the change in $\vec{\mathbf{v}}$ in circular motion. Finally, we study *non-uniform* circular motion in which *both* the magnitude and direction of $\vec{\mathbf{v}}$ change.

A. OBJECTIVES

1. To understand Newton's second law as applied to uniform or nearly uniform circular motion:
   a. water in a bucket rotated in a vertical circle,
   b. the moon orbiting the earth,
   c. the conical pendulum.

2. To understand "tension" in ropes, cables, or strings (as in the conical pendulum).

3. To understand frictional forces and Newton's second law for:
   a. linear motion in the case of:
      (1) pushing a massive box across a table, sliding of the box on a cart which is accelerating with respect to the lab frame of reference;
      (2) the tablecloth-slip-out trick;
   b. circular motion of a body on a rotating turntable.

4. To derive expressions from Newton's laws for the critical angular frequency ω_c:
   a. below which water drops out of bucket rotated in a vertical circle,
   b. for a conical pendulum as the angle between the string and the vertical approaches zero,
   c. for sliding of a body placed on a rotating turntable.

5. To judge whether or not expressions for physical parameters (such as the ω_c's above) are *physically reasonable* by considering dimensions and predicted magnitudes for both realistic and extreme limiting conditions.

6. To devise and execute experimental tests of expressions for physical parameters (the ω_c's above).

7. To understand the relationship of angular velocity $\vec{\omega}$, tangential acceleration $\vec{\mathbf{a}}_t$, radial acceleration $\vec{\mathbf{a}}_r$, and the net acceleration $\vec{\mathbf{a}}$ for a body in circular motion with both zero and non-zero angular acceleration $\vec{\alpha}$.

8. To understand, devise, and execute controlled-variable experiments.

9. To gain an appreciation for the value of orthographic views in diagraming physics problems.

10. To study motion as it appears to observers in accelerating reference frames (Appendix).

B. HOW TO PREPARE FOR THIS LAB

1. **Review "Ground Rules for SDI Labs," Sec. I-C of SDI Lab #0.1**

2. Before you begin, it will also help to study the following sections in the course text *Physics*, 4th ed. by Douglas Giancoli (or similar material in whatever text you are using): Chap. 4, Sec. 4.8, "Problems Involving Friction, Inclines"; Chap. 5, "Circular Motion, Gravitation"; Chapter 8, Sec. 8.1, "Angular Quantities." In addition please review ALL the material covered in lectures and in homework on circular motion and frictional forces.

II. FORCE $\vec{\mathbf{F}}$, VELOCITY $\vec{\mathbf{v}}$, ACCELERATION $\vec{\mathbf{a}}$, AND ANGULAR VELOCITY $\vec{\omega}$ FOR A BODY IN UNIFORM CIRCULAR MOTION.

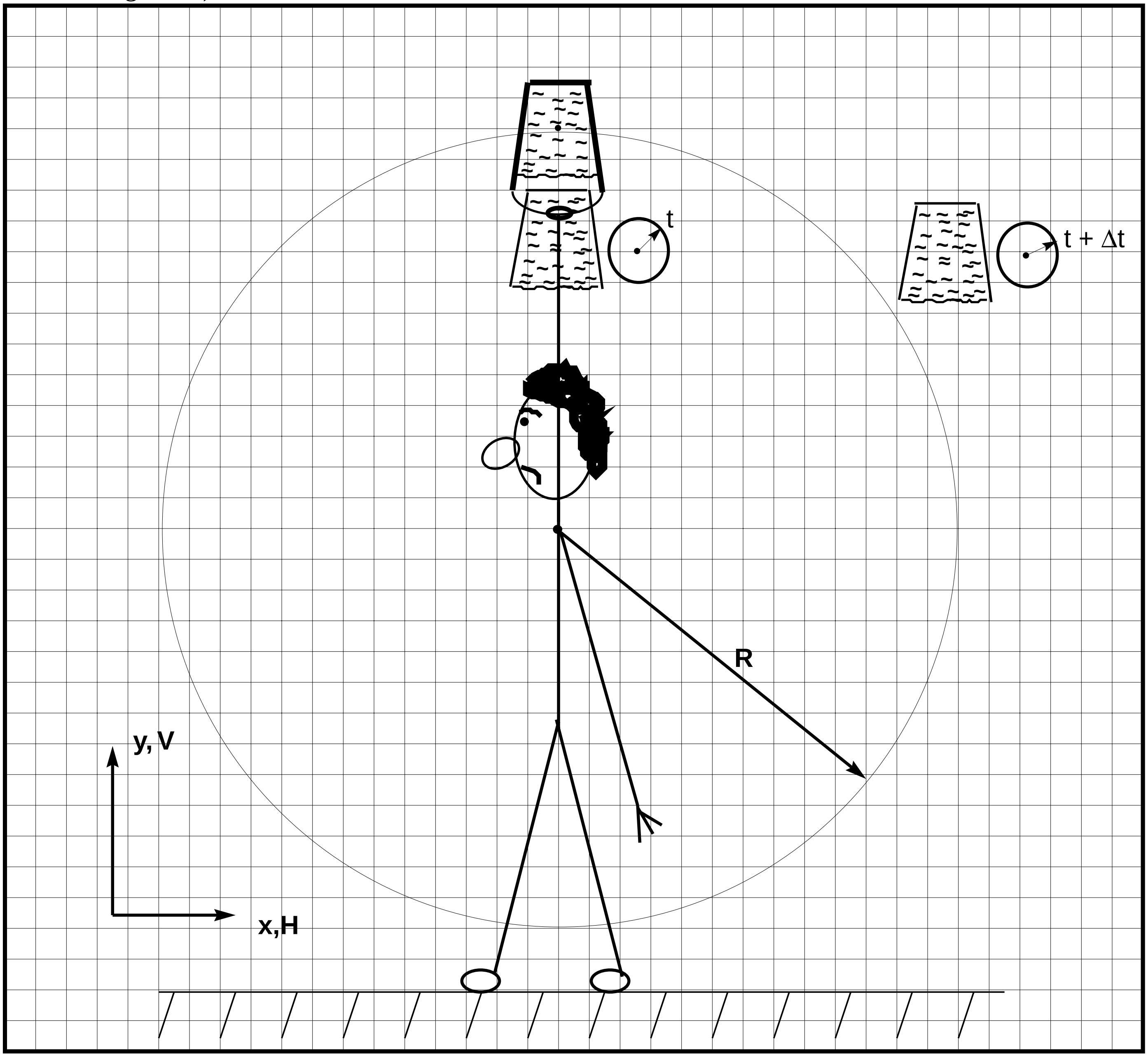
A. BUCKET OF WATER†

1. Fill one of the plastic buckets at your table about half full of water. Go to an area of the lab where there's plenty of clearance and no other students. (Have a mop and wringer-pail at the ready.) Hold the bucket *inverted* and *stationary* over your head. (You may wish to don a bathing suit – no dry labing!! Forgot your suit? OK, do this as a thought experiment.)

The sketch on the next page shows you, the bucket, the water, and the floor at an instant in time t when the *water* is partway between the bucket and your head and the velocity of the water is $\vec{\mathbf{v}}_i$, and *just the water* at a slightly later time t + Δt (displaced to the side so vectors which you are to draw don't overlap) when the velocity of the water is $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \Delta\vec{\mathbf{v}}$. The subscripts "i" ("f") indicate the initial (final) velocities at the beginning (end) of the interval Δt, consistent with the definition $\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0}(\Delta\vec{\mathbf{v}}/\Delta t) = \lim_{\Delta t \to 0}[(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)/\Delta t]$. In the diagram on the next page show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$ vectors for the water at t; the $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ vectors at t + Δt; and the $\Delta\vec{\mathbf{v}}$ vector for Δt.

† For the less venturesome, rubber balls are a good substitute for water.

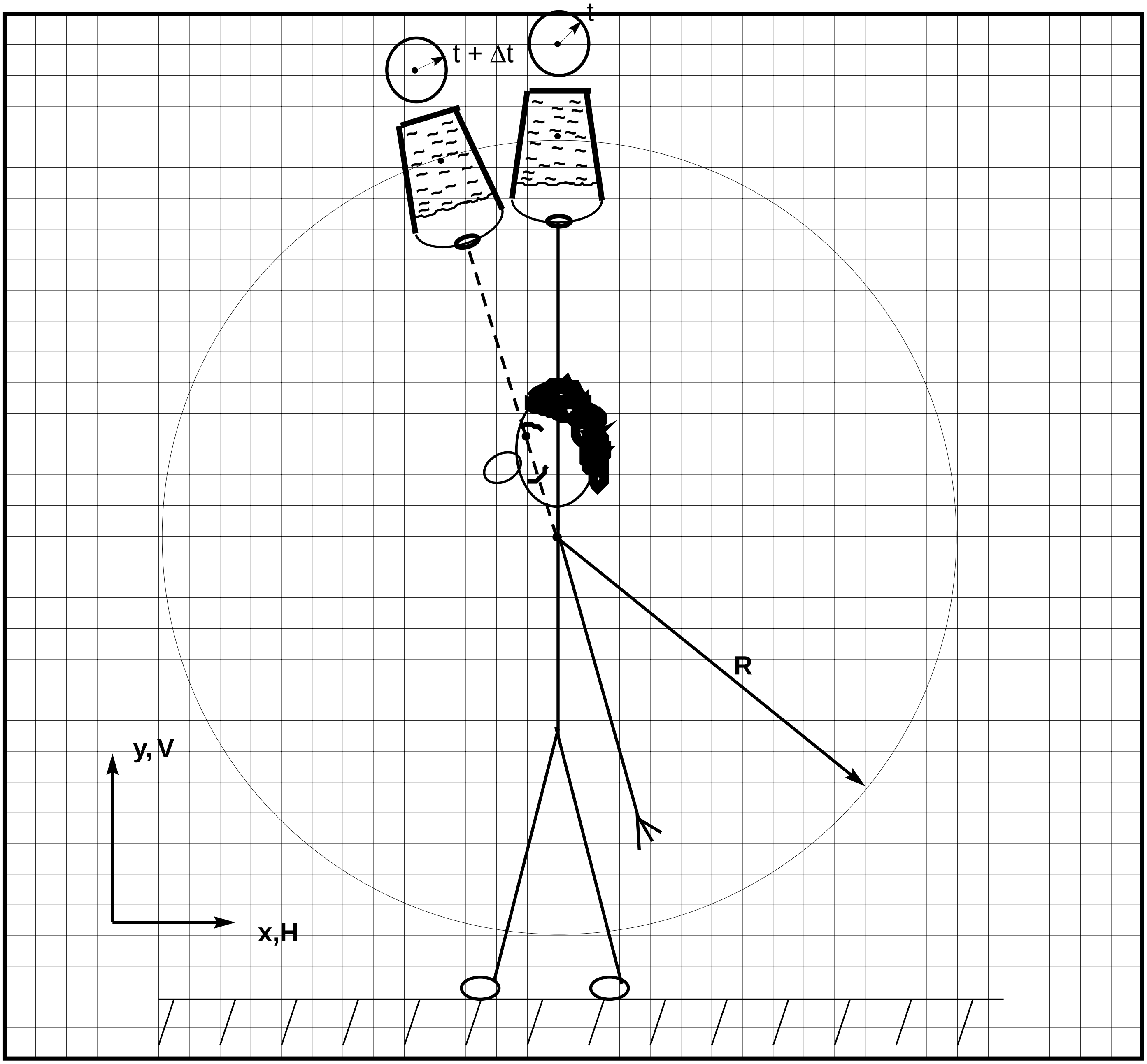
YOU HOLD A STATIONARY BUCKET OF WATER UPSIDE DOWN OVER YOUR HEAD SO THAT THE WATER **DOES FALL OUT** OF THE BUCKET (for simplicity we take artist's license with the shape of the falling water)



2. Do you understand why the water **does** fall out of the bucket? {Y, N, U, NOT} [HINT: Here and below, "why" asks for the Newtonian interpretation. Consider Newton's second law (N2), $\vec{\mathbf{F}}_{\text{net on body}} = m_{body}\vec{\mathbf{a}}_{body}$, and the *definition* of acceleration $\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0}(\Delta\vec{\mathbf{v}}/\Delta t) = \lim_{\Delta t \to 0}[(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)/\Delta t]]$, where $\vec{\mathbf{v}}_f$ and $\vec{\mathbf{v}}_i$ are, respectively, the velocities at the end and beginning of the time increment Δt.]

3. Repeat "1" above, except now *swing the bucket of water* ***very rapidly*** *in a vertical circle*. To simplify the situation, try to swing the bucket with a nearly constant angular velocity $\vec{\omega}$. The sketch below shows you, the bucket, the water, and the floor at an instant in time t when the *water* is at the top of its path with velocity $\vec{\mathbf{v}}_i$ and at a slightly later time (t + Δt) when the velocity of the water is $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \Delta\vec{\mathbf{v}}$, consistent with the definition $\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0}(\Delta\vec{\mathbf{v}}/\Delta t) = \lim_{\Delta t \to 0}[(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)/\Delta t]$. Show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$ vectors for the water at t; the $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ vectors at t + Δt; and the $\Delta\vec{\mathbf{v}}$ vector for Δt.

YOU ROTATE A BUCKET OF WATER **VERY RAPIDLY** OVER YOUR HEAD AT A NEARLY CONSTANT ANGULAR VELOCITY $\vec{\omega}$ IN A VERTICAL CIRCLE SO THAT THE WATER **DOES NOT FALL OUT** OF THE BUCKET



4. Do you understand why the water **does NOT** fall out of the bucket? {Y, N, U, NOT}

HINT #1: Study the statement of "2" above and your answer.

HINT #2: Just as in "2," consider N2 and the definition of $\vec{\mathbf{a}}$.

HINT #3: Can "2" and "4" be explained on the same basis? {Y, N, U, NOT}

5. Repeat "3," but now pay attention to the force $\vec{\mathbf{F}}_{\text{on hand by bucket}}$ felt by you as you rotate the bucket around your head. How does the $\vec{\mathbf{F}}_{\text{on hand by bucket}}$ at the bottom of the path compare with $\vec{\mathbf{F}}_{\text{on hand by bucket}}$ at the top of the path?

6. It can be shown by application of N2 and N3 that for the case of a light plastic bucket $\vec{\mathbf{F}}_{\text{on bucket by hand}} \approx \vec{\mathbf{F}}_{\text{on water by bucket}}$.† How, then, from your observations in "5," does $\vec{\mathbf{F}}_{\text{on water by bucket}}$ at the bottom of the path compare with $\vec{\mathbf{F}}_{\text{on water by bucket}}$ at the top of the path?

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† According to N2, $\vec{\mathbf{F}}_{\text{net on bucket}} = m_{\text{bucket}} \vec{\mathbf{a}}_{\text{bucket}}$..(1)

But in this situation $m_{\text{bucket}} \vec{\mathbf{a}}_{\text{bucket}} \approx 0$ because m_{bucket} is very small in comparison with m_{water}. Therefore $\vec{\mathbf{F}}_{\text{net on bucket}} = \vec{\mathbf{F}}_{\text{on bucket by hand}} + \vec{\mathbf{F}}_{\text{on bucket by water}} + m_{\text{bucket}} \vec{\mathbf{g}} \approx 0$. Now since the last term is negligible, $\vec{\mathbf{F}}_{\text{on bucket by hand}} + \vec{\mathbf{F}}_{\text{on bucket by water}} \approx 0$, which implies that $\vec{\mathbf{F}}_{\text{on bucket by hand}} \approx -\vec{\mathbf{F}}_{\text{on bucket by water}}$ ....(2)

Applying N3 to the right-hand side of (2) yields $\vec{\mathbf{F}}_{\text{on bucket by hand}} \approx \vec{\mathbf{F}}_{\text{on water by bucket}}$ ....QED*

This is similar to the situation where one pulls on a low-mass string tied to a ball with a force $\vec{\mathbf{F}}_{\text{on string by hand}}$ and the tension T in the string, equal to $|\vec{\mathbf{F}}_{\text{on string by hand}}|$, is transmitted undiminished through the string to produce a force $\vec{\mathbf{F}}_{\text{on ball by string}} = \vec{\mathbf{F}}_{\text{on string by hand}}$. (See, e.g., the discussion for the conical pendulum in Sec. II-D5.)

