Measuring Magnetic Fields

I. Objectives

In this lab, you will become familiar with several alternative techniques that can be used to measure absolute magnetic field strengths. Each method is based on a different fundamental physics principle; thus, each has different advantages and disadvantages, and each involves the use of very different pieces of equipment. You will also use your measurements to characterize some of the properties of the magnetic field created between the pole tips of an electromagnet.

II. Physics background

A quick look through any introductory modern physics text will show that the magnetic field \( \mathbf{B} \) enters into a large number of our physical laws, and as such plays an important role in a wide range of diverse physical phenomena. To study these phenomena quantitatively, it is therefore crucial that one have accurate knowledge of both the magnitude and the direction of the magnetic field present in the area where the experiment is being conducted. On the other hand, if these same physical principles are understood sufficiently well, they can also be ‘turned around’ and used instead to determine \( \mathbf{B} \), once the apparatus has been calibrated properly against known magnetic field strengths.

In this lab, you will use two different physical principles (the Hall Effect and Faraday’s Law) and the equipment associated with each (the Hall probe plus Gaussmeter, and a flip-coil plus current integrator, respectively) to measure some properties of the magnetic field established by an electromagnet around an iron yoke. Note the lack of similarity between these two laws of physics! For the former, one relies on the Lorentz force law for charges moving in the presence of a magnetic field to produce a transverse (to both \( \mathbf{B} \) and \( \mathbf{v} \)) potential difference \( \Delta V \); while Faraday’s Law describes the electromotive force \( \varepsilon_{\text{EMF}} \) induced in a coil due to a changing (time-dependent) magnetic flux (see Appendix I). A third physical principle, that of nuclear magnetic resonance (NMR), is also commonly used to determine magnetic field strengths, often to very high accuracy (as high as a few parts per billion!), and is the subject of a separate lab in this course.

Before starting this lab, it would be a good idea to refresh your memory on the physics behind the Hall effect and Faraday’s induction law. Any rigorous introductory-level text (e.g., Halliday and Resnick) or a good E & M book should suffice – choose your favorite.
III. Experimental Equipment

There are three distinct systems of equipment that you will need to understand in order to carry out the procedures suggested below. These are: (1) the magnet itself (coils and yoke), with its attendant power supply (Harrison Labs 6267A) and a current-measuring device (shunt resistor and DVM); (2) the Hall probe and Gaussmeter, which are more or less self-contained units; and (3) the flip-coil, which must be connected to some sort of a current-integrating device. In olden times, this last function was often carried out using a ‘ballistic galvanometer,’ which involves mirrors and a torsion string, and which can be now be found only in science museums ..., or on top of the beta spectrometer. For this lab, you will use a modern (and much more reliable) device, based on the JFET operational amplifier, or ‘op amp’, described in Appendix I.

The magnetic field you will study in this lab is produced by direct (‘D.C.’) current flowing through coils which are wrapped around an iron yoke. Make sure you understand the concepts of magnetic saturation and hysteresis as they apply to an electromagnet with an iron core. Our primary interest here will be in measuring properties of the magnetic field that is established in the air gap between the tips of the cylindrical pole pieces.

IV. Suggested Measurements

1. Be sure you understand all aspects of the field-generating equipment. Draw the circuit, showing all interconnections. Record the size of the shunt resistance in ohms (Ω).

2. Run up the power supply, making sure it is in ‘current regulation mode’, and not voltage mode. The supply should not be run above about 5 A. At several settings, check that your value for the current (as determined from the shunt resistor and DVM) agrees with that of the power supply meter to within reasonable tolerances.

3. Locate the Gaussmeter and Hall probe. After letting it warm up for a few minutes, figure out a way to zero the Gaussmeter. (Hint – note that the probe is sensitive to only a single component of the vector B.) Once properly zeroed, use the calibrated magnetic field (~200 G) in the large heavy box to adjust the gain. You may want to scan the operating manual for some suggested procedures.

4. With the Gaussmeter now calibrated, position the Hall probe in the center of the magnet gap and orient it for maximum field strength. Carefully map out B vs. I_{coil} for I first increasing, then decreasing. Check for indications of saturation and hysteresis in the magnet iron. Again, you should not let I_{coil} exceed about 5 A.

5. Keeping I_{coil} fixed, investigate the homogeneity of the field in the gap area. Assuming cylindrical symmetry, two useful measurements would be looking at how B_{z} varies as one moves along either the pole axis (z direction) or radially outward in the midplane.

6. Finally, measure B at several values of I_{coil}, using a flip-coil and the current-integrating circuit. See Appendix I for details.
V. Analysis Ideas

- Discuss possible errors in your zeroing and gain calibration procedures for the Hall probe and Gaussmeter, and estimate their sizes.

- Do you see any indications of saturation in your \( B \) vs. \( I \) curves? What about hysteresis?

- What is the extent of the homogeneous region between the pole tips, for a given tolerance in \( B_z \)? Use your data to estimate the fractional inhomogeneities \( (1/B_z)dB_z/dz \) and \( (1/B_z)dB_z/dr \). Based on your studies of saturation, would you expect your results to be strongly dependent on the value of \( I_{coil} \) that you chose?

- Make a list of all of the uncertainties present in your measurements obtained using the flip-coil. Keeping these in mind, compare these measurements to those taken with the Gaussmeter. In your discussions, make sure you distinguish between absolute errors (those common to all of your flip-coil readings), and relative errors (uncertainties that will vary from measurement to measurement).

Appendix I – Current integration techniques

Faraday’s Law tells us that an electromotive force \( \varepsilon \) is induced in a coil whenever the magnetic flux \( \Phi \) through the coil is changing: \( \varepsilon = -(d\Phi/dt) \). If a coil consisting of \( N \) turns, each of area \( A \), is rotated by 180° while in a magnetic field \( B \), the integrated EMF is

\[
\int \varepsilon \, dt = \Delta \Phi