Welcome to the Accelerator Physics
S.Y. Lee, Indiana University

This course is an introduction to the physics, technology, design, and operation of high energy particle accelerators. Topics include accelerator magnets, history and introduction of various types of particle accelerators, single particle transverse and longitudinal motion; emittance; effects of linear magnet errors; chromatic effects and their correction; effects of nonlinearities; basic beam manipulations; RF systems, diagnostic systems; and introduction to accelerator lattice design. Other topics such as synchrotron radiation excitation and damping; beam-beam interaction; collective effects and instabilities; linear accelerators will also be discussed. Future perspectives of high energy accelerators and colliders are discussed. [http://physics.indiana.edu/~shylee/p570/](http://physics.indiana.edu/~shylee/p570/)

Prerequisites: Electromagnetism and classical mechanics.

In 1911, Rutherford employed α particles escaping the Coulomb barrier of Ra and Th nuclei to investigate the inner structure of atoms. He demonstrated, to the surprise of many physicists, the existence of a positively charged nucleus with a diameter less than 10⁻¹³ m. This led to the introduction of Bohr’s atomic model, and the revolution of quantum mechanics in the early 20th century. In 1919, Rutherford also used α particles to induce the first artificial nuclear reaction, α + ¹⁴N → ¹⁷O + H. This discovery created an era of search for high-voltage sources for particle acceleration that can produce high-intensity high-energy particles for the study of nuclear transmutation.

Historical prospective

- Historically, many major discoveries in sciences can be associated with innovative new experimental tools. Some of these tools involves particle beams, which have been very useful to research in high energy and nuclear physics (high energy colliders), condensed matter physics (SR and Neutron sources), biomedical sciences, etc.
- Accelerators has many applications: Medical treatments (Loma Linda, MGH, HIMAC, GSI, etc), nuclear medicine, industrial processing (polymerization, sterilization, ion beam implantation), semiconductor industry, fossil-dating (AMS), defense industry, etc.
- Cockcroft-Walton electrostatic high voltage source and Van de Graaff high voltage source – limited to about 1 MV/m in air.
- Induction accelerators – betatron, induction linac, induction synchrotron
- RF accelerators: linacs (DTL, CCL, SC, electron linacs, …), synchrotrons, RFQ, recirculating linacs.
- Cyclotrons, AVF (isochronous), separate sector cyclotrons,
- Phase focusing, AG focusing.
- Beam Cooling (stochastic cooling, electron cooling, laser cooling, ionization cooling, SR damping, etc.)
- Storage rings and Colliders
- Applications of superconductivity in accelerators (SC cavity, SC magnets)
- New acceleration schemes: laser, plasma wake field, etc.
- NEW IDEAS!
I.2 Electrostatic Accelerators

1. X-ray tubes: William David Coolidge in 1926 achieved 900-keV electron beam energy by using three X-ray tubes in series. The cascade type of X-ray tube is called the Coolidge tubes.

2. Cockcroft-Walton electrostatic accelerator: In 1930, John Douglas Cockcroft and Ernst Thomas Sinton Walton developed a high voltage source by using high-voltage rectifier units. In 1932, they reached 400-kV terminal voltage to achieve the first man-made nuclear transmutation: \( p + ^7\text{Li} \rightarrow ^2\text{He} \). The maximum achievable voltage was limited to about 1 MV because of sparking in air. Since then, Cockcroft-Walton accelerators have been widely used as the first-stage ion-beam accelerator. Recently, they are being replaced by more compact, economical, and reliable radio frequency quadrupole (RFQ) accelerators.

3. Van de Graaff and tandem accelerators: In 1931, R.J. Van de Graaff developed the electrostatic charging accelerator. In the Van de Graaff accelerator, the rectifier units are replaced by an electrostatic charging belt, and the high-voltage terminal and the acceleration tube are placed in a common tank with compressed gas for insulation, which increases the peak acceleration voltage. Placement of the high-voltage terminal at the center of the tank, the charge-exchange process on a negatively charged atomic beams can provide tandem acceleration to the stripped positively-charged nuclei. Today the voltage attained in tandem accelerators is about 25 MV.

Cockcroft-Walton’s experiments

i. verified the tunneling of Gamow;
ii. verified Einstein’s equivalence of mass and energy: \( E = mc^2 \);
iii. the first nuclear transmutation.

\[ p + ^7\text{Li} \rightarrow ^2\text{He} \]

*it’s the first step that counts!* Rutherford


1. Electro-static Accelerators
   a. Cockcroft Walton
   In 1932, they reached 400-kV terminal voltage to achieve the first man-made nuclear transmutation: \( p + ^7\text{Li} \rightarrow ^2\text{He} \). The maximum achievable voltage was limited to about 1 MV because of sparking in air. Since then, Cockcroft-Walton accelerators have been widely used as the first-stage ion-beam accelerator.

b. Van de Graaff
   Tandem accelerator
   \[ E \sim 1 \text{ MV/m} \]