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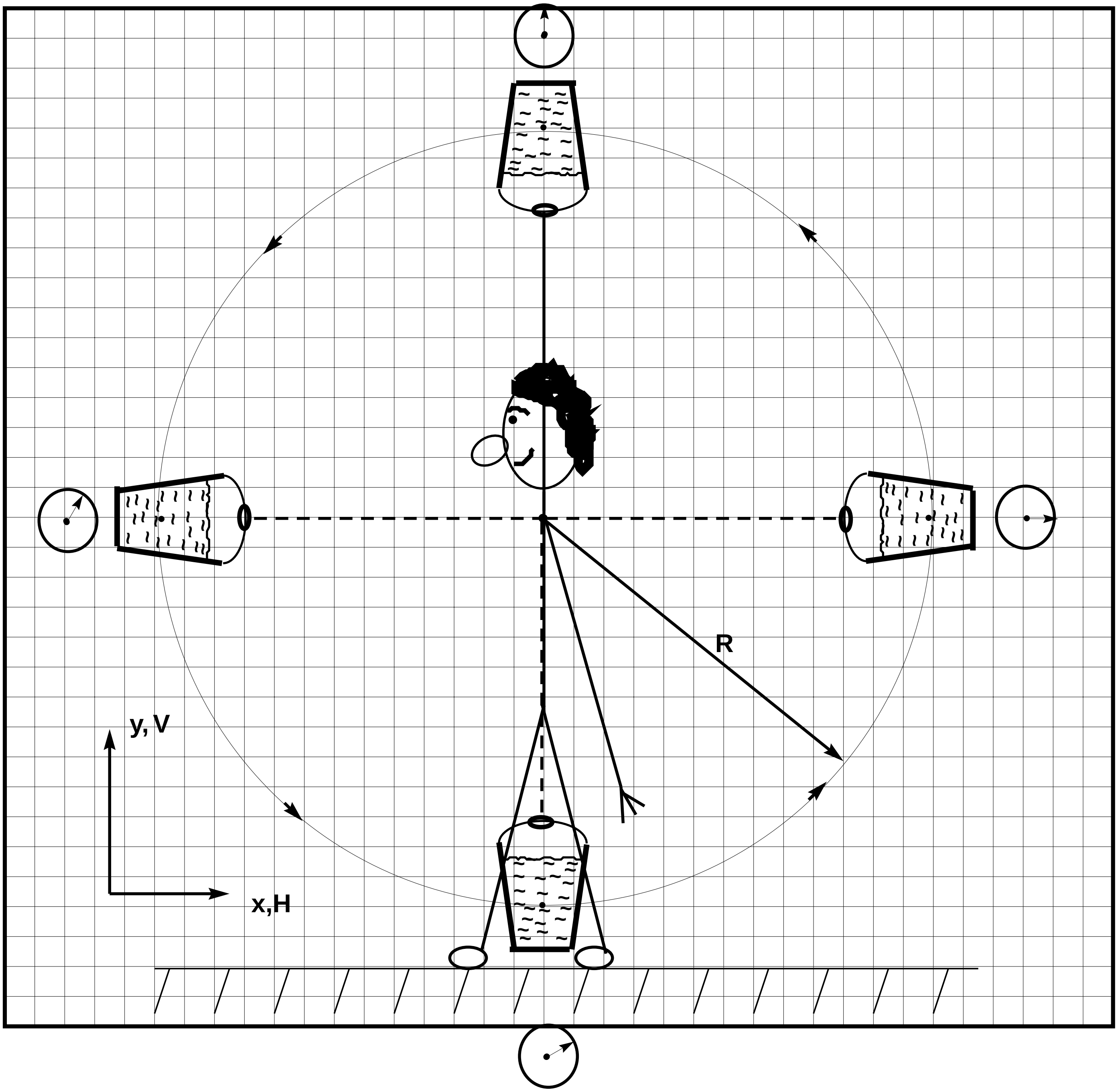
*QED is a common mathematical abbreviation for the Latin "quod erat demonstrandum" meaning "which was to be demonstrated".

7. Gradually diminish the tangential speed of the bucket of water until a few drops of water spatter on you or the floor. The sketch below shows you, the bucket, the water, and the floor at four instants of time for this **critical condition** when the *water* at the top of its path *is on the verge of spilling out of the bucket.* **You are rotating the bucket at a constant angular velocity $\vec{\omega}$ so that its tangential *speed*** v **is constant in time.** Show ALL the $\vec{F}$, $\vec{v}$, $\vec{a}$, and $\vec{\omega}$ vectors for the water at the top of its path. Label the tangential and angular velocity vectors $\vec{v}_c$ and $\vec{\omega}_c$ where the subscript "c" stands for "critical."

YOU ROTATE A BUCKET OF WATER AT ***CONSTANT*** *ANGULAR VELOCITY* $\vec{\omega}$ SO THAT THE WATER IS *ON THE VERGE* OF SPILLING OUT OF THE BUCKET AT THE TOP OF THE PATH



8.   The motion in "7" above, where the angular velocity $\vec{\omega}$ is constant, is called "uniform circular motion." Using Newton’s laws, derive an expression for the *critical angular velocity* ω_c for the *motion of the water* in terms of g (the acceleration due to gravity) and R (the radius of the circular motion).
[HINT: In the SI system, the angular velocity ω is just the rotational frequency in radians per second. Since there are 2π radians in a complete revolution, ω and the frequency f in revolutions/sec are related as $\omega = 2\pi f$.]

9. Is your expression for ω_c physically reasonable? {Y, N, U, NOT} [HINT: Is your expression dimensionally correct? Does your expression yield physically reasonable magnitudes for the parameter of interest (in this case ω_c) for both realistic and extreme limiting conditions of other variables?]

10. In the drawing of part 7, show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$, and $\vec{\omega}$ vectors for the *motion of the water* at the other three instants of time. Are the drawings at the top and bottom of the path consistent with your conclusions in part "6" above? {Y, N, U, NOT}

a. Can you determine the magnitude and direction of the **net** *horizontal* force on the water when the water is half-way between the top and bottom of the path? {Y, N, U, NOT} [HINT: Here and below, consider Newton's second law (N2), $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$.]

b. Can you determine the magnitude and direction of the **net** *vertical* force on the water when the water is half-way between the top and bottom of the path? {Y, N, U, NOT}

c. Would it be possible to rotate the bucket in a vertical circle *at constant tangential speed* (hence constant $\vec{\omega}$) if the bucket were tied to a rope and the rope were pivoted about the center of the circle? {Y, N, U, NOT}

d. Can you determine the magnitude and direction of the **net** *horizontal* force on the water when the water is at the bottom of the path? {Y, N, U, NOT}

e. Can you determine the magnitude and direction of the **net** *vertical* force on the water when the water is at the bottom of the path? {Y, N, U, NOT}

B. BUCKET OF WATER - Computer Investigation (Optional)

Please complete the preceding Section A "Bucket of Water" and discuss it with an instructor before starting this section.

Ask your instructor to introduce you to the Force-Motion-Vector Animation (FMVA) *Bucket*.† Play around with the program until you understand the various controls and readouts.

1. Pull down the Options Menu and select: *Trajectory, Velocity*, A*cceleration, All Forces.* Use the sliders to set the Radius at 2.00 m and the Mass of the water at 1.00 kg.

a. Set the Angular Velocity at its minimum value ω_{min} and record this value here ω_{min} = ________rad/sec.

b. The program is designed so that for any value of the radius R, the minimum angular velocity setting will be just the critical value ω_c for the water to be on verge of falling out of the bucket when it is directly overhead. Using the expression derived in Sec. II-A8, calculate ω_c for R = 2.00 m.

c. Does the above calculated ω_c agree with the computer's ω_{min}? {Y, N, U, NOT} If not "Y" please discuss with your partners or an instructor.

d. Pull down the Action Menu, select "Swing Bucket," and watch the animation.

(1) Is the velocity $\vec{\mathbf{v}}$ of the water constant in time? {Y, N, U, NOT}

(2) Is the speed $|\vec{\mathbf{v}}|$ of the water constant in time? {Y, N, U, NOT}

(3) Is the acceleration $\vec{\mathbf{a}}$ of the water constant in time? {Y, N, U, NOT}

(4) Is the magnitude $|\vec{\mathbf{a}}|$ of the acceleration of the water constant in time? {Y, N, U, NOT}

† Written by Randall Bird for *Project Socrates*. Bird's animations, running only on Power Macs, are available on 3.5-in HD disks by request to R.R. Hake. Similar animations running on a variety of platforms are commercially available as "Interactive Physics" from Knowledge Revolution.

e. Use the time slider to set the position of the bucket at the four positions of your drawings on p. 7. At each position do the computer's $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$ vectors agree with those shown in your drawings? {Y, N, U, NOT} If not "Y", discuss with your partners or an instructor.

f. From the Options Menu "deselect" *All Forces* and select *Net Force.* Pull down the Action Menu, select "Swing Bucket," and watch the animation.

(1) Is the animation in accord with Newton's Second Law $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$? {Y, N, U, NOT}

(2) Suppose you were to change the mass of the water from 1.00 kg to 0.50 kg, leaving the angular velocity, the radius, and the time unchanged. Can you ***predict*** what would happen to the:

(a) acceleration? {Y, N, U, NOT}

(b) net force? {Y, N, U, NOT}

(c) minimum angular velocity $\omega_{min} = \omega_c$? {Y, N, U, NOT}

(3**)** Change the mass of the water from 1.00 kg to 0.50 kg, leaving the angular velocity, the radius, and the time unchanged. Are the above predictions confirmed for the:

(a) acceleration? {Y, N, U, NOT} If not "Y", discuss with your partners or an instructor.

(b) net force? {Y, N, U, NOT} If not "Y", discuss with your partners or an instructor.

(c) minimum angular velocity $\omega_{min} = \omega_c$? {Y, N, U, NOT} If not "Y", discuss with your partners or an instructor.

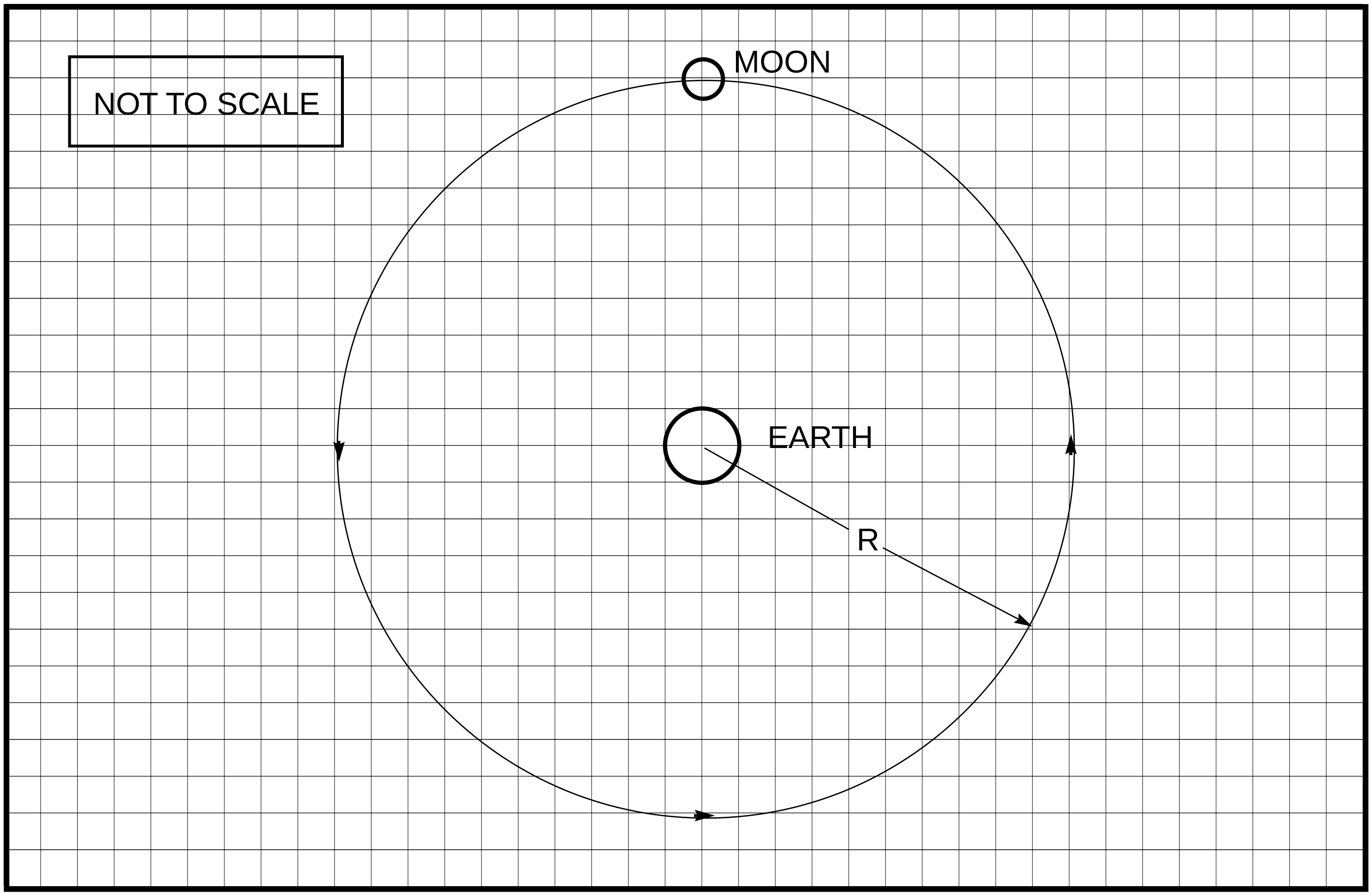
C. THE MOON

The motion of the bucket of water twirled in a vertical circle at constant angular velocity $\vec{\omega}$ is analogous to the *motion of the Moon* around the Earth.

1. Do you understand why the Moon **does NOT** fall out of its orbit around the Earth? {Y, N, U, NOT} [HINT: Consider your answer to part "A4" above. Try to arrange to see the splendid *Mechanical Universe* video *The Law of Falling Bodies*.

2. What’s the approximate period T for motion of the moon around the Earth?

3. The Moon-Earth system is shown on the next page as seen by an observer in the Earth Frame (the observer corrects his/her observations for the once-a-day rotation of the Earth about its axis). Regard the Earth as stationary in space, with the Moon revolving in a circle around the Earth’s center of mass. Ignore forces on the Moon and on the Earth by the Sun. Show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$, and $\vec{\omega}$ vectors for the approximately uniform circular *motion of the moon* around the Earth. Show in your drawing both $\vec{\mathbf{F}}_{\text{on moon by Earth}}$ and $\vec{\mathbf{F}}_{\text{on Earth by moon}}$. Write an expression for the magnitude of each of these forces in terms of other parameters of the Earth-Moon system. [HINT: Consider Newton’s Universal Law of Gravitation.]



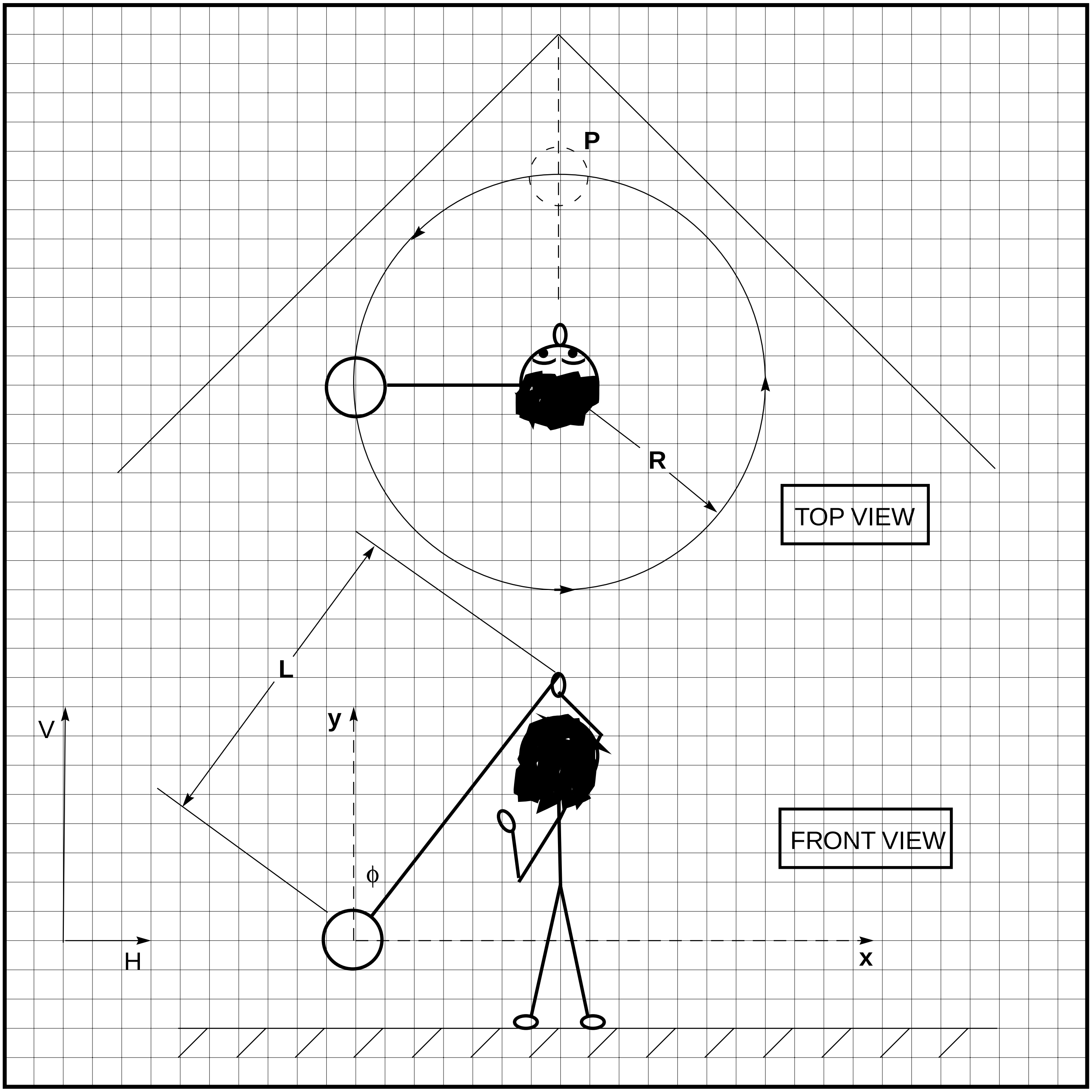
D. CONICAL PENDULUM

1. Take a rubber ball tied to the end of an approximately 5.5-ft (1.7 m) long string to the *cleared* corner of the lab. Face the corner, hold the end of the string, and twirl the ball around your head *in a horizontal circle* with a constant tangential speed. (This arrangement is called a "conical pendulum" because the string sweeps out a cone.) In both TOP and FRONT views shown on the next page, show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$, and $\vec{\omega}$ vectors for the *motion of the ball* with constant tangential speed v in a horizontal plane.

Views that show only two dimensions of an object are called "orthographic views."
The TOP VIEW shows the object's width and depth as seen by an observer looking vertically downward.
The FRONT VIEW shows the object's width and height as seen by an observer looking horizontally.
The SIDE VIEW shows the object's depth and height as seen by an observer looking horizontally.

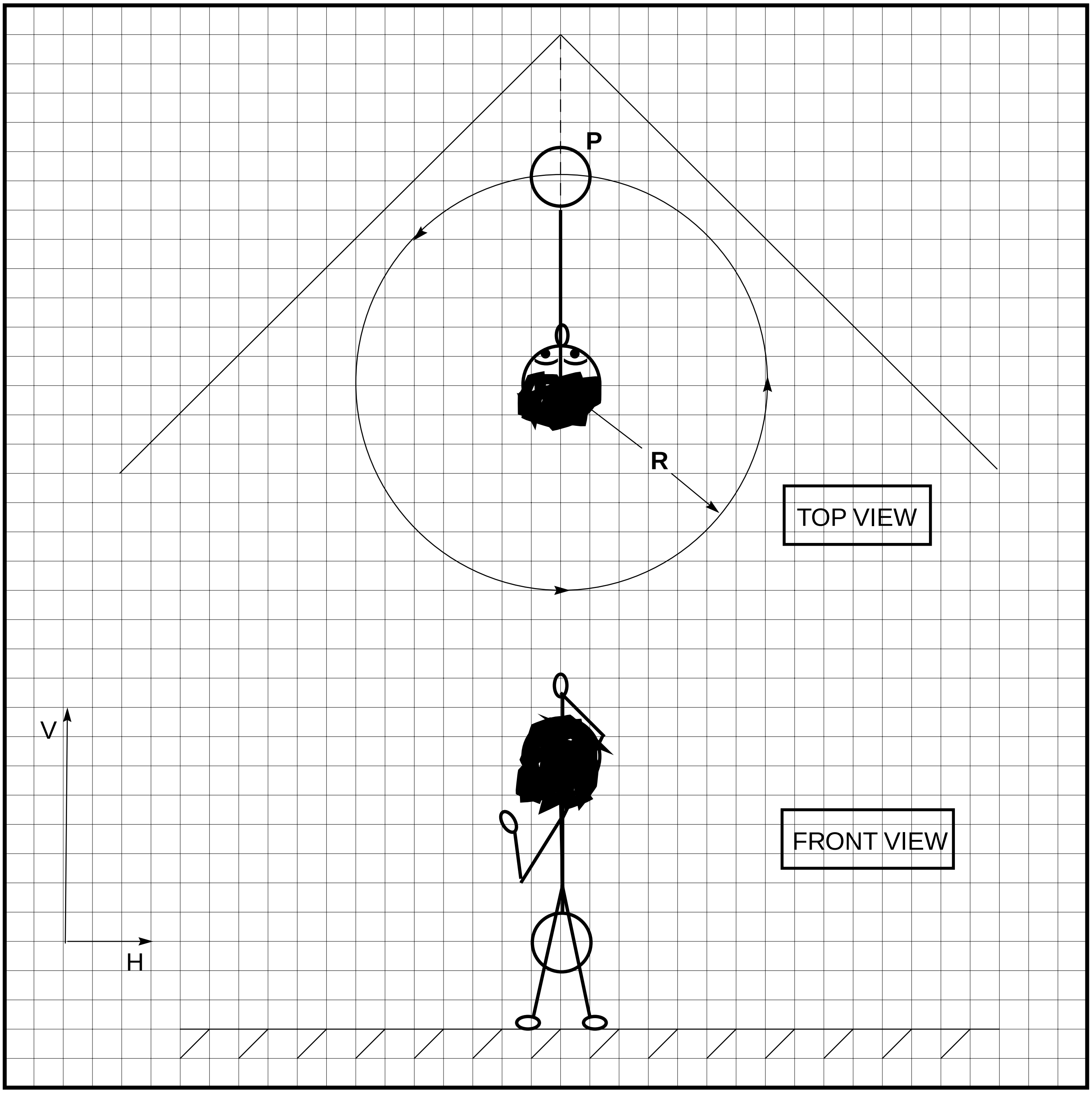
In contrast, perspective and "isometric" views show all three dimensions of an object, but are more difficult to draw and do not show dimensions in a clear-cut way.

TOP AND FRONT VIEWS OF A CONICAL PENDULUM



2. As you twirl the ball, predict the flight of the ball if you were to let the string go *at the instant the ball lies at* P *on a straight line between you and the corner of the room.* Sketch the predicted path with dashed lines – – – – – – – in top and front views on the next page. Now do the experiment and show the actual path in the two diagrams by solid lines ————. Are your results in accord with your predictions? {Y, N, U, NOT}

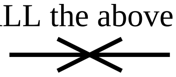
THE PATH OF A CONICAL PENDULUM BALL AFTER RELEASE FROM ITS SUPPORT

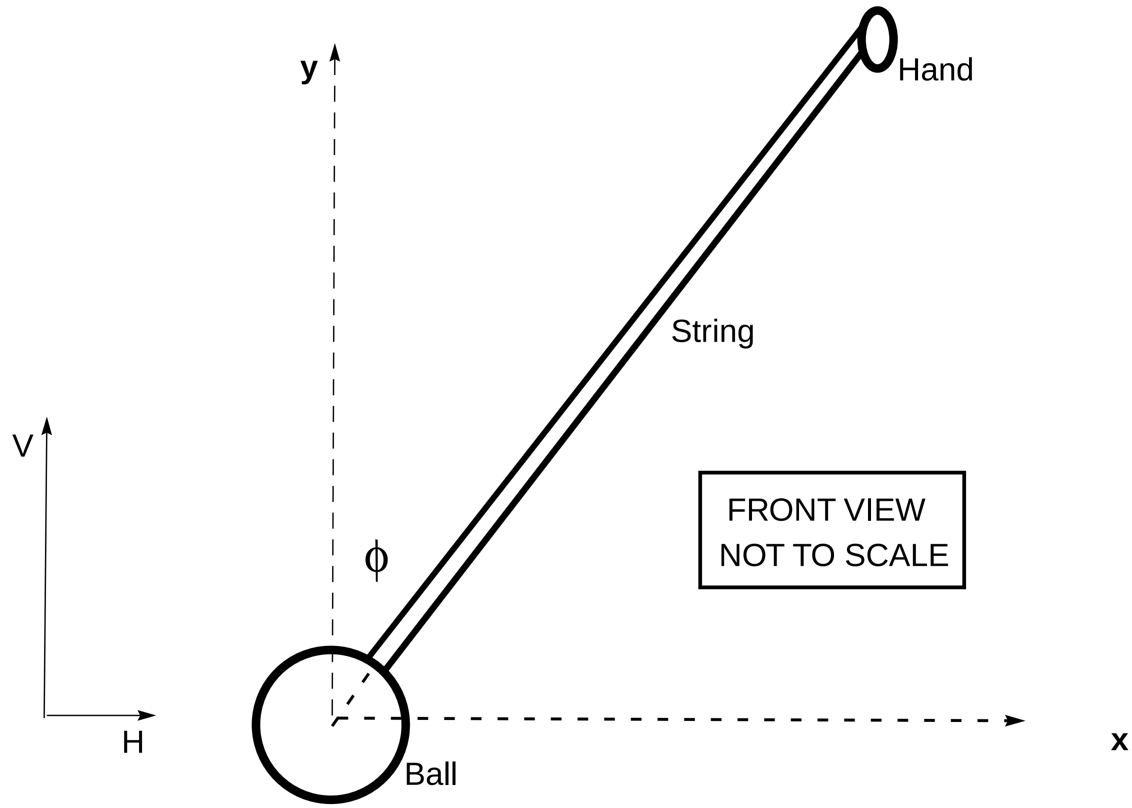


3. Is the above observed path of the ball (*after* you let go the string but *before* the ball hit the wall) in accord with Newton's second law, $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$? {Y, N, U, NOT}

4. Let φ be the angle between the string and the vertical as you twirl the ball around your head at an approximately constant tangential speed v. Obtain a relationship for T (the tension in the string) in terms of φ and W (the weight of the ball). Is your expression physically reasonable? {Y, N, U, NOT} [HINT: Consider dimensions and the predicted magnitude of T for realistic and extreme limiting conditions.] Will the string break as $\varphi \rightarrow 90°$ (i.e., as φ approaches 90°) ? {Y, N, U, NOT}

TENSION T AS A FUNCTION OF WEIGHT W AND ANGLE φ FOR A CONICAL PENDULUM

5. Do you understand the relationship between T (the tension in the string), and the forces $\vec{\mathbf{F}}_{\text{on string by hand}}$, $\vec{\mathbf{F}}_{\text{on hand by string}}$, $\vec{\mathbf{F}}_{\text{on ball by string}}$, and $\vec{\mathbf{F}}_{\text{on string by ball}}$? {Y, N, U, NOT} Show ALL the above forces in the enlarged diagram below. Indicate the tension T by the conventional symbol  in the middle of the string.



For the four forces diagrammed above:

a. Which constitute a Newton's-Third-Law pair?

b. Which is (are) equal in magnitude according to Newton's Second Law? (Assume that the string is massless.)

c. How do the magnitudes compare with one another?

d. How do the magnitudes compare with the tension T?

e. Which, if any, is (are) directly felt by you?

6. Test the relationship for T in terms of φ and W (part 4 above) by twirling the ball around your head with several φ values from $\varphi \approx 0°$ to $\varphi \approx 90°$. Is the relationship *qualitatively* obeyed? {Y, N, U, NOT} [HINT: Since this is a only a qualitative test you can judge by the *feel*, knowing from part 5 (above) how T is related to the force felt by you.]

7. (You might wish to consider this **O**ut-of-**L**ab **P**roblem **OLP** #2 , following OLP#1 of SDI#0.1) Apply Newton's second law, $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$, to the motion of the ball so as to obtain an expression for T in terms of m (the mass of the ball), ω (the angular velocity), and L (the length of the string). [HINT: Write equations for the motion in the vertical and horizontal directions in terms of the angle φ between the string and the vertical, ω (the angular velocity), and L (the length of the string).]

8. (You might wish to do this as **O**ut-of-**L**ab **P**roblem **OLP #3**.) Obtain from the analysis of part 7, above, an expression for the angular velocity ω in terms of g, L and ϕ. Is your expression physically reasonable? {Y, N, U, NOT}

9. (You might wish to do this as out-of-lab problem **OLP #4**.) From your analysis in part 8, above, obtain an expression for the *critical* angular velocity ω_c of the conical pendulum as $\varphi \rightarrow 0$ (i.e., as φ approaches zero).

10. OPTIONAL. Devise a method to experimentally check the ω (g, L, φ) expression you obtained in part 8 above. Do the experiment, extract ω_c from the data, and compare it with the theoretical ω_c of part 9 above. NOTE: "ω (g, L, φ)" is a standard abbreviation for "ω as a function of g, L, and φ." [HINT: Take data *in tabular form.* Then *make a graph* which will (a) test your expression, (b) determine ω_c.]

E. CONICAL PENDULUM - Computer Investigation - Optional

Please complete the preceding Section IID on the conical pendulum and discuss it with an instructor before starting this section.

Ask your instructor to introduce you to the Force-Motion-Vector Animation (FMVA) *Conical Pendulum*.† Play around with the program until you understand the various controls and readouts. Note that there are four views shown in the animation: top left – TOP VIEW, top right – SIDE VIEW, bottom left – FRONT VIEW, bottom right – isometric view showing all three dimensions.

The adjustable angles theta (θ), phi (ϕ), and psi (ψ) at the bottom of the screen are so-called Euler angles which specify the spatial orientation of the four views (click on a view to see the angles for that view). For the present experiments, please leave these angles at their default settings:

TOP VIEW:	θ = 1.57rad, ϕ = 1.57rad, ψ = 0.00rad
FRONT VIEW:	θ = 1.57rad, ϕ = 0.00rad, ψ = 0.00rad
SIDE VIEW:	θ = 3.14rad, ϕ = 0.00rad, ψ = 3.14rad
ISOMETRIC VIEW:	θ = 1.57rad, ϕ = 0.50rad, ψ = 0.00rad

1. Pull down the Options Menu and select: *Trajectory*, *Velocity*, *Acceleration*, and *All Forces*. Use the sliders to duplicate typical values for the lab experiments of the preceding Sec. IID: length of the string L ≈ 1.7 m, angular velocity $\omega \approx$ 4.0 radians/sec, mass of the ball m ≈ 0.50 kg (this is larger than the actual mass of the ball but yields a more easily observed weight vector), and time t such that the position of the ball in the TOP VIEW is the same as the first diagram in Sec. IID (i.e., to the left of viewer).

2. Do the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$, vectors for the motion of the ball as shown by the computer agree with those on your diagram in Sec. IID? {Y, N, U, NOT} If not "Y" discuss with your partners or an instructor.

3. Pull down the option menu. De-select velocity and select angular velocity. Does the $\vec{\omega}$ vector shown by the computer agree with that on your diagram in Sec. IID {Y, N, U, NOT} If not "Y" discuss with your partners or an instructor.

4. Return to the options and settings in "1." Pull down the Action Menu, select *Swing Pendulum*, and watch the animation in the four views.

. a. Is the velocity $\vec{\mathbf{v}}$ of the ball constant in time? {Y, N, U, NOT}

b. Is the speed $|\vec{\mathbf{v}}|$ of the ball constant in time? {Y, N, U, NOT}

c. Is the acceleration $\vec{\mathbf{a}}$ of the ball constant in time? {Y, N, U, NOT}

d. Is the magnitude $|\vec{\mathbf{a}}|$ of the acceleration of the ball constant in time? {Y, N, U, NOT}

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† Written by Randall Bird for *Project Socrates.* Bird's animations, running only on Power Macs, are available on 3.5-in HD disks by request to R.R. Hake. Similar animations (but lacking the orthographic and isometric views of this one) running on a variety of platforms are commercially available as "Interactive Physics" from Knowledge Revolution.

5. After viewing the animation several times, you may wish to use the time slider so that you can set the ball at different orientations and study the various views. [In order to reinforce the meaning of the four views it may be helpful for you to see three of the views rotate into any one view. Select one view by clicking on it so that it becomes shaded. Then pull down the Action Menu and select *Rotate into Section.*]

6. PREDICTION - Net Force on the Ball

Here and below, *please discuss your prediction with your partners, then underline your prediction* ***in black*** *and justify it on the basis of Newton's laws or kinematic principles.* With the settings as indicated in "1," if you were to select *Net Force* and deselect *All Forces* in the Options Menu, would the:

a. Net force vector as shown in the FRONT VIEW (bottom left) be along the string, vertically up, vertically down, horizontal and to the right, horizontal and to the left, perpendicular to and into the screen, perpendicular to an out of the screen, none of these? (Underline one and justify your answer.)