I.3 Induction Accelerators

Faraday’s law

\[ \oint_\gamma \mathbf{E} \cdot d\mathbf{s} = \hat{\Phi}, \quad \Phi = \int_s \mathbf{B} \cdot d\mathbf{s}. \]

- Induction linac: developed by N.C. Christofilos in the 50’s for the acceleration of high-intensity beams. A linear induction accelerator (LIA) employs a ferrite core arranged in a cylindrically symmetric configuration to produce an inductive load to a voltage gap. Each LIA module can be viewed as a low-Q 1:1 pulse transformer. When an external current source is discharged through the circuit, the electric field at the voltage gap along the beam axis is used to accelerate the beam. A properly pulsed stack of LIA modules can be used to accelerate high-intensity short-pulse beams with a gradient of about 1 MeV/m and a power efficiency of about 50%.

A. Induction LINAC

\[ \oint_\gamma \mathbf{E} \cdot d\mathbf{s} = \hat{\Phi}, \quad \Phi = \int_s \mathbf{B} \cdot d\mathbf{s}. \]

Early History of Induction Accelerators (in the USA)

<table>
<thead>
<tr>
<th></th>
<th>Kinetic energy</th>
<th>Beam current</th>
<th>Pulse length</th>
<th>Average rep rate (max)</th>
<th>Burst rep rate (# of pulses)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astron Inj, LLNL(1963) Upgrade (1968)</td>
<td>3.7 MeV</td>
<td>350 A</td>
<td>300 ms</td>
<td>60 Hz</td>
<td>1440 Hz (100)</td>
</tr>
<tr>
<td></td>
<td>6.0 MeV</td>
<td>800 A</td>
<td>300 ms</td>
<td>60 Hz</td>
<td>800 Hz (100)</td>
</tr>
<tr>
<td>NBS Prototype (1971)</td>
<td>0.8 MeV</td>
<td>1,000 A</td>
<td>2,000 ns</td>
<td>&lt; 1 Hz</td>
<td>--</td>
</tr>
<tr>
<td>ERA Inj, LBL (1971)</td>
<td>4 MeV</td>
<td>1,000 A</td>
<td>45 ns</td>
<td>5 Hz</td>
<td>--</td>
</tr>
<tr>
<td>ETA, LLNL (1979)</td>
<td>4.5 MeV</td>
<td>10,000 A</td>
<td>30 ns</td>
<td>2 Hz</td>
<td>900 Hz (5)</td>
</tr>
<tr>
<td>FXR, LLNL (1982)</td>
<td>18 MeV</td>
<td>3,000 A</td>
<td>70 ns</td>
<td>0.3 Hz</td>
<td>--</td>
</tr>
<tr>
<td>ATA, LLNL (1983)</td>
<td>45 MeV</td>
<td>10,000 A</td>
<td>60 ns</td>
<td>5 Hz</td>
<td>1,000 Hz (10)</td>
</tr>
</tbody>
</table>

\[ P = 45 \times 10^6 \times 10^3 \times 60 \times 10^{-9} \times 5 = 135 \text{ kW} \]

B. Betatron

Faraday’s law

\[ \oint_\gamma \mathbf{E} \cdot d\mathbf{s} = \hat{\Phi}, \quad \Phi = \int_s \mathbf{B} \cdot d\mathbf{s}. \]

- Betatron: The betatron principle states that the guide field \( B_g \) is equal to 1/2 of the average field \( B_{av} \) (R. Wiederoe in 1928).

\[ \oint \mathbf{E} \cdot d\mathbf{s} = 2\pi \rho \mathbf{E} = \pi \rho^2 B_{av}, \quad \text{or} \quad \mathbf{E} = \frac{1}{2} B_{av} \rho. \]

\[ \dot{\rho} = e\mathbf{E} \quad \rho = \frac{1}{2} e B_{av} \rho = e B_g \rho, \quad \text{or} \quad B_g = \frac{1}{2} B_{av}. \]

14In 1922, Joseph Slepian patented the principle of applying induction electric field for electron beam acceleration in the U.S. patent 1645304.
The stability of transverse motion was solved in 1941 by D. Kerst and R. Serber. In 1940 D. Kerst was the first to operate a betatron to achieve 2.3 MeV. In 1949 he constructed a 315-MeV betatron at the University of Chicago with the parameters $\rho = 1.22$ m, $B_g = 9.2$ kG, $E_{inj} = 80 - 135$ keV, $I_{inj} = 13$ A. The magnet weighed about 275 tons and the repetition rate was about 6 Hz. The limitations of the betatron principle are

1. synchrotron radiation loss; and
2. the transverse beam size limit due to the intrinsic weak-focusing force.