6'. COMPUTER TEST - Net Force on the Ball

a'. Perform the operation indicated in "6" above. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

b'. Pull down the Action Menu, select *Swing Pendulum,* and watch the animation in the four views. Is the animation in accord with Newton's Second Law $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}} \vec{\mathbf{a}}_{\text{body}}$? {Y, N, U, NOT}

7. PREDICTION - Increasing the Mass of the Ball

With the options and settings initially as indicated in "1," if you were to increase the mass m of the ball, leaving L and ω unchanged, would the:

a. Vertical force $\vec{\mathbf{W}}_{\text{on ball by Earth}}$ decrease, remain the same, increase, or none of these? (Underline one and justify your answer.)

b. Magnitude of the force along the string (same as the tension T in the string) decrease, remain the same, increase, or none of these ? (Underline one and justify your answer.)

c. *Direction* of the acceleration $\vec{\mathbf{a}}$ of the ball remain the same or change? (Underline one and justify your answer.)

d. *Magnitude* of the acceleration $\vec{\mathbf{a}}$ of the ball decrease, remain the same, increase, or none of these? (Underline one and justify your answer.)

e. The angle ϕ between the string and the vertical decrease, remain the same, increase, or none of these? (Underline one and justify your answer.)

7'. COMPUTER TEST - Increasing the Mass of the Ball.

Perform the operation in "7" above. Underline the results in red in "7a,b,c,d,e" above, then compare your prediction with the computer results in 7a',b',c',d',e' below.

a'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

b'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

c'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

d'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

e'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

8. PREDICTION - *Increasing* the Angular Velocity of the Ball (as in the experiment of Sec. IID-6, p. 18). With the options and settings initially as indicated in "1," if you were to *increase* the angular velocity ω of the ball, leaving the length L of the string and the mass m of the ball unchanged, would the:

a. Vertical force $\vec{\mathbf{W}}_{\text{on ball by Earth}}$ decrease, remain the same, increase, or none of these ? (Underline one and justify your answer.)

b. Magnitude of the force along the string (same as the tension T in the string) decrease, remain the same, increase, or none of these ? (Underline one and justify your answer.)

c. *Direction* of the acceleration $\vec{\mathbf{a}}$ of the ball remain the same or change? (Underline one and justify your answer.)

d. *Magnitude* of the acceleration $\vec{\mathbf{a}}$ of the ball decrease, remain the same, increase, or none of these? (Underline one and justify your answer).

e. The angle ϕ between the string and the vertical decrease, remain the same, increase, or none of these (Underline one and justify your answer).

8'. COMPUTER TEST - *Increasing* the Angular Velocity of the Ball.

Perform the operation in "8" above. Underline the results in red in "8a,b,c,d,e" above, then compare your prediction with the computer results in 8a',b',c',d',e' below.)

a'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

b'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

c'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

d'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

e'. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result.

9. PREDICTION - *Decreasing* the Angular Velocity of the Ball

With the options and settings initially as indicated in "1," if you were to *decrease* the angular velocity ω of the ball, leaving the length L of the string and the mass m of the ball unchanged, as ω approaches zero, the trajectory would become: an ellipse, a point, unstable at some critical value ω_c, none of these.

9'. COMPUTER TEST - *Decreasing* the Angular Velocity of the Ball.

Perform the operation in "9" above. Underline the results in red above. Does your prediction agree with the computer result? {Y, N, U, NOT} If not "Y" either show that the computer is wrong or else justify the computer result. How does ω_{min} at which the computer picture vanishes compare with the value of ω_c calculated from the formula derived in Sec. IID-9.

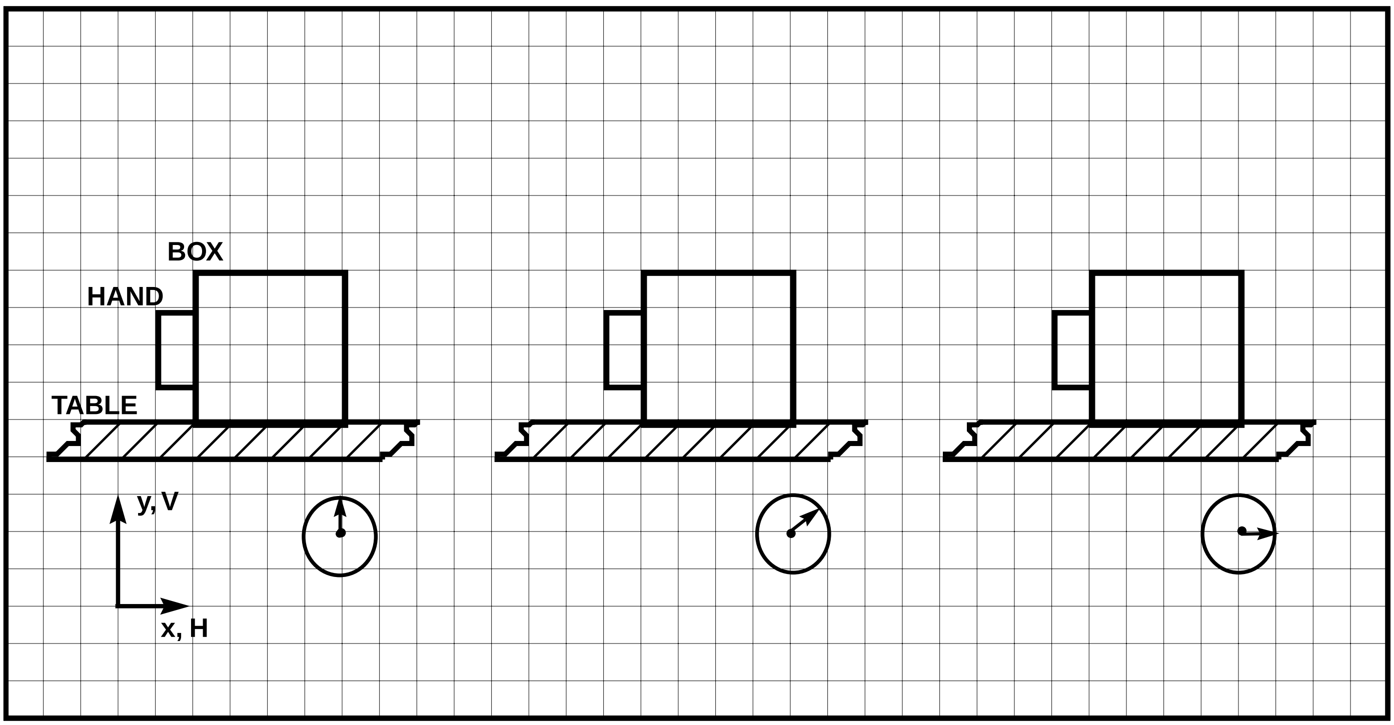
III. FRICTIONAL FORCES IN LINEAR MOTION

Before considering the relatively complicated situation of frictional force in *circular* motion (Sec. IV), it might be worthwhile to review frictional forces in *linear* motion.

A. EXPERIMENTS WITH A MASSIVE BOX

1. Place a relatively massive (W ≈ 50 lb and thus m ≈ 23 kg) box of copier paper on your table. With one hand exert a *horizontal* force $\vec{\mathbf{F}}_{\text{on box by hand}} \equiv \vec{\mathbf{F}}_{bh}$ in such a way that $\vec{\mathbf{F}}_{bh}$ *builds up slowly in time* to a magnitude which is *just below* the *critical* value F_{bh}' required to set the box in motion. In the space below show **ALL** the force vectors acting **on the box** at three instants of time such that $\vec{\mathbf{F}}_{bh}$ is (a) slightly greater than zero, (b) about half $\vec{\mathbf{F}}_{bh}'$, (c) slightly less than $\vec{\mathbf{F}}_{bh}'$. Draw acceleration and velocity vectors (if they exist) in each of the snapshot sketches.

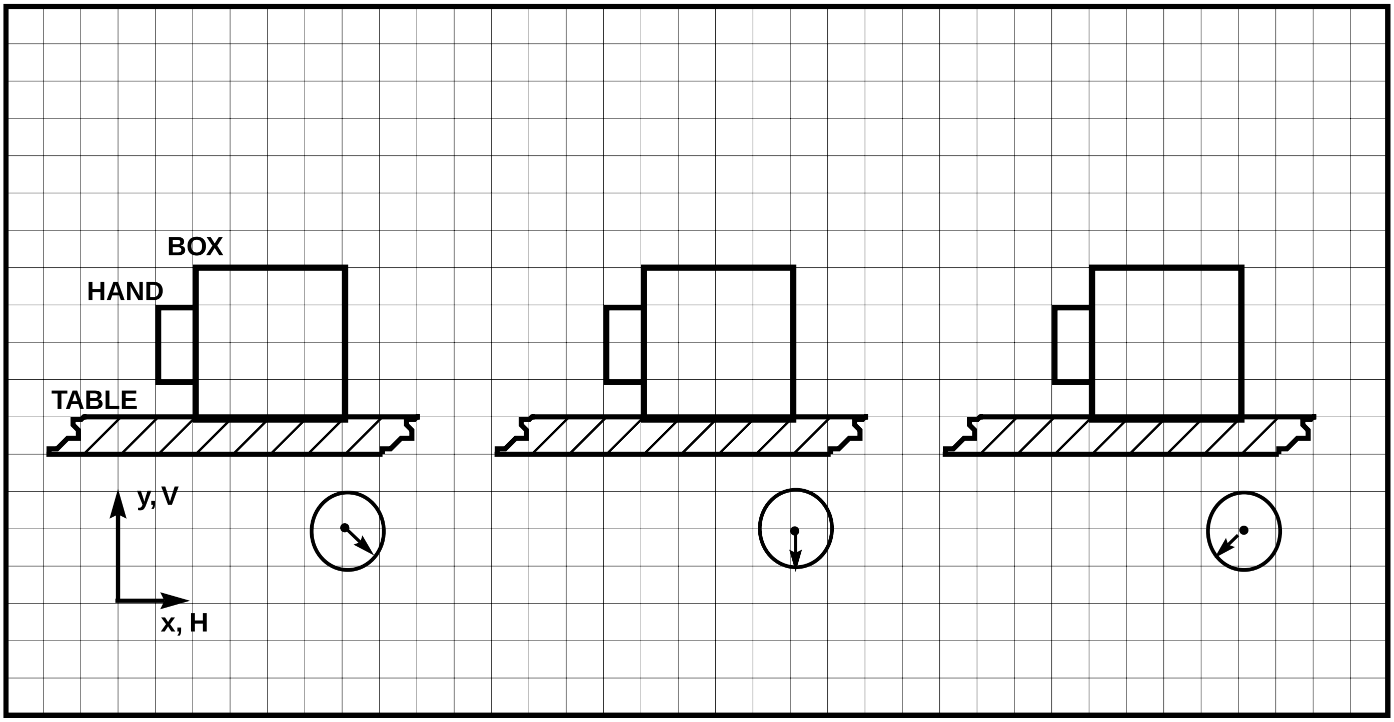
A STATIONARY MASSIVE BOX PUSHED WITH SUCCESSIVELY GREATER FORCES



a. Can you write down a mathematical relation between F_{bh}, m, g, and the coefficient of static friction μ_s between the box and the table which is obeyed *in each* of the snapshot sketches? {Y, N, U, NOT}

2. Now continue increasing the horizontal force $\vec{\mathbf{F}}_{bh}$ until it becomes slightly *greater* than the critical value F_{bh}' required to set the box in motion and the box starts to accelerate. Quickly adjust your applied force $\vec{\mathbf{F}}_{bh}$ to a value F_{bh}'' so that the box moves *at constant velocity* $\vec{\mathbf{v}}$ *across the table*. In the space below, show **ALL** the force vectors acting **on the box** at three instants of time during this constant $\vec{\mathbf{v}}$ motion. Draw acceleration and velocity vectors (if they exist) for the box.

A MASSIVE BOX PUSHED SO THAT IT MOVES WITH CONSTANT VELOCITY ON A TABLE



a. Can you write down a mathematical relation between F_{bh}'', m, g, and the coefficient of kinetic coefficient μ_k between the box and the table which is obeyed during the constant $\vec{\mathbf{v}}$ motion? {Y, N, U, NOT}

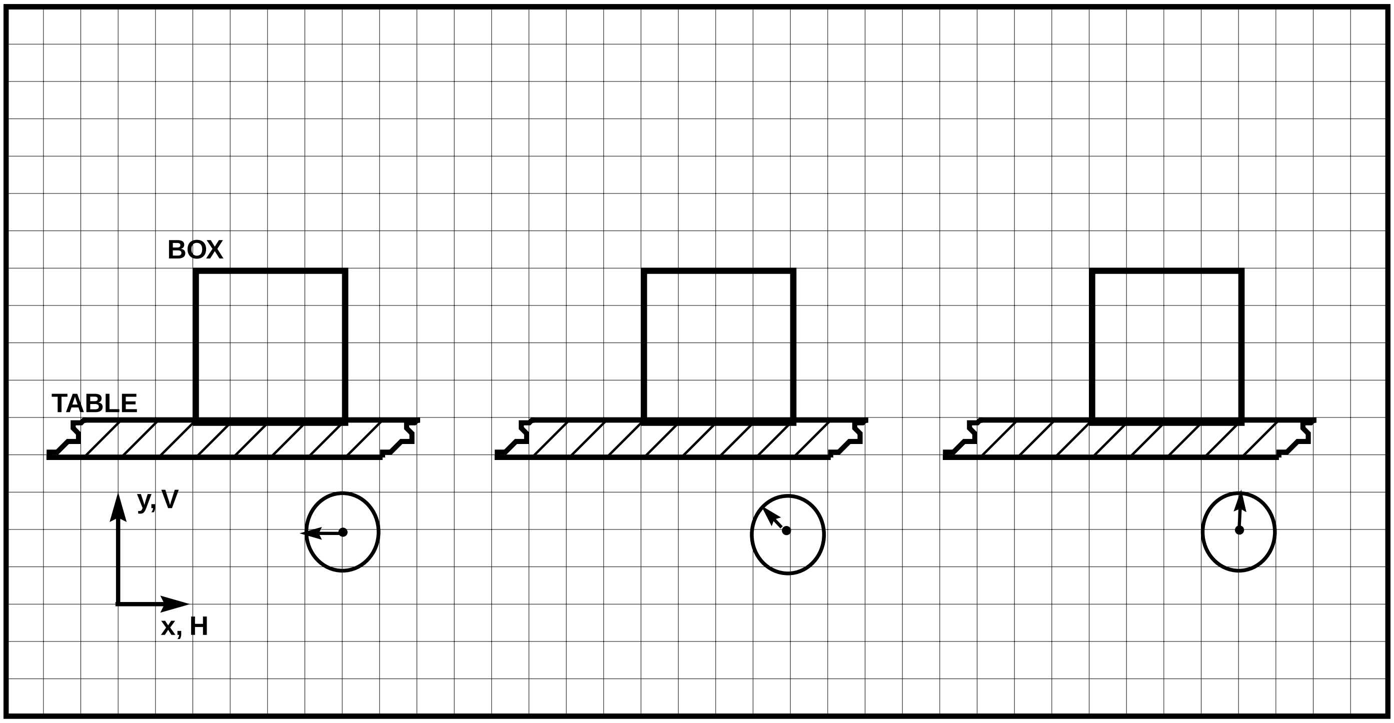
b. Do you feel any difference between this situation and that of SDI #1 and #2 when you pushed a 50-lb dry ice block so that it moved a constant velocity on a glass-top table? {Y, N, U, NOT} (Give a Newtonian justification for your answer.)

3. Place the box on a bathroom scale and record its weight W = mg. Now place the box on a table and place your hand on the box so as to push it horizontally across the table. Insert the bathroom scale between your hand and the box. Apply a *gradually* increasing horizontal force and record the scale reading F_{bh}' when the box *starts to move* and the scale reading F_{bh}'' as the box *moves at nearly constant speed* across the table. Repeat the experiment five or more times and calculate the averages $\langle F_{bh}' \rangle$ and $\langle F_{bh}'' \rangle$, and the ratio $\langle F_{bh}'' \rangle / \langle F_{bh}' \rangle$. The ratio is (underline one) [greater than, equal to, less than] one. Can you explain this result on some physical basis?

a. Can you express the ratio $(F_{bh}'')/(F_{bh}')$ in terms of the coefficients of static μ_s and kinetic μ_k friction between the box and the table? {Y, N, U, NOT}

4. After the box is moving at constant velocity $\vec{\mathbf{v}}$, suddenly *remove your hands from the box*. The figure below shows the box at 3 positions *after* it has left your hand and *while it is in motion*: near the start, middle, and end of its slide. Show **ALL** the force vectors acting ON the box at these 3 positions. Draw velocity vectors and acceleration vectors (if they exist) at each of the three positions.

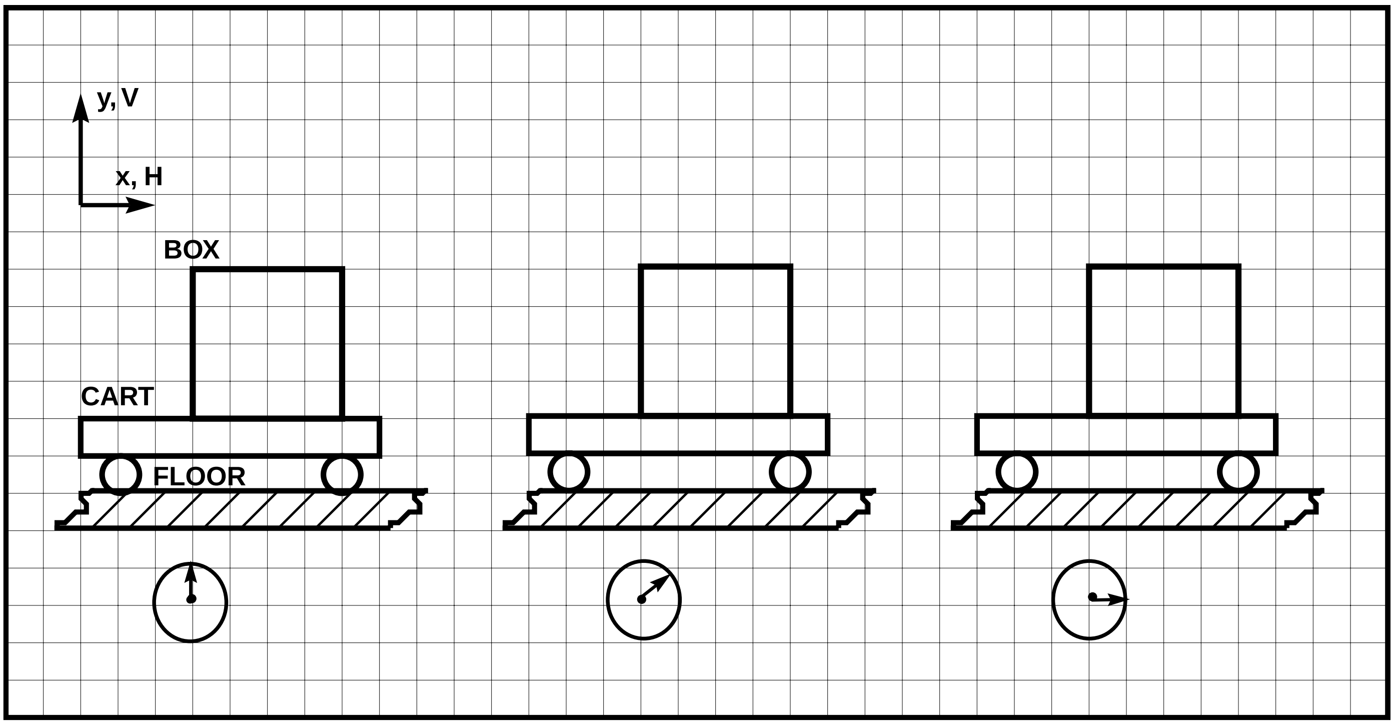
A MASSIVE BOX SLIDING TO REST ON A TABLE



a. Can you express the acceleration of the box in terms of g and the coefficient of kinetic friction μ_k between the box and the floor? {Y, N, U, NOT}

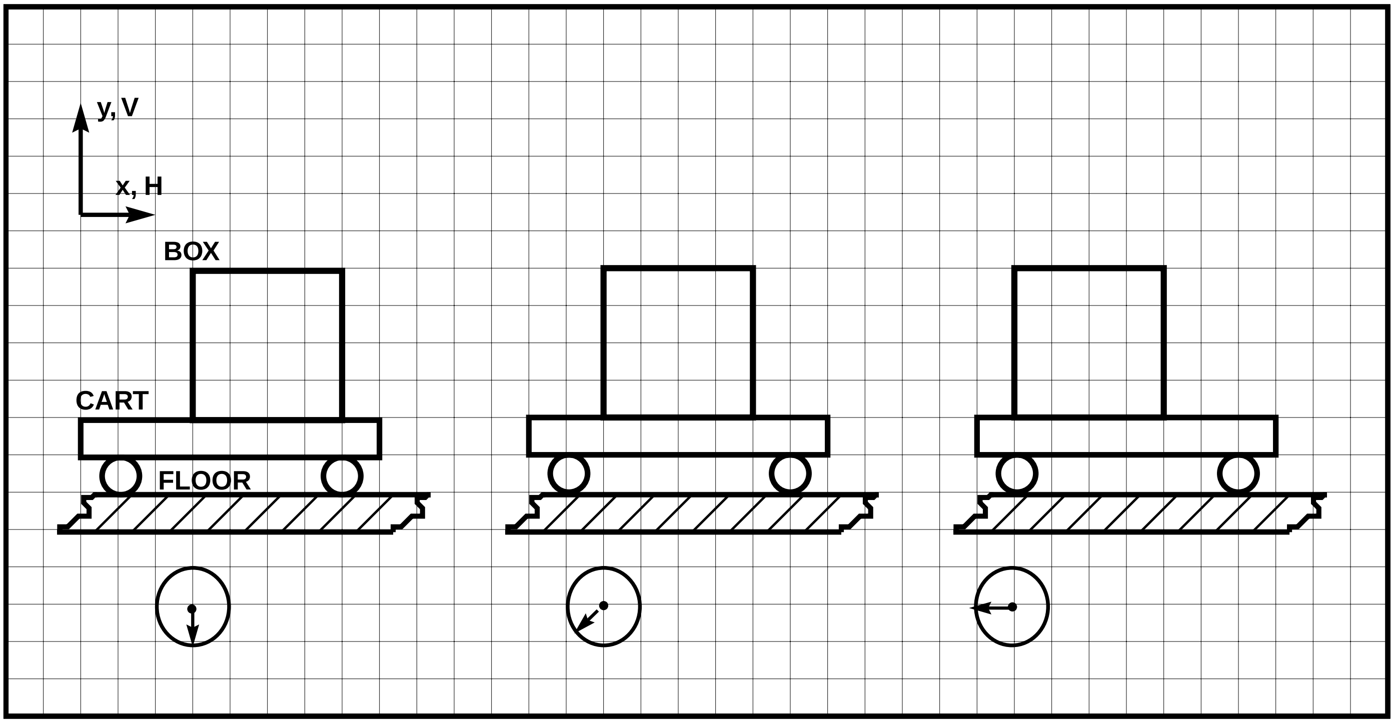
5. Place the box on a metal lab cart. Push the cart so that the box and cart move together with a low and nearly constant *acceleration* $\vec{\mathbf{a}}$ (as measured in the lab frame) across the floor. At low acceleration the box will be stationary *with respect to the cart.* In the figure below show **ALL** the force vectors acting **on the box** at the three instants of time. Draw acceleration and velocity vectors (if they exist) for the box and for the cart *as seen from the lab inertial reference frame.*

A MASSIVE BOX AT REST WITH RESPECT TO A CART WHICH ACCELERATES TO THE RIGHT IN THE LAB REFERENCE FRAME, ALL AS SEEN BY AN OBSERVER IN THE LAB FRAME



6. Starting with the box and cart at rest, jerk the lab cart *quickly* towards you so that the box *moves about 10 cm with respect to the cart:*

A MASSIVE BOX SLIDING WITH RESPECT TO A CART WHICH ACCELERATES TO THE RIGHT IN THE LAB REFERENCE FRAME, ALL AS SEEN BY AN OBSERVER IN THE LAB FRAME

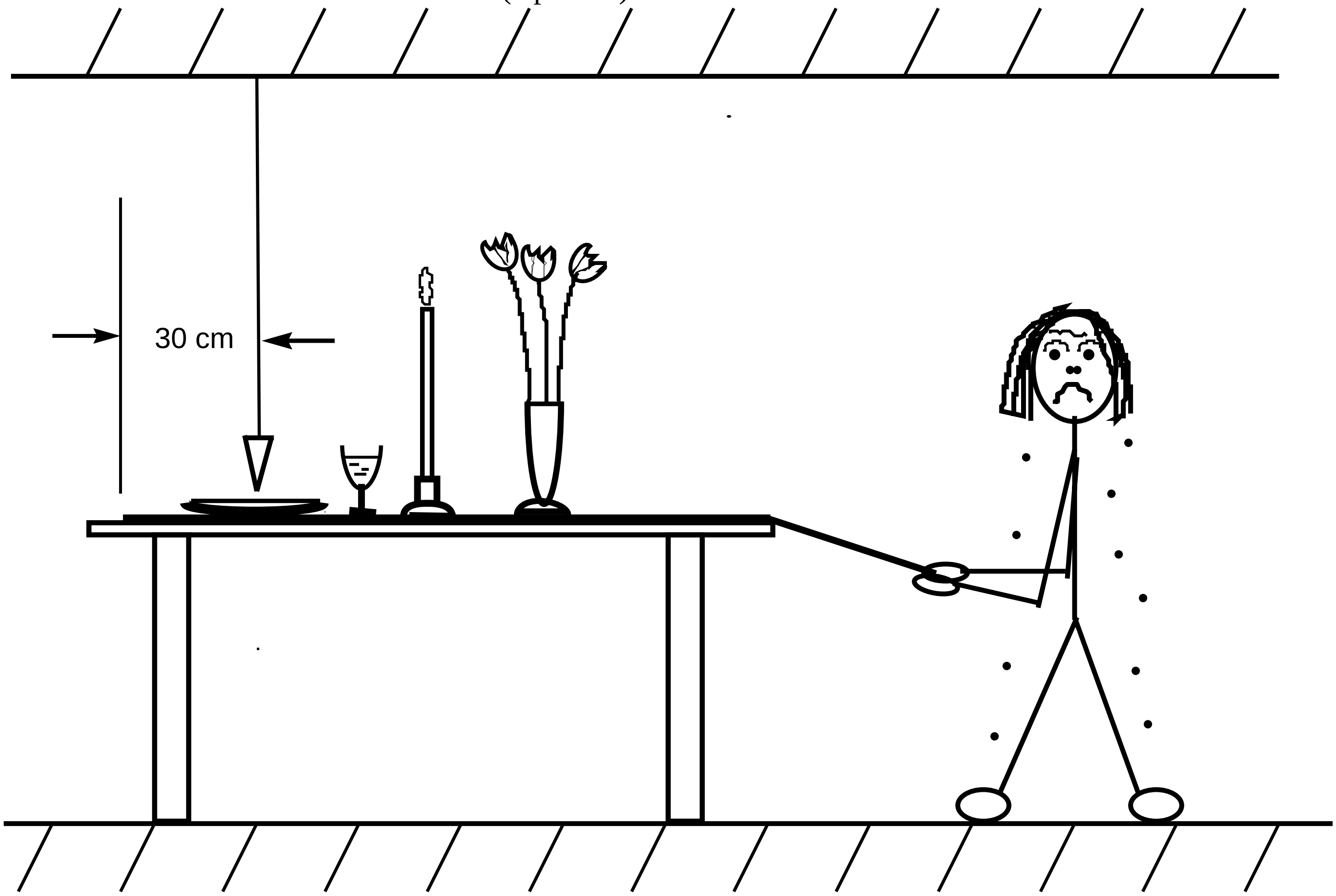


a. In the snapshot sketches above show the box at three times when it is sliding with respect to the cart. Show **ALL** the force vectors acting **on the box** at these times. Draw acceleration and velocity vectors (if they exist) for the box and for the cart at these times.

b. *Just before* the box slipped on the cart, the box and the cart were accelerating together at some maximum acceleration a_{max}. Can you express a_{max} in terms of g and the coefficient of static friction u_s between the box and the cart?

c. Can you express the acceleration a of the box while it is sliding on the cart in terms of g and the coefficient of kinetic friction μ_k between the box and the cart? {Y, N, U, NOT}

B. TABLECLOTH-SLIP-OUT TRICK (Optional)

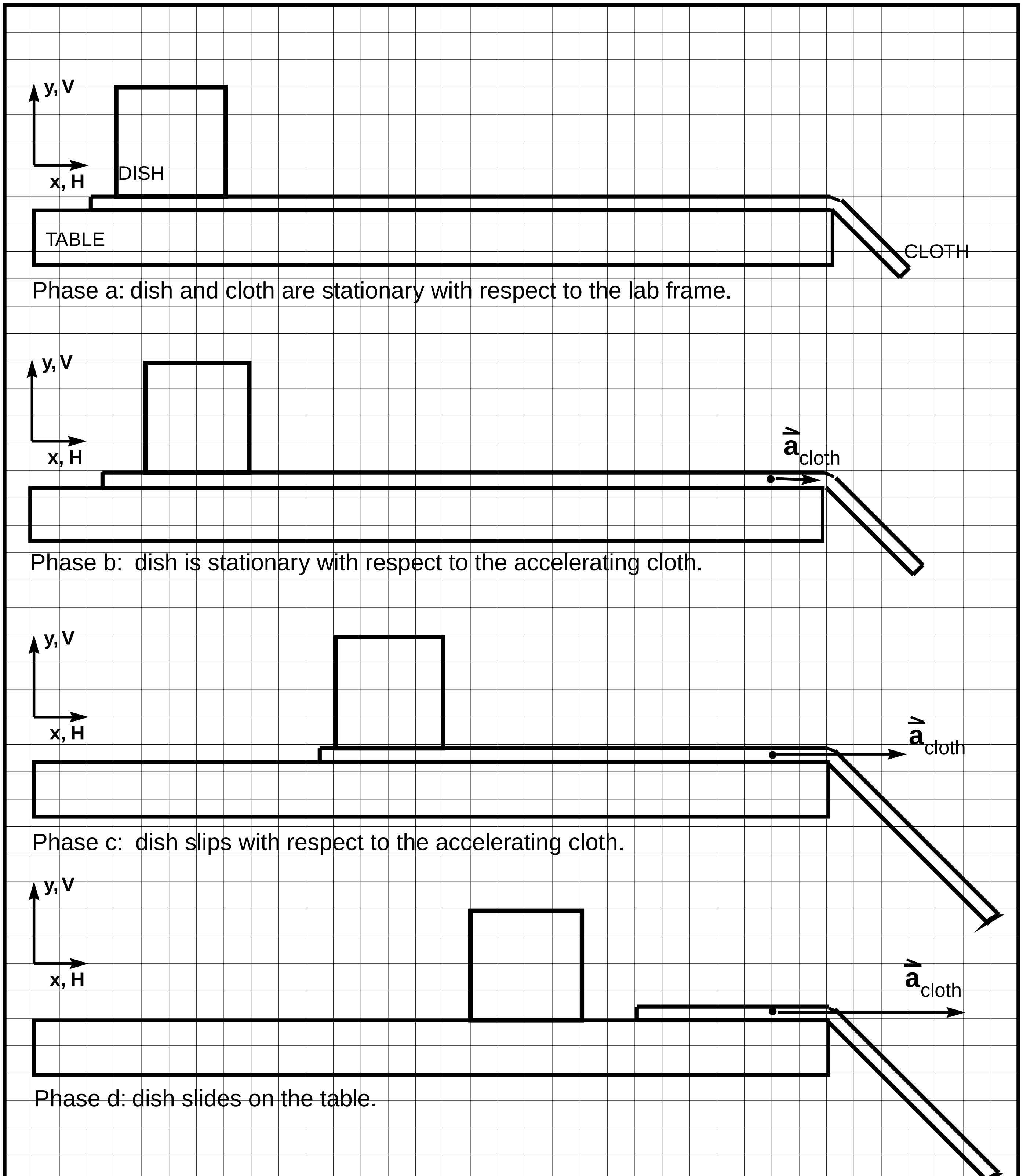


1. Tablecloth-slipout Contest

Center the plate at a position *30.0 cm from the edge of the cloth opposite the hang-over edge* as shown above. Then *center the plumb bob at the center of polar graph paper* taped to a cardboard sheet taped to the top of the plate. Have the waiter light the dinner candle, pour the $500/bottle wine into the $1000 wine glass, arrange the flowers, and dim the lights. Grasp the hang-over edge of the tablecloth. Then with a smooth and very fast downward pull...................................... ZIP ⟶ the cloth out from under the dishes!! Measure the displacement δ of the plate by reading the position of the plumb bob on the polar graph paper: δ = __________inches (record this also along with your name on the sheet taped to the front table). *A FANTASTIC award will be made to the student setting the low-δ record.* [NOTE: If you break a few dishes don't worry– we have lots more. But it's important for you to keep trying until you succeed. Only then will you have confidence in Newton's laws and in your ability to zip the tablecloth out from under your family's Thanksgiving dinner.]