For the first time, a bunch of protons in the synchrotron has been accelerated by an induction method (K Takayama et al. 2004). The technique may overcome certain effects that normally limit the intensity achieved in a synchrotron beam, and could prove to be an important advance for future proton colliders. The concept of an "induction synchrotron" was first proposed about five years ago by K. Takayama and Jun-ichi Kishiro of KEK and the Japan Atomic Energy Research Institute (Takayama and Kishiro 2000). The idea was to overcome shortcomings of the RF synchrotron, in particular the limited longitudinal phase-space available for the acceleration of charged particles - in other words the distribution in energy and position around the ring of the particles being accelerated. In a conventional synchrotron, the particles are accelerated when they pass through an RF cavity, a device that contains the oscillating radio wave. The electric field naturally concentrates the particles into bunches in the direction of motion (i.e. longitudinally).

Transverse Motion in circular accelerators

$$B_z = B_0 \left( \frac{R}{r} \right)^n = \frac{B_0}{(1 + x/R)^n} = B_0 - \frac{nB_0}{R} x$$

Electron (-/+), proton (+/-)

$$\frac{dp}{dt} - \gamma m \theta^2 = -e \dot{\theta} B_z, \quad \frac{d\theta}{dt} = e \theta B_{rz},$$

$$B_z = B_0 \left( \frac{R}{r} \right)^n \approx B_0 \left( 1 - \frac{nR - R}{R} + \cdots \right),$$

$$B_{rz} = z (\frac{dB_z}{dr})_{r=R} = -\frac{nB_0}{R^2} z + \cdots$$

$$\xi = (r-R)/R \quad \text{and} \quad \zeta = z/R, \quad \theta = v/r \quad \omega_0 = v/R = eB_0/\gamma m$$

$$\ddot{\xi} + \frac{\omega_0^2}{1 - n} \xi = 0, \quad \ddot{\zeta} + \omega_0^2 n \zeta = 0,$$

$$\begin{align*}
\frac{d^2 z}{dt^2} + \omega^2 n z &= 0, \\
\frac{d^2 x}{dt^2} + \omega^2 (1 - n) x &= 0.
\end{align*}$$

Summary

1. What are the advantages and disadvantages of electrostatic accelerator?
2. What are the advantages and disadvantages of induction accelerators?
3. What is the maximum field index $n$ for stable transverse motions?

Momentum Rigidity (magnetic rigidity)

$$F = q(E + v \times B).$$

$$B\rho [T \cdot m] = \frac{p}{q} = \frac{A}{Z} \times 3.33564 \times 10^3 \times p [\text{GeV/c}].$$

1. What is the momentum rigidity of proton beam with momentum 3 GeV/c?
2. What is the momentum rigidity of electron beam with momentum 3 GeV/c?
3. What is the momentum rigidity of deuteron beam with momentum 3 GeV/c/u?

<table>
<thead>
<tr>
<th>$e$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>3000.00</td>
</tr>
<tr>
<td>2.21</td>
<td>3000.00</td>
</tr>
<tr>
<td>10.01</td>
<td>10.01</td>
</tr>
</tbody>
</table>
4. RF accelerators

In 1925 G. Ising pointed out that particle acceleration can be achieved by using a radio-frequency field. In 1928 R. Wiederoe reported the first working rf accelerator, using a 1MHz, 25-kV oscillator to produce 50-kV potassium ions.

In 1931 D.H. Sloan and E.O. Lawrence built a linear accelerator using a 10-MHz, 45 kV oscillator to produce 1.26 MV Hg+ ion. Lawrence’s interest turned to cyclotron.

Since the distance between adjacent drift tubes is \( \beta \lambda / 2 = \beta c / (2 f_{\text{rf}}) \), it would save space by employing higher frequency rf sources.

However, the problem associated with a high frequency structure is that it radiates rf energy at a rate of \( P = \omega_{\text{rf}} CV_{\text{rf}}^2 \), where \( \omega_{\text{rf}} \) is the rf frequency, \( C \) is the gap capacitance, and \( V_{\text{rf}} \) is the rf voltage. The rf radiation power loss increases with the rf frequency. To eliminate rf power loss, the drift tube can be placed in a cavity so that the electromagnetic energy is stored in the form of a magnetic field (inductive load). At the same time, the resonant frequency of the cavity can be tuned to coincide with that of the accelerating field.

1948: Louis Alvarez and W.K.H. Panofsky constructed the first 32-MV drift tube linac (DTL or Alvarez linac) for protons.

1945: E. M. McMillan and V. Veksler discovered the phase-focusing principle.
Alvarez designed the first proton linear accelerator at the University of California at Berkeley where he was a Professor of Physics. He also provided the first published proposal for charge-exchange accelerators, discovered the K capture process of He(3), received a Nobel Prize in 1968 for his work with liquid hydrogen bubble chambers (and working with his son Walter proposed the comet-impact origin of the extinction of species).