Please blow out the candle and replace the cloth, plumb bob, and dishes exactly as they were when you started the experiment.

2. Newtonian Physics of the Tablecloth-Slip-Out Trick (Note: the physics is very similar to that of the massive-box experiments in part IIIA. There the box played the role of the dish and the cart played the role of the cloth.)



During the downward pull on the tablecloth, the force applied to the tablecloth by the puller increases rapidly in time. There are four phases to the motion of the dish just before and during the pull as indicated above.

In the figure on the preceding page show **ALL** the force vectors acting ON the dish during the four phases a, b, c, and d of the motion. Draw velocity and acceleration vectors (if they exist) for the dish.

3. (You might wish to do this as out-of-lab problem **OLP #5**.) Eq. (1) for the distance δ moved by the plate was derived by Uri Haber-Schaim and John Dodge.

$$\delta = (1/2)\, \mu_{kc}\, g\, \tau^2 \,[\, 1 + (\mu_{kc}/\, \mu_{kt})]. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Can you derive this expression? {Y, N, U, NOT} In Eq. (1), g is the acceleration due to gravity; τ is the time required to pull the cloth out from under the plate (i.e., the time duration of the "phase-c motion" (see above); and μ_{kc} and μ_{kt} are the coefficients of kinetic friction for, respectively, the plate on the cloth and the plate on the table. [HINT: Use Newton's second law, $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$, to obtain accelerations in terms of μ and g for the phases b and c of the motion diagrammed above. (Assume that the distance moved during phase b is negligible.) Then use the "Famous Five" constant-acceleration kinematic equations† to calculate displacements s_c while the dish is on the cloth and s_t while the dish is on the table.

† $v_f = v_o + at$

$v_f^2 = v_o^2 + 2as$

$s = v_o t + (1/2)at^2$

$\langle v \rangle = (v_o + v_f)/2$

$s = \langle v \rangle\, t$

where the displacement s(t=0) = 0 and $v(t=0) = v_o$

4. (You might wish to do this as out-of-lab problem **OLP #6**.) Is Eq. (1) physically reasonable? {Y, N, U, NOT} [HINT: Consider dimensions and the predicted magnitude of δ for both realistic and extreme limiting conditions]

5. (You might wish to do this as out-of-lab problem **OLP #7**.) According to Eq. (1), the displacement δ of the dish is independent of the mass m of the dish and the area A of contact of the dish on the cloth or the table. Is this physically reasonable? {Y, N, U, NOT} As usual, justify your answer.

6. (You might wish to do this as out-of-lab problem **OLP #8**.) Plug the measured values of δ, and coefficients of friction roughly measured by the staff (μ_{kc} = 0.1, μ_{kt} = 0.2) into Eq. (1) so as to yield τ. Do you think this τ is physically reasonable?{Y, N, U, NOT} Can you think of ways to measure τ directly? {Y, N, U, NOT}

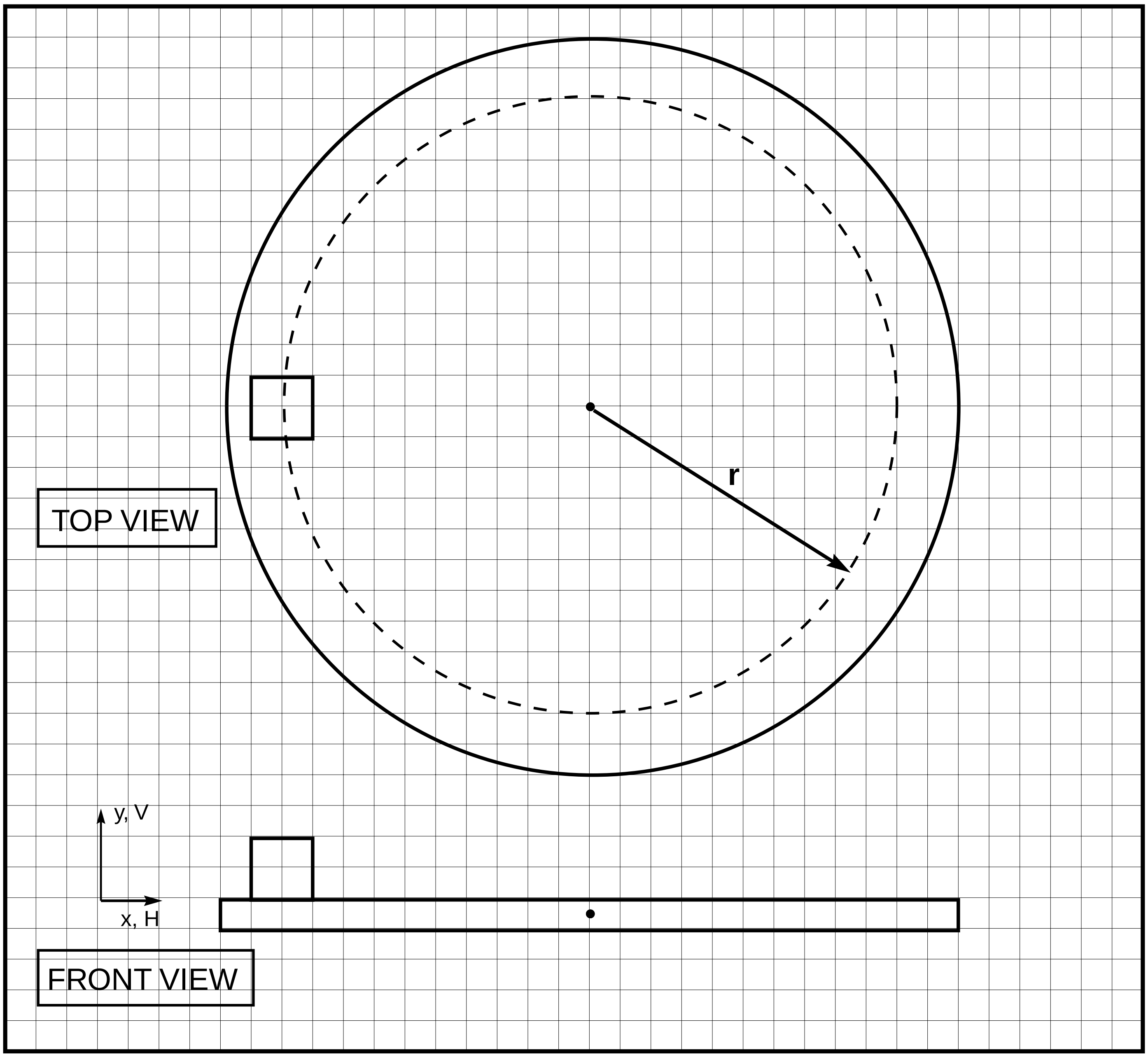
7. (You might wish to do this as out-of-lab problem **OLP #9**.) So far, in our work on mechanics, we have assumed that extended objects can be treated as point particles. Thus, in the derivation of Eq. (1) we ignored "torques" which might act to *rotate* the objects on the tablecloth. Explain why it's a good idea to avoid placing tall high-center-of-mass objects such as the flower-vase-plus-flowers on the slip-out-trick tablecloth, as shown in the figure at the start of this section.

IV. FRICTIONAL FORCES IN CIRCULAR MOTION

A. FRICTIONAL FORCE ON A CUBE ROTATING ON A TURNTABLE

1. Place a small aluminum cube on the hand-powered turntable, as shown below. Rotate the turntable *at some nearly constant angular velocity* ω so that the cube remains *stationary with respect to (wrt) the turntable*. In the TOP and FRONT views below show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$, $\vec{\alpha}$, and $\vec{\omega}$ vectors for the circular motion of the cube *as seen by an observer in the lab frame.*

A CUBE AT REST WRT A TABLE ROTATING WITH *CONSTANT* ANGULAR VELOCITY $\vec{\omega}$



2. Is there a frictional force $\vec{\mathbf{f}}$ acting on the cube? {Y, N, U, NOT} If so, is it a kinetic or a static frictional force?

B. CRITICAL VALUE ω_c FOR FLY-OFF

1. Repeat the experiment above, except *very graduall*y *and uniformly* increase the magnitude of the angular velocity ω of the turntable until the cube flies off the table. Is there any relationship between this experiment and the letting go of the conical pendulum string in Sec. IIC-2? {Y, N, U, NOT}

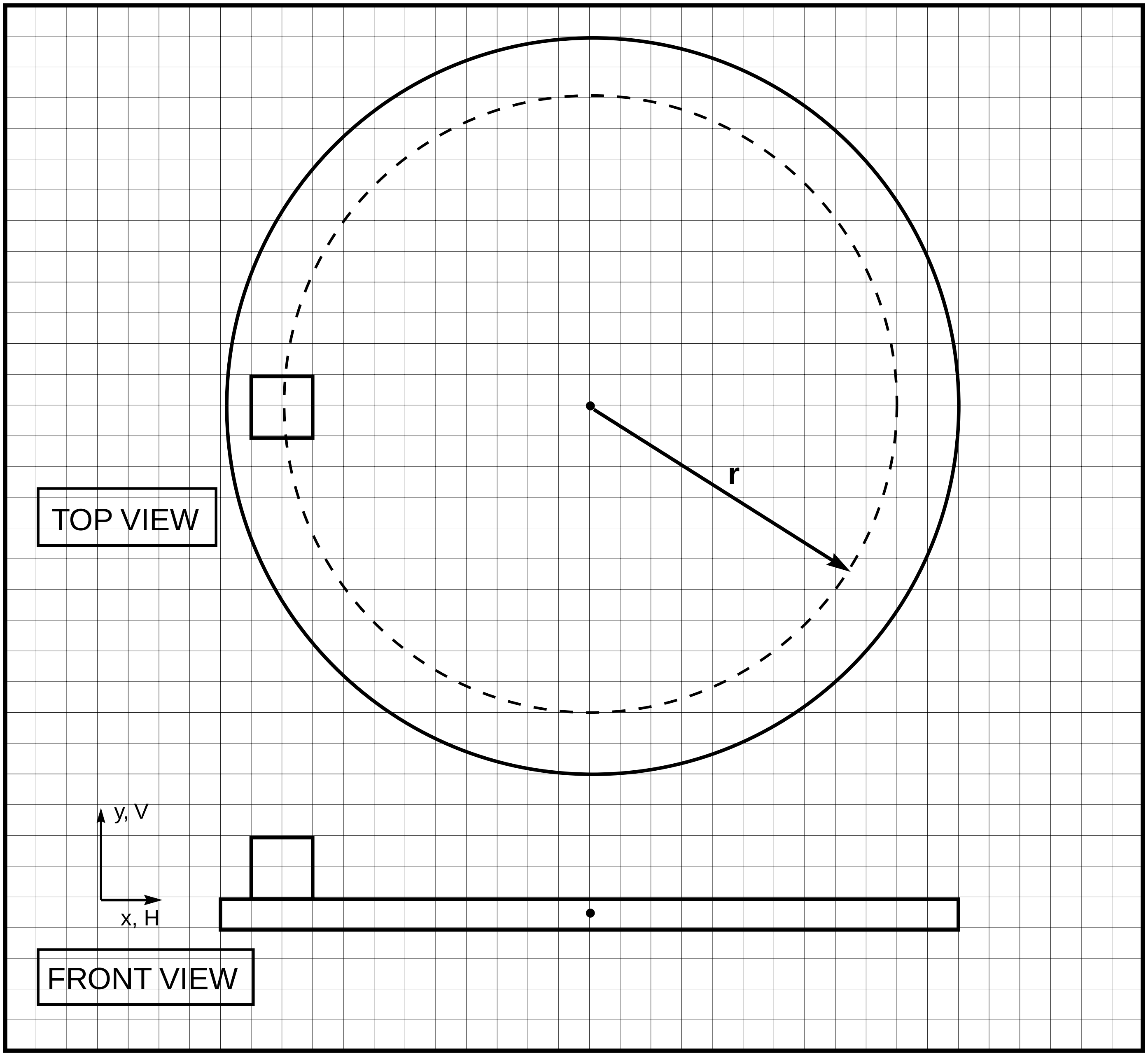
2. What is the path of the cube *with respect to the laboratory reference frame* while it is in the air after leaving the turntable?

a. Is the path in a vertical plane? {Y, N, U, NOT}

b. Is the path in accord with Newton's second law, $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$? {Y, N, U, NOT}

3. Consider part 1 above, when ω is increasing but *before the cube moves with respect to (wrt) the table*. In the TOP and FRONT views below show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$, $\vec{\alpha}$, and $\vec{\omega}$ vectors for the circular motion of the cube *as seen by an observer in the lab frame*. [HINT: It helps to consider the tangential a_t and radial a_r components of the acceleration $\vec{\mathbf{a}}$.] For equal ω, is the NET acceleration vector $\vec{\mathbf{a}}$ for the cube in this experiment the same as in Sec. IVA-1 where ω was constant? {Y, N, U, NOT}

A CUBE AT REST WRT A TABLE ROTATING WITH *INCREASING* ANGULAR VELOCITY $\vec{\omega}$



C. VARIABLES AFFECTING THE CRITICAL ANGULAR VELOCITY ω_c FOR FLY-OFF

1. Surface Roughness

a. A clever method (due to Brenda Murray, Indiana University Q202 student, Fall 1981) to examine the effect of surface roughness on ω_c is to tape the composite plastic-cork-rubber-sandpaper surface to the turntable as shown below:

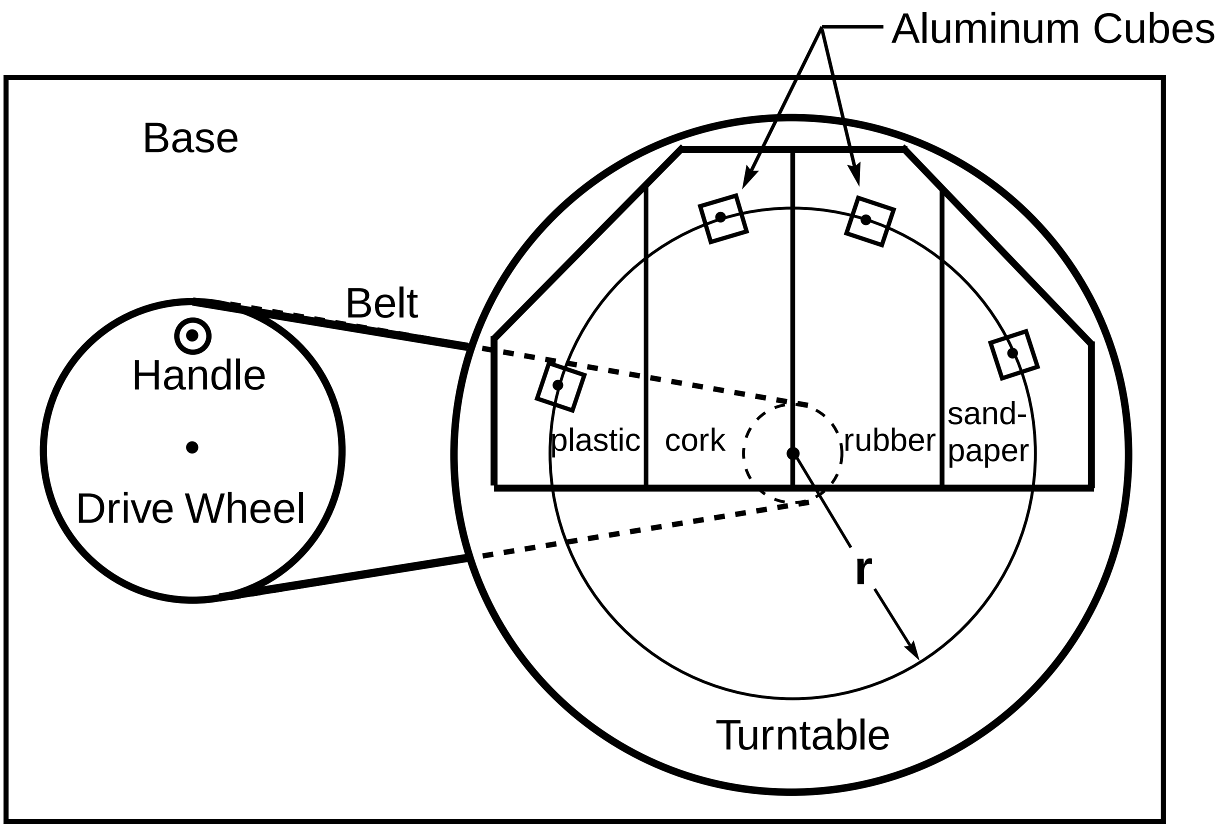
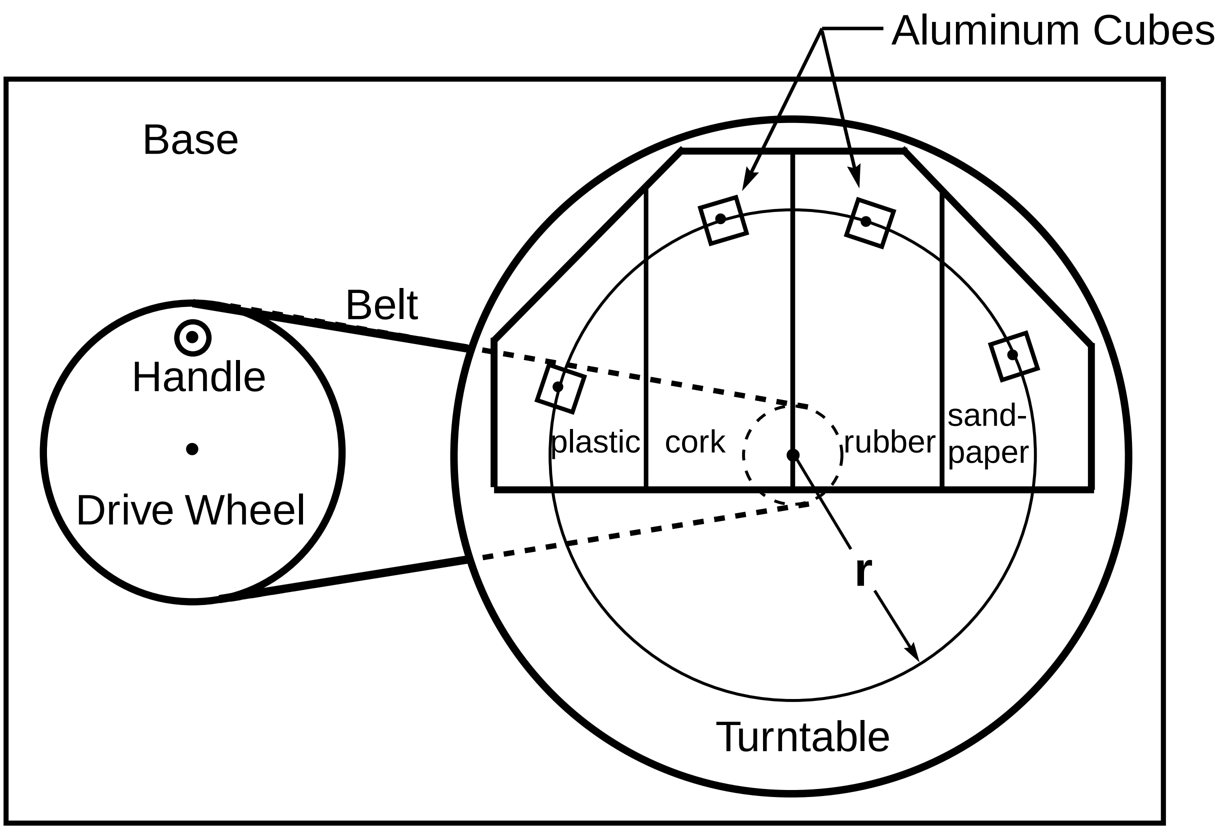


Fig. 1 Four *identical* aluminum cubes placed on four different surfaces at *equal* radii from the center of rotation of a hand-powered turntable.

b. Rotate the turntable at some *low* and *constant* angular velocity $\vec{\omega}$, *well below that required to throw any cube off the table*. In the TOP VIEW below show ALL the $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$, $\vec{\alpha}$, and $\vec{\omega}$ vectors for the circular motion of the 4 cubes *as seen by an observer in the lab frame*. Show only the horizontal forces $\vec{\mathbf{F}}$ acting on each of the four cubes.

FOUR IDENTICAL CUBES (ON FOUR DIFFERENT SURFACES) ALL AT REST (WITH RESPECT TO THE SURFACES) AT EQUAL DISTANCES FROM THE CENTER OF A TURNTABLE ROTATING WITH CONSTANT ANGULAR VELOCITY $\vec{\omega}$



c. How do the relative magnitudes of the frictional forces $\vec{\mathbf{f}}$ applied to the four cubes compare when the turntable is rotated as above?

d. Derive an expression for the magnitude of $\vec{\mathbf{f}}$ in terms of m, ω, and r.

e. Very *gradually and uniformly* increase the angular velocity ω of the turntable. Write down the time order in which the cubes fly off the table. [HINT: Repeat this experiment several times to make sure that your results are reproducible.] Can you state, then, any qualitative relationship between the critical value ω_c for fly-off of a cube and the surface roughness? {Y, N, U, NOT} Do you know what parameter is used in mechanics to characterize the surface roughness at an interface? {Y, N, U, NOT}

2. Other Variables Affecting the Critical Angular Velocity ω_c

a. Make a list of variables other than surface roughness which might affect the critical angular velocity ω_c at which an object flies off the turntable.

b. Do you know what is meant by the term "controlled variable experiment." {Y, N, U, NOT} [HINT: For an example, see Fig. 1, p. 42]

c. Carry out controlled variable experiments that allow you to to test the effect (or lack of effect) of the variables you have listed above. A variety of various objects which you may want to place on the turntable is available at the front table. Describe your experiments and the results. [HINT: One labeled sketch is worth a teraword.]

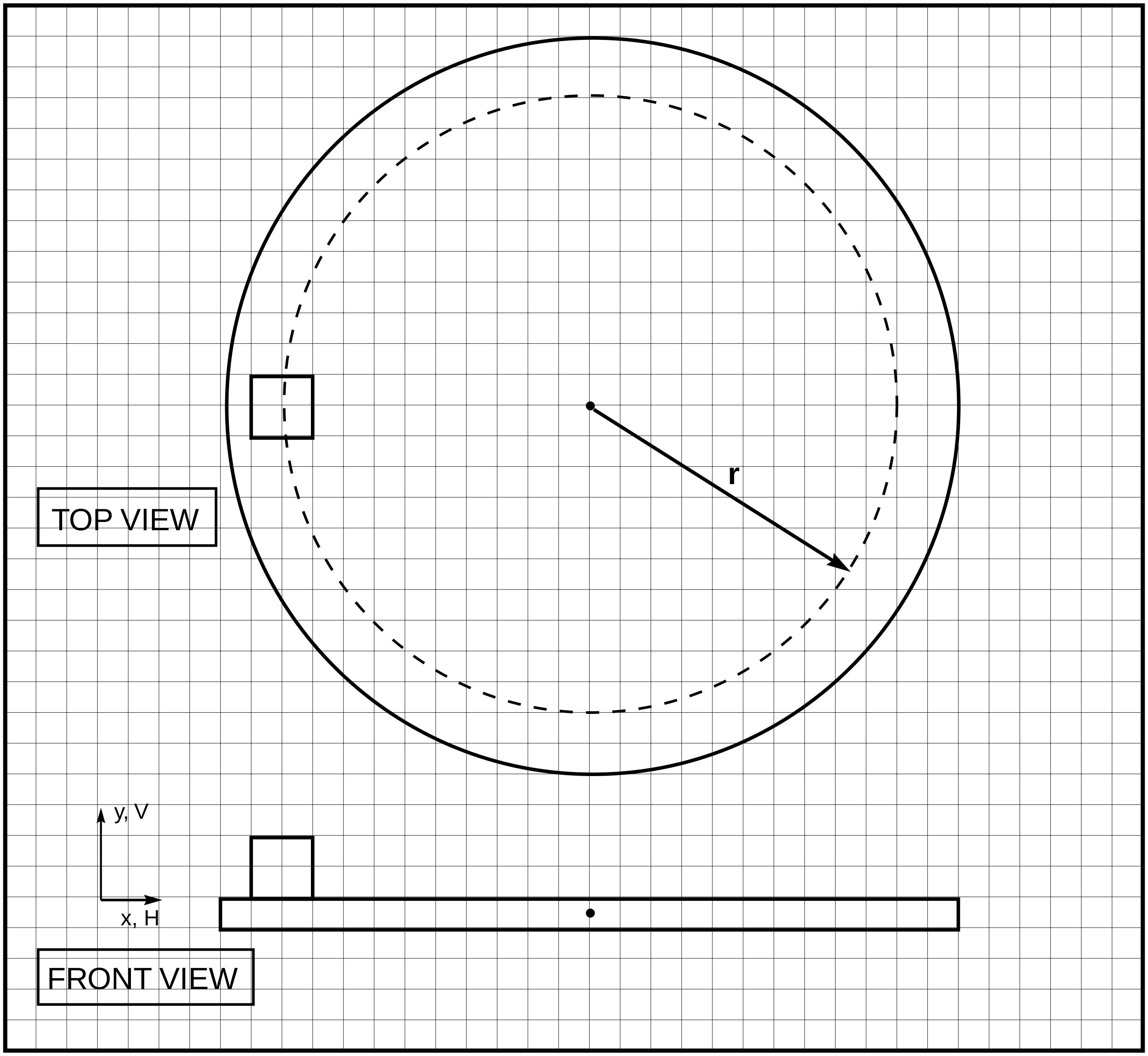
CONTROLLED VARIABLE EXPERIMENTS

CONTROLLED VARIABLE EXPERIMENTS

D. LIMITING CONDITIONS

1. In science, as in other areas, it is often helpful to consider *extreme limiting condition*s. Suppose you were to place a "perfectly smooth" object on a "perfectly smooth" turntable. In the TOP and FRONT views below show ALL the $\vec{\mathbf{F}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{a}}$, $\vec{\alpha}$,and $\vec{\omega}$ vectors motion of the object *as seen by an observer in the lab frame.*

THE MOTION OF A PERFECTLY SMOOTH OBJECT ON A PERFECTLY SMOOTH TURNTABLE



2. Describe the motion of the object as you gradually increase the angular velocity ω of the turntable from zero.

3. Can you think of a way to test your answers to "1" and "2" above experimentally? {Y, N, U, NOT} [HINT: Although there are no "perfectly smooth" surfaces in nature, one can experiment with surfaces or conditions which approach this ideal and then extrapolate the results.]

E. SUPER SPLASH SPINNER- A Thought Experiment

At the *National Physics-Fun Amusement Park* you and the rest of the class pile on the "Super Splash Spinner" shown below, a giant motorized version of the turntable used in this lab. The spinner's angular velocity ω is *very gradually and uniformly increased* from zero. One by one your classmates fly off the spinner and splash into the surrounding pool. (Would it be wise to wave goodbye to them?) As ω increases you consider the Newtonian physics of the critical value ω_c for your fly-off from the spinner. The coefficient of static friction between you and the spinner is μ_s. Your mass is m. Ignore air friction. Is the physics essentially the same as in Sec. IVB, where ω is gradually increased until an aluminum cube flies off a hand-powered turntable? {Y, N, U, NOT}

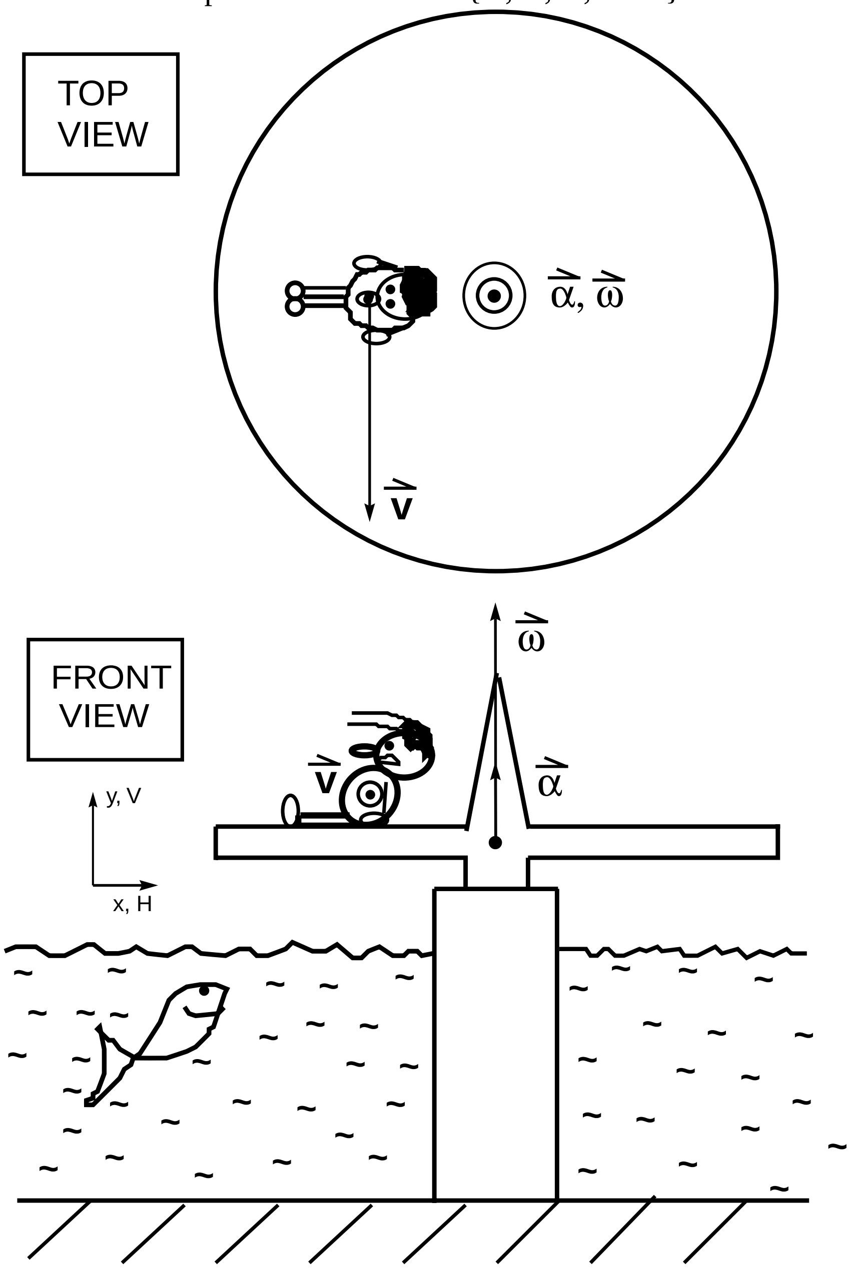


Fig. 2. A student struggles to stay aboard the Giant Super-Splash Spinner (not to scale). α is the angular acceleration.

1. (You may wish to do this as an out-of-lab problem **OLP #10** .) Derive a general expression for your critical fly-off angular velocity ω_c in terms of relevant parameters of the problem. (Assume that your *angular* acceleration is very low and thus your tangential acceleration is negligible in comparison with your radial acceleration.) Is your expression physically reasonable? {Y, N, U, NOT}

F. EXPERIMENTAL MEASUREMENT OF μ_s and ω_c

1. Can you devise and carry out a method to measure μ_s, the coefficient of static friction for an aluminum-turntable interface? {Y, N, U, NOT} [HINT: Use the aluminum cylinder (diam. ≈ 2 in, height ≈ 1.8 in) at your table.]

2. Devise and carry out a method to measure the critical fly-off angular velocity ω_c for the above aluminum cylinder on the turntable to within, say, $\pm$20%. Calculate the discrepancy between ω_c (experiment) and ω_c (theory).

3. Considering your expression for ω_c derived and experimentally tested above and the situation of Fig. 3, what might you do to promote your chances of winning the $100 prize awarded to the last person to be thrown off the Super Splash Spinner?

ACKNOWLEDGEMENTS: This lab has benefited from (a) helpful comments of Professors Fred Lurie and James Sowinski who served as SDI lab instructors during the Spring semesters of 1993-94, (b) valuable suggestions by Laboratory Coordinator Ray Wakeland, (c) feedback from the experiments, writing, discussion, drawing, and dialogue of (1) the 1263 Indiana University introductory-physics-course students who have taken SDI labs as a major part of their regular lab instruction, (2) the 22 student volunteers from P201 classes who consented to be videotaped while working through SDI labs on Saturday mornings in the Fall of 1993-94.