<table>
<thead>
<tr>
<th>FNAL Accelerators</th>
<th>Frequency Range (MHz)</th>
<th>Cavity Volume Control</th>
<th>Cavity Tuning System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low energy Linac (DTL)</td>
<td>201.25</td>
<td>tuning slug/cavity temp</td>
<td>n/a (fixed frequency)</td>
</tr>
<tr>
<td>High energy Linac (SCL)</td>
<td>805</td>
<td>cavity temp</td>
<td>n/a (fixed frequency)</td>
</tr>
<tr>
<td>Booster</td>
<td>37.8 – 52.8</td>
<td>cavity temp</td>
<td>3 ferrite tuners per cavity</td>
</tr>
<tr>
<td>Main Injector</td>
<td>52.8 – 53.1</td>
<td>cavity temp</td>
<td>2 ferrite tuner per cavity</td>
</tr>
<tr>
<td>Tevatron</td>
<td>53.103 – 53.104</td>
<td>cavity temp</td>
<td>none (∆f small)</td>
</tr>
</tbody>
</table>

After World War II, rf technology had advanced far enough to make magnetron and klystron amplifiers that could provide rf power of about 1 MW at 3 GHz (S band). Today, the highest energy linac has achieved 50-GeV electron energy operating at S band (around 2.856 GHz) at SLAC, and has achieved an acceleration gradient of about 20 MV/m, fed by klystrons with a peak power of 40 MW in a 1-μs pulse length. To achieve 100 MV/m, about 25 times the rf power would be needed.

Electron Linacs

Superconducting cavities have also become popular in recent years. At the Continuous Electron Beam Accelerator Facility (CEBAF) at the Thomas Jefferson National Accelerator Laboratory in Virginia, about 160 m of superconducting cavity was installed for attaining a beam energy up to 4 GeV in 5 paths using 338 five-kW CW klystrons. During the LEP-II upgrade more than 300 m of superconducting rf cavity was installed for attaining an almost 100-GeV beam energy. Many accelerator laboratories, such as KEK in Japan, Cornell and Fermilab in the U.S. and DESY in Germany, are collaborating in the effort to achieve a high-gradient superconducting cavity for a linear collider design called the International Linear Collider (ILC). Normally, a superconducting cavity operates at about 5–10 MV/m. Wall conditioned single-cell cavities have reached far beyond 25 MV/m.
In 1970, I.M. Kapchinskij and V.A. Teplyakov invented a low energy radio-frequency quadrupole (RFQ) accelerator – a new type of low energy accelerator. Applying an rf electric field to the four-vane quadrupole-like longitudinally modulated structure, a longitudinal rf electric field for particle acceleration and a transverse quadrupole field for focusing can be generated simultaneously. RFQ provides the solution for high intensity beam focusing and acceleration at the same time without the complication of high voltage.

Accelerators for low energy particles

Low energy beams encounter many difficulties, such as space charge force, focusing force, emittance growth, etc.

The DC accelerators (Cockcroft-Walton and Van de Graaff) has the difficulty in maintaining high voltage in electrodes. (see also Ex. 1.16: paraxial equation).

Coulomb force between charged particles is strong at low energy! Methods to combine acceleration and focusing are critical for the transport of high quality beams!

In 1970, I.M. Kapchinskij and V.A. Teplyakov invented a low energy radio-frequency quadrupole (RFQ) accelerator – a new type of low energy accelerator. Applying an rf electric field to the four-vane quadrupole-like longitudinally modulated structure, a longitudinal rf electric field for particle acceleration and a transverse quadrupole field for focusing can be generated simultaneously. RFQ provides the solution for high intensity beam focusing and acceleration at the same time without the complication of high voltage.

Energy and momentum kinematics

The momentum and kinetic energy of the extracted particle are

\[ p = m c \beta \gamma, \quad KE = mc^2 (\gamma - 1) = \frac{p^2}{m(\gamma + 1)}, \quad p = m \omega R \gamma \]

\[ KE = 4\pi^2 m \frac{\gamma^2}{\gamma + 1} R^2 \left( \frac{f_{\text{rev}}}{h} \right)^2 = \frac{e^2 B_0^2 R^2}{m_e (\gamma + 1)} \left( \frac{Z}{A} \right)^2 \]

\[ KE = K \frac{Z^2}{A} \]

These formula can be used to calculate the kinetic energy KE of the extracted beam. The radius is the orbiting radius that is centered onto the magnetic field. For a given magnetic field, the kinetic energy per amu is proportional to \((Z/A)^2\), i.e., \(KE/A = K(Z/A)^2\). The factor \(K\) is called the \(K\)-value of the cyclotron magnet.

At a fixed magnetic field, the period of revolution is

\[ t = \frac{2\pi R}{v} = \frac{2\pi m}{eB} \gamma \]

C: Cyclotron

In 1925 G. Ising pointed out that particle acceleration can be achieved by an alternating radio-frequency field. In 1928, R Wiederöe built the first working rf accelerator.

In 1930, Ernest Lawrence published his idea for a circular particle accelerator – the basics of the cyclotron - in a short article in Science.

\[ \omega_{\text{syn}} = \frac{eB_0}{\gamma m} \quad \frac{\gamma}{m} \quad \omega_{\text{cycl}} = \frac{eB_0}{m} \]

On January 2, 1931 M.S. Livingston demonstrated the cyclotron principle by accelerating protons to 80 keV in a 4.5” cyclotron, where the rf potential applied across the accelerating gap was only 1000 V.

In 1932 Lawrence’s 11” cyclotron reached 1.25-MeV proton kinetic energy that was used to split atoms, just a few months after this was accomplished by the Cockcroft-Walton electrostatic accelerator.

References:
1. CERN Accelerator School, CERN 96-02 on “Cyclotrons, Linacs and their applications.”
2. Proceedings of International Cyclotron Conferences
3. E.O.Lawrence and N.E.Edlefsen, Science, 72 376 (1930)

Difficulty: The maximum kinetic energy was limited by the kinetic mass effect. Because the relativistic mass effect can destroy particle synchronism, the upper limit of proton kinetic energy attainable in a cyclotron is about 12 MeV [see H. Bethe and M. Rose, Phys. Rev. 52, 1254 (1937)]. Two ideas proposed to solve the dilemma are the isochronous cyclotron and the synchrocyclotron. From the cyclotron principle, the magnetic field is related to the orbit radius \(r\) by

\[ r = \frac{p}{eB} = \frac{p}{m \omega} \frac{e}{\omega} \left( 1 - \frac{1}{\gamma^2} \right)^{1/2}, \quad B = \frac{m \omega}{e} \gamma(r) = \frac{m \omega}{e} \sqrt{1 - \left( \omega/r e \right)^2} \]

However, when the required magnetic field produces a focusing index with \(n < 0\) and the vertical betatron motion becomes unstable.

Solution1: In 1938, R.H. Thomas [Phys. Rev. 54, 580 (1938)] pointed out that, by using an azimuthal varying field, the orbit stability can be retained while maintaining the isochronism. The isochronous cyclotron is also called the azimuthal varying field (AVF) cyclotron. The breakthrough came in 1957, when the first AVF cyclotron in DELFT (Netherlands) produced a beam.

Solution2: Synchro-cyclotron: synchronization between cyclotron frequency and rf frequency can be achieved by using rf frequency modulation (FM). FM cyclotrons can reach 1-GeV proton kinetic energy.
The iron saturates at a field of about 1.8 T (depending slightly on the quality of iron and magnet design). The total volume of iron-core is proportional to the cubic power of the beam rigidity $B_{\rho}$. Thus the weight of iron-core increases rapidly with its $K$-value: Weight of iron $= W \sim (B_{\rho})^3 \sim K^{1.5}$. Where $B_{\rho}$ is the beam rigidity. Typically, the magnet for a K-100 cyclotron weighs about 160 tons. The weight problem can be alleviated by using superconducting cyclotrons.

Beam extraction systems in cyclotrons is challenging. Let $V_0$ be the energy gain per revolution. The kinetic energy at N revolutions is $K_N = eN V_0 = e^2 B_{\rho} r^2/2m$, where $e$ is the charge, $m$ is the mass, $B$ is the magnetic field, and $r$ is the beam radius at the N-th revolution. The radius $r$ of the beam at the N-th revolution becomes

$$r = \frac{1}{B} \left( \frac{2mV_0}{e} \right)^{1/2} N^{1/2}$$

The orbiting radius increases with the square root of the revolution number $N$. The beam orbit separation in successive revolutions may become small, and thus the septum thickness becomes a challenging design problem.

1. Cyclotron has evolved through several generations of improvements in the orbit stability, ease of beam extraction, and better beam quality. The design evolves from classical cyclotron to AVF (isochronous cyclotron), synchrocyclotron (FM modulated cyclotron), separate sector cyclotron (ring cyclotron), separate orbit cyclotron (did not take off), FFAG (fixed field alternate gradient, dead in 1960’s but are reviving), superconducting cyclotron, etc.

2. Cyclotron is not a dead field in accelerator design. It is a versatile machine that can have many applications. There are many cyclotrons with various sizes and capabilities. The highest power cyclotron deliver more than 1.3 MW of beam power (2.2 mA 590 MeV) at PSI. In the near future, it is projected to deliver 1.8 MW of beam power. This is a very powerful beam source. If one can build a cyclotron that can deliver 5-10 MW, it will become a machine of choice of future nuclear reactor.

3. Cyclotrons have become the major power source for the production of radionuclides. There are many companies that produce cyclotrons for radionuclide production. For energy less than 240 MeV, cyclotrons have many advantages in radiation therapy, while synchrotrons have advantage for high energy machines.

Synchrotron equation of motion

In 1945, McMillan an Veksler discovered the phase focusing principle, i.e. the coupled energy phase oscillation of synchrotron motion.

$$\Delta \omega = -\frac{h}{\omega_0} \Delta \phi$$

$$\Delta f = \frac{\Delta v}{v_0} \frac{\Delta r}{r_0} = -\left( \frac{\alpha_e}{\gamma^2} - \frac{1}{\gamma^2} \right) \frac{\Delta p}{p} = -\frac{\Delta p}{p}$$

$$\frac{d}{dt} (\phi - \phi_0) = -h \Delta \omega = -h \omega_0 \frac{\Delta f}{f_0} = h \omega_0 \eta \frac{\Delta p}{p} = h \omega_0^2 \eta \Delta E / \beta^2 E_0$$

$$\dot{E} = f_0 e V \sin \phi, \quad \dot{E}_0 = f_0 e V \sin \phi_0, \quad \frac{d}{dt} \left( \frac{\Delta E}{\omega_0} \right) = \frac{1}{2\pi} e V (\sin \phi - \sin \phi_0)$$

$$\frac{d^2}{dt^2} (\phi - \phi_0) \approx \frac{h \omega_0^2 \eta e V \cos \phi_0}{2\pi \beta^2 E_0} (\phi - \phi_0)$$
E. Synchrotrons

After E.M. McMillan and V. Veksler discovered the phase focusing principle of the rf acceleration field in 1945, a natural evolution of the cyclotron was to confine the particle orbit in a well-defined path while tuning the rf system and magnetic field to synchronize particle revolution frequency. The first weak-focusing proton synchrotron, with focusing index $0 < n < 1$, was the 3-GeV Cosmotron in 1952 at BNL. A 6-GeV Bevatron constructed at LBNL in 1954, led to the discovery of antiprotons in 1955.

In 1952, Courant, Snyder and Livingston discovered the alternate-gradient-focusing concept, which was patented by a US engineer, N. Christofilos, working in Greece. Since then, synchrotrons are preferred for high energy accelerators. However, cyclotron can still have advantage in low energy accelerators for higher duty cycle! The largest synchrotron with a circumference of 27 km is located at CERN in Geneva.

The Cosmotron

In April 1948, the Atomic Energy Commission approved a plan for a proton synchrotron to be built at Brookhaven, called the Cosmotron. The Cosmotron reached its full design energy of 3.3 GeV in January 1953. It was also the first synchrotron to provide an external beam of particles for experimentation outside the accelerator itself. Early on, the intensity of the beam extracted for experiments was $10^{10}$ protons per pulse. By 1966, intensity had been increased nearly 100 times. Behind these energies and intensities was a 75-foot diameter machine, weighing 2,000 tons and composed of 288 C-shaped magnets that guided the protons in a circular path. After one second of acceleration in the Cosmotron, the protons had traveled 135,000 miles and had reached an energy of about 3 GeV. With cloud chambers, or with other detectors, the Cosmotron was the first machine to produce all the types of negative and positive mesons known to exist in cosmic rays, making possible the discoveries of the $K_L$ meson and the first vector meson. It was also the first accelerator to produce heavy unstable particles, called "V" particles, and the experimental confirmation of the theory of associated production of strange particles. After fourteen years of service to the physics research community, the Cosmotron ceased operation in 1966 and was dismantled in 1969. Knowledge gained from the Cosmotron would lead to revolutionary design improvements that would overcome these limitations and pave the way for construction the Alternating Gradient Synchrotron.

D. Microtron

As accelerating rf cavities are expensive, it would be economical to use the rf structure repetitively: microtrons, originally proposed by V. Veksler in 1944, are designed to do this. Repetitive use requires synchronization between the orbiting and the rf periods.

Some operational microtrons are the three-stage MAMI microtron at Mainz, Germany, and the 175-MeV microtron at Moscow State University. Several commercial models have been designed and built by Scanditron. The weight of the microtron also increases with the cubic power of beam energy.

$$n \lambda_{rf} = 2 \pi \frac{\Delta E}{ecB}$$

$$\frac{m \nu}{e} = B \rho, \quad t = \frac{2 \pi \rho}{\nu} = \frac{2 \pi m}{eB},$$

path difference $= c \Delta t = \frac{2 \pi \Delta E}{ecB} = n \lambda_{rf}$

$$\gamma = \gamma_0 > \gamma_0, \quad \eta > 0$$

$$\gamma = \gamma_0 < \gamma_0, \quad \eta < 0$$
Alternating Gradient Synchrotron
The AGS reached its design energy of 33 billion electron volts (GeV) on July 29, 1960. In the original configuration of the AGS, protons started on their journey to high energies via a Cockroft-Walton generator (see right), which provided an initial energy of 750,000-electron volts to the protons. They were then injected into a 110-foot long, 50-million electron volt (MeV) linear accelerator (or linac), which in turn injected the protons into the AGS. In 1972, a much more powerful 537-foot, 200-MeV linac began operation, greatly increasing the AGS's efficiency and maximum intensity. At the beginning of its operational history, the AGS's peak intensity (the total number of protons in the beam) stood at $3 \times 10^{11}$ protons per pulse, a factor of 30 over the original design intensity. By 1986, intensity had been increased more than 1,800 times over the design specifications as the AGS reached a record intensity of $1.9 \times 10^{13}$ protons per pulse. In 1984, the AGS began providing researchers with polarized protons (protons with spins aligned in the same direction) at the highest energy ever for polarized protons - 16.5 GeV. Beam energies have increased beyond this level through continued machine upgrades.