SDI Lab #3 Appendix: **ROTATING REFERENCE FRAMES**

A. PLAYING CATCH ON A MERRY-GO-ROUND†

1. With a partner of similar weight, sit on the Merry-Go-Round (MGR) as shown below in Fig. 3. (The heavier partner may have to sit in towards the center. If so, have a helper reclamp the backrest as shown to the right in Fig. 3.)

Fig. 3. Two students sit at opposite ends of a Merry-Go-Round.

2. Have an observer in the Lab Reference Frame (LRF) set the MGR into *counterclockwise* rotation ($\vec{\omega}$ vertically up), thus simulating the Earth's rotation in the northern hemisphere.

CAUTION!! - a. Do not spin with such a high $\vec{\omega}$ that the riders become sick or dizzy.
b. When the MGR is rotating, observers must stand outside the red circle marked on the floor, otherwise their ankles may be broken.

Try to play catch with your partner. While you are throwing the balls observe their paths. Do the balls thrown by you appear to curve to your left, to your right, or neither left nor right (*underline one*).

Have an observer in the LRF observer the paths of the thrown balls. The LRF observer can more reliably see the paths *with respect to the lab frame* by hiding the MGR from view with a sheet of paper held close to the eyes. For balls moving away from the lab observer do the paths appear to curve to the left, to the right, or neither left nor right (*underline one*).

3. Fig. 4 shows a top views of a student S riding on the MGR who throws a ball to a partner S'. The motion of the ball is different for observers in the stationary lab reference frame (Fig. 4a) and in the rotating Merry-Go-Round frame (Fig. 4b). The latter is a *non-inertial frame* similar to the accelerating truck frame in Sec. XI, "Motion of a kid in a truck- revisited," of SDI Lab #2.

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† Sections A1-2 are adapted from L. Evans, "The Coriolis Effect and other Spin-Off Demonstrations," *The Physics Teacher* **20**, 102 (1982).

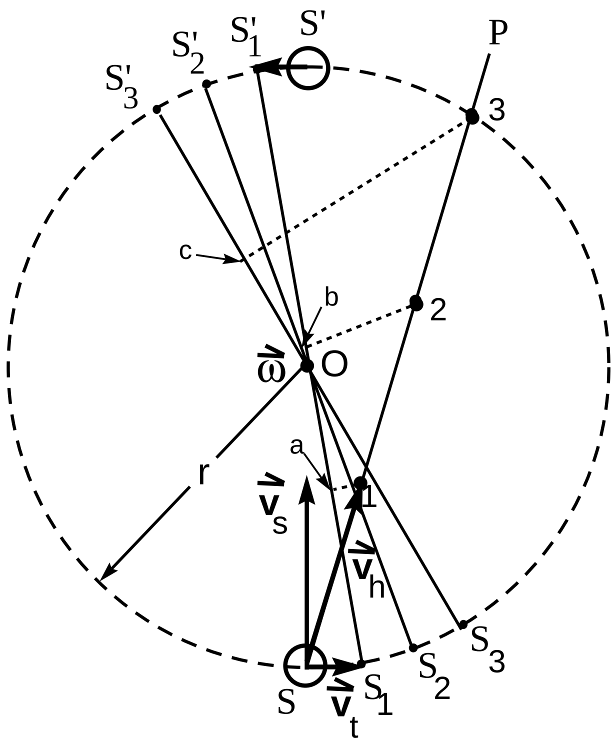

Fig. 4a. As seen by an observer in the stationary lab reference frame for $\vec{\omega}$ counterclockwise.

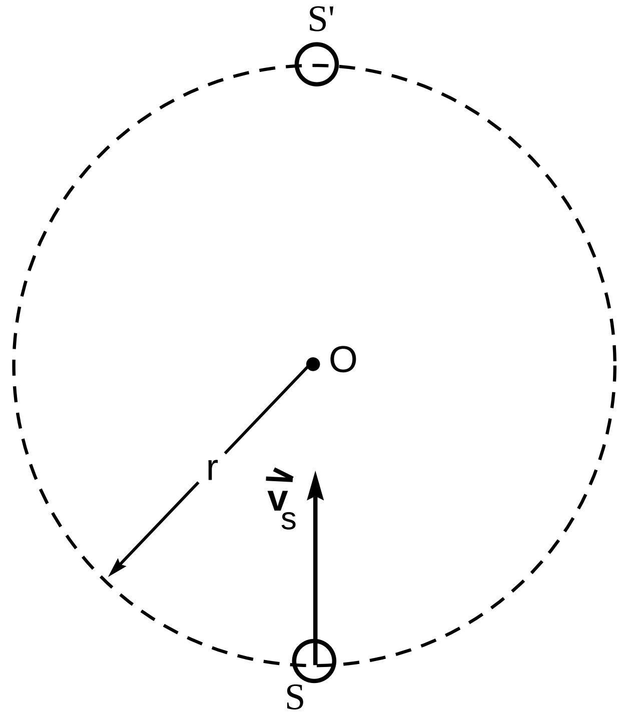

Fig. 4b. As seen by an observer in the rotating Merry-Go-Round frame for $\vec{\omega}$ counterclockwise.

To an observer in the stationary frame, the MGR has an angular velocity $\vec{\omega}$. Student S and her partner S' are at distances r from the center O of the MGR, and both have tangential velocities of magnitude $v_t = \omega r$. Suppose S throws a ball radially inward with a velocity $\vec{v}_s$ towards a partner S'. After the ball leaves S's hand, it will have a horizontal velocity $\vec{v}_h = \vec{v}_t + \vec{v}_s$. That is, the horizontal velocity of the ball, as seen by the stationary observer, is the vector sum of the tangential velocity of the ball before it was released and the radial velocity imparted by S to the ball. Since (ignoring the very small air friction) there are no horizontal forces acting on the ball, the velocity $\vec{v}_h$ will be constant as long as the ball is in the air. The path of the thrown ball will be the usual parabola, but in the top view the path will appear to the stationary observer to be a straight line along SP and $\vec{v}_h$, since the parabola is in a vertical plane. Is this path consistent with the observations of the lab observer in "2" above? {Y, N, U, NOT} At successive equal intervals of time Δt , S moves to S_1, then to S_2, and then to S_3. During the same intervals of time the ball moves to position 1, then to position 2, and then to position 3 along the line SP.

To an observer in the rotating frame, the MGR, S, S', and the diameter SOS' are all stationary, while the lab rotates clockwise. At the end of the first time interval Δt , the ball has moved a distance S-1 from S, and is at a perpendicular distance 1-a to the right of the diameter SOS'. At the end of the second time interval Δt, the ball has moved a distance S-2 from S, and is at a perpendicular distance 2-b to the right of the diameter SOS'. At the end of the third time interval Δt , the ball has moved a distance S-3 from S, and is at a perpendicular distance 3-c to the right of the diameter SOS'. Despite the fact that there are no true horizontal interaction forces of the type $\vec{\mathbf{F}}_{\text{on ball by X}}$, observer S in the rotating frame can apply Newtonian mechanics to the horizontal motion of the ball by inventing a "pseudo force" (here called the "Coriolis force") to account for the apparent sideways deflection of the ball. In Fig. 4b, can you draw in the path of the ball as it appears to observer S in the rotating frame? {Y, N, U, NOT} Is the path you have drawn in qualitative accord with your observations as an observer in the rotating frame? {Y, N, U, NOT}

4. Have an observer in the Lab Reference Frame (LRF) set the MGR into *clockwise* rotation ($\vec{\omega}$ vertically down), thus simulating the Earth's rotation in the southern hemisphere. Try to play catch with your partner. While you are throwing the balls observe the paths of the balls. Do the balls thrown by you appear to curve to your left, to your right, or neither left or right (underline one). Are your observations in accord with the explanation in "3" above as amended for a clockwise rotation of the MGR? {Y, N, U, NOT}

B. CYCLONE AND BATHTUB VORTICES

Satellite surveillance photographs show that winds flowing inward towards low pressure areas, as in cyclones and hurricanes, tend to move in counterclockwise vortices in the Northern Hemisphere and clockwise vortices in the Southern Hemisphere. Likewise, water draining from a bathtub in the northern-hemisphere was consistently observed to rotate *counterclockwise* (as observed from above) by Shapiro[1] at MIT, as expected if the rotation is determined by the Coriolis force. Water draining from a bathtub in the southern-hemisphere was consistently observed to rotate *clockwise* by Trefethen *et al.*[2] at the University of Sydney, again in harmony with the Coriolis-force interpretation. The heroic efforts required for these bathtub-vortex observations are summarized by Walker.[3]

For the less heroic, a plastic-pail bathtub may be placed on a Merry-Go-Round with an angular velocity $\vec{\omega}$ about 10^4 greater than that of the Earth as shown in Fig. 5. Rotations simulating northern and southern hemispheres then result in water vortices around the drain which are respectively counterclockwise and clockwise.

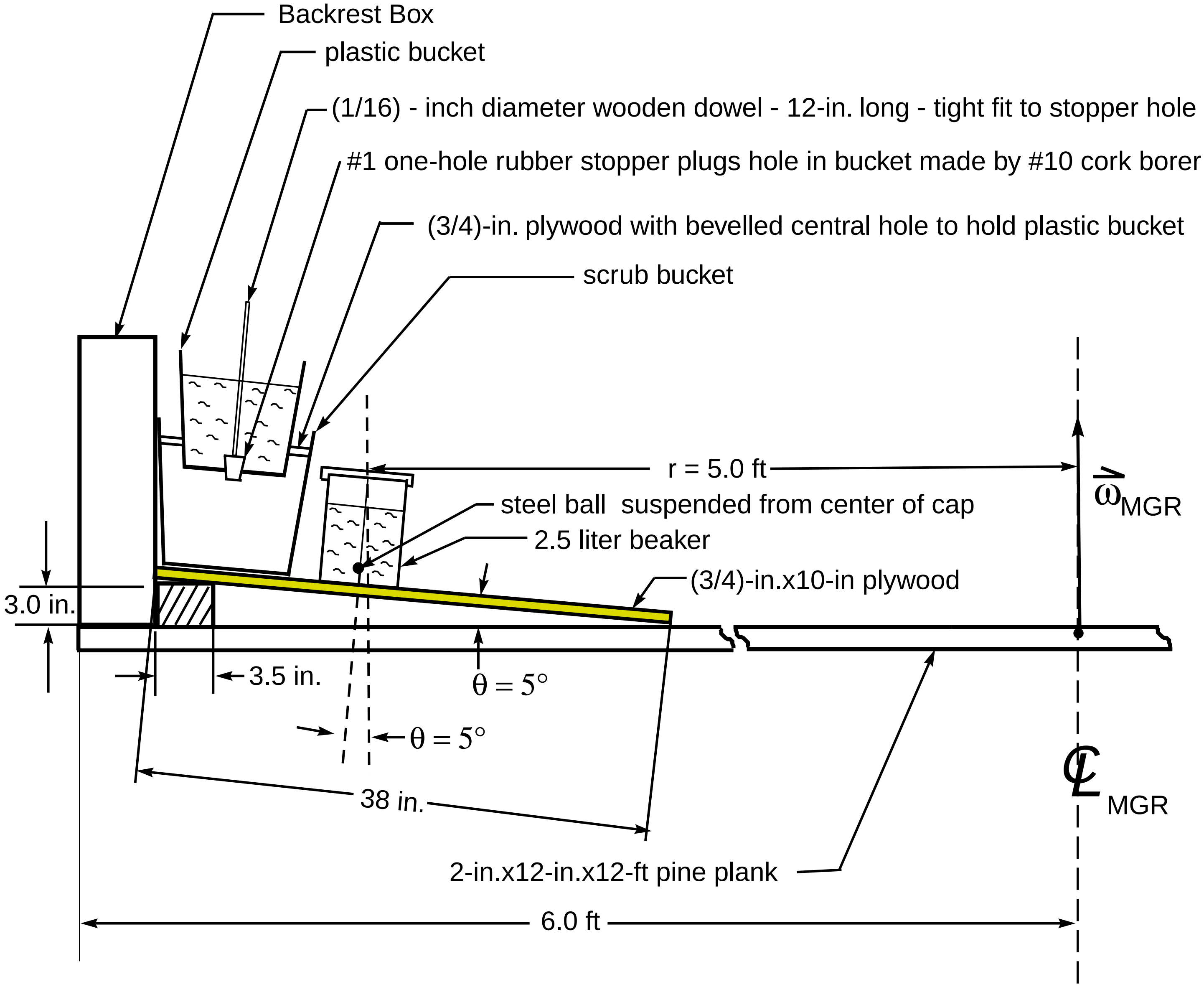

Fig. 5. A plastic bucket "bathtub" is placed on a Merry-Go-Round with an angular velocity $\vec{\omega}$ about 10^4 greater than that of the Earth.

1. Demonstrate the "bathtub vortex" in the Northern hemisphere. Place the double-bucket assembly on one end of the the MGR as shown in Fig. 3. *Stand outside the red circle so as to avoid having your ankles broken.* Place a few drops of food coloring in the water of the bucket so as to enhance the visibility of any swirling motion. Now rotate the MGR *counterclockwise (* $\vec{\omega}_{MGR}$ vertically up as shown in Fig. 5.) After a steady $\vec{\omega}_{MGR}$ is attained with the string holding the steel ball along the center line of the beaker, pull the stopper out of the hole in the bottom of the bucket by pulling on the long slender dowel attached to the stopper. Is the angular velocity $\vec{\omega}_v$ of the vortex clockwise, counterclockwise, or neither? (underline one). Repeat this experiment several times. Are the results reproducible? {Y, N, U, NOT}

2. Repeat the experiment in "1" above, except rotate the MGR clockwise *(* $\vec{\omega}_{MGR}$ vertically down, opposite of that shown in Fig. 5.) so as to simulate the rotation of the Earth as observed in the Southern hemisphere. Is the angular velocity $\vec{\omega}_v$ of the vortex clockwise, counterclockwise, or neither? (underline one). Repeat this experiment several times. Are the results reproducible? {Y, N, U, NOT}

3. From the geometry of Fig. 5, can you obtain the period T_θ of the MGR from θ? {Y, N, U, NOT}

4. Directly measure the period T_m of the MGR when the string suspending the steel ball is along the center line of the beaker. Compare this with T_θ as obtained above in "3". [HINT: Increase the accuracy by measuring the time for ten rotations and then divide by 10 to obtain the period.]

5. What is the period of rotation T_E of the Earth? What is the ratio T_m / T_E ? What is the ratio ω_m / ω_E ?

C. CONNECTION BETWEEN PLAYING CATCH ON THE MERRY-GO-ROUND AND CYCLONE/BATHTUB VORTICES

Fig. 6 represents cyclone wind patterns in the Northern Hemisphere. Can you see the connection between these patterns and the apparent deflection of balls thrown on the Merry-Go-Round?
{Y, N, U, NOT}

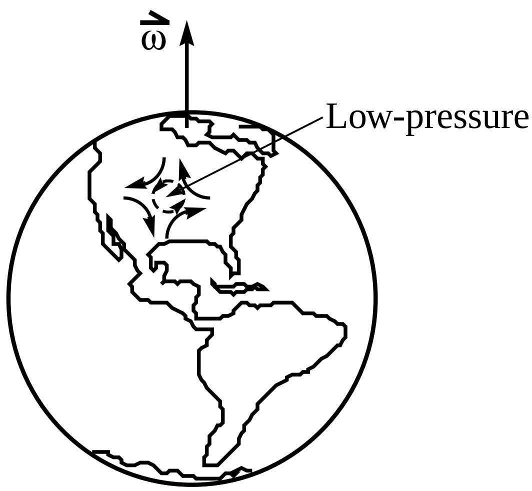

Fig. 6. Cyclone wind patterns in the Northern hemisphere.

References

1. A. H. Shapiro, "Bathtub Vortex," *Nature* **196**, 1080 - 1081 (1962); film *Vorticity*, Educational Services, Inc., Watertown, Mass., 1961); film loop #FM-15, *The Bathtub Vortex, ibid.*, 1963.

2. L.M. Trefethen, R.W. Bilger, P.T. Fink, R.E. Luxton, and R.I. Tanner, "Bathtub Vortex in the Southern Hemisphere," *Nature* **207**, 1084 - 1085 (1965).

3. J. Walker, *The Flying Circus of Physics* (With Answers) (John Wiley & Sons, New York, 1975) p. 95. Walker gives other references germane to the experiments.