Important Discoveries
Three discoveries made at the AGS -- the muon-neutrino, CP violation and the J particle -- captured the Nobel Prize in physics. Other important particles found there include the omega-minus (1964) and the charmed lambda (1975).

1.5 Colliders and Storage Rings
The total center-of-mass energy obtainable by having an energized particle smash onto a stationary particle is limited by the kinematic transformation (see Exercise 1.6). To boost the available center-of-mass energy, two beams are accelerated to high energy and made to collide at interaction points. Since the lifetime of a particle beam depends on the vacuum pressure in the beam pipe, stability of the power supply, intrabeam Coulomb scattering, Touschek scattering, quantum fluctuations, collective instabilities, nonlinear resonances, etc., accelerator physics issues have to be evaluated in the design, construction, and operation of colliders. Beam manipulation techniques such as beam stacking, bunch rotation, stochastic beam cooling, invented by S. Van de Meer, electron beam cooling, invented by Budker in 1966, etc., are essential in making the collider a reality.

The first proton-proton collider was the intersecting storage rings (ISR) at CERN completed in 1969. ISR was the test bed for physics ideas such as stochastic beam cooling, high vacuum, collective instabilities, beam stacking, phase displacement acceleration, nonlinear beam-beam force, etc. It reached 57 A of single beam current at 30 GeV. It stopped operation in 1981.

The first electron storage ring (200 MeV) was built by B. Touschek et al. in 1960 in Rome. It had only one beam line and an internal target to produce positrons, and it was necessary to flip the entire ring by 180° to fill both beams. Since the Laboratoire de l’Accelerateur Lineaire (LAL) in Orsay had a linac, the storage ring was transported to Orsay in 1961 to become the first e+e− collider. The Stanford-Princeton electron-electron storage ring was proposed in 1956 but completed only in 1966. The e−e− collider moved from Moscow to Novosibirsk in 1962 began its beam collision in 1965. Since the 1960’s, many e+e− colliders have been built. Experience in the operation of high energy colliders has led to an understanding of beam dynamics problems such as beam-beam interactions, nonlinear resonances, collective (coherent) beam instability, wakefield and impedance, intrabeam scattering, etc.

Proton-antiproton colliders include the Tevatron at Fermilab and SppS at CERN. The discovery of type-II superconductors led to the successful development of superconducting magnets, which have been successfully used in the Tevatron to attain 2-TeV c.m. energy, and in HERA to attain 820-GeV proton beam energy. At present, the CERN LHC (14-TeV c.m. energy) and the BNL RHIC (200-GeV/u heavy ion c.m. energy) are under construction. The (40-TeV) SSC proton collider in Texas was canceled in October 1993. Physicists are contemplating a very large hadron collider with about 60–100 TeV beam energy.
Collider vs fixed target experiments

Fixed target experiment: \[ P_1 = (p_1, E_1 = \gamma mc^2) \quad P_2 = (0, E_2 = mc^2) \]
\[ s_{\text{FT}} = (E_1 + E_2)^2 - (p_1 + 0)^2 c^2 = 2(\gamma + 1)m^2c^4 \]
1 TeV p + fixed target p = 45 GeV available energy

Minimum energy for \[ p + p \rightarrow p + p + p + \bar{p} \] is 6 mc^2.

Luminosity: \[ \mathcal{L} = \frac{dN}{dt} \times (\text{target thickness: atoms/cm}^2) = \frac{dN}{dt} \times \rho \]

Collider \[ P_1 = (p, E = \gamma mc^2) \quad P_2 = (-p, E = \gamma mc^2) \]
\[ s = (E_1 + E_2)^2 - (p_1 - p_1)^2 c^2 = 4\gamma^2 m^2 c^4 \]
1 TeV + 1 TeV = 2 TeV available cm energy
\[ 2(\gamma_{\text{FT}} + 1)m^2c^4 = 4\gamma_{\text{cm}}^2 m^2 c^4 \]

Luminosity: \[ \mathcal{L} = \frac{fN_1N_2}{4\pi\sigma_x \sigma_z} \]

The counting rate of a reaction with cross-section \( \sigma \) is \[ \frac{dN}{dt} = \mathcal{L}\sigma \]

Electron in circular accelerators

Larmor’s formula:
\[ P = \frac{1}{4\pi \varepsilon_0} \frac{2e^2\dot{\varphi}^2}{3e^3} = \frac{1}{4\pi \varepsilon_0} \frac{2e^2}{3m^2e^3} \frac{dp_{\mu}}{dt} \frac{dp_{\mu}}{dt} \]

- 1898: Liénard derived an expression for electromagnetic radiation in a circular orbit.
- 1947: the synchrotron radiation was observed from electron beam in 70-MeV General Electric electron synchrotron (John Blewett) [F.R. Elder et al., Phys. Rev. 71, 829 (1947); ibid. 74, 52 (1948); J. Appl. Phys. 18, 810 (1947)].

\[ P = \frac{2r_0}{3mc} \gamma^2 \omega^2 |p|^2 = \frac{2r_0}{3mc} \gamma^2 |F_\perp|^2 = \beta^4 c^4 E^4 \frac{d\mu}{\gamma} \frac{d\mu}{\gamma} = \beta^4 e^2 c^2 \frac{d\mu}{\gamma} \frac{d\mu}{\gamma} - C_e E^2 B^2, \]

\[ C_e = 4\pi \frac{r_0}{3 (mc^2)^3} = \begin{cases} 8.846 \times 10^{-5} \text{m/(GeV)}^3 & \text{for electrons} \\ 4.840 \times 10^{-14} \text{m/(GeV)}^3 & \text{for muons} \\ 7.783 \times 10^{-18} \text{m/(GeV)}^3 & \text{for protons} \end{cases} \]
I.6 Synchrotron Radiation Storage Rings
Since the discovery of synchrotron light from a then high energy (80-MeV) electron synchrotron in 1947, the synchrotron light source has become an indispensable tool in basic atomic and molecular physics, condensed-matter physics, material science, biological, chemical, and medical research, and material processing. Worldwide, about 70 light sources are in operation or being designed or built.

II Layout and Components of Accelerators
II.1 Acceleration Cavities
The rf acceleration cavity provides high longitudinal electric field at an rf frequency that ranges from a few hundred kHz to 10–30 GHz. For a particle with charge e, the energy gain/loss per passage through a cavity gap is

\[ \Delta E = e \Delta V, \]

where \( \Delta V = V_0 \sin(\omega_{rf}t + \phi) \) is the effective gap voltage, \( \omega_{rf} \) is the rf frequency, \( V_0 \) is the effective peak accelerating voltage, and \( \phi \) is the phase angle.

II.2 Accelerator Magnets
Dipole magnets are used to guide charged particle beams along a desired orbit. From the Lorentz force law, the bending angle \( \theta \) in passing through a dipole, and the integrated dipole field in a ring are

\[ \theta = \frac{e}{p_0} \int_{r_1}^{r_2} B dl = \frac{1}{B_d} \int_{\rho}^{\rho + B_d} B dl, \]

The total length of magnet of a synchrotron is \( L = 2\pi B_d / B_{dip} \)

Dipole field: \( B = \mu_0 NI / g \)
Inductance:
\[ L = \mu_0 N^2 \ell w / g = \mu_0 n^2 \times \text{volume of the dipole} \]

The 2D magnetic field at a distance \( r \) from a infinitely-long straight wire carrying current \( I \) is \( B = \mu_0 I / 2\pi r \) (Ampere’s law), or the 2D magnetic field at location \( y = x + jz \) induced by a long straight wire at \( y_0 = x_0 + jz_0 \) is (Beth representation):

\[ B_x(x,z) + jB_z(x,z) = \frac{\mu_0 I}{2\pi(y-y_0)} \]

The magnetic field between two sheets of parallel iron-plates with infinite permeability separated by a gap \( g \) is

\[ B_z + jB_x = \frac{\mu_0 I}{4g} \left[ \frac{\pi(y-y_0)}{2g} + \coth \frac{\pi(y-y_0)}{2g} \right] \]
Septa (plural of septum) provide a way of modifying beam trajectory using an electrostatic or magnetic field.

The electrostatic septum can be used in switchyard to split a beam of charged particles into two separate beams. Particles are directed towards the grounded wires found along the length of a septum, and the electric fields formed between the wires and the high voltage plates cause the stream of particles to be split into two separate streams. Usually one needs a string of septa to accomplish the required splitting force.

The magnetic septum may be made of **current-sheet** or a **Lambertson** septum.

With septa, switchyard provides beam to a number of different experiments simultaneously.

A chopper is an electrostatic device that deflects a portion of a continuous stream of beam in a desired path. Choppers is used to create time gap in a continuous beam (750 keV chopper). Connecting a pair of conductive plates to a high voltage power supply, one can switch if the plates are to be charged to high voltages supplies.

In this introduction, I hope that you get some ideas on the following questions:

1. What are advantages and disadvantages of rf accelerators?
2. What is the advantage of cyclotrons vs synchrotrons?
3. What is the advantage of linacs over synchrotrons for very high energy electrons
4. What is the total length of dipoles for a circular accelerator?
5. How to determine the Circumference of a circular accelerator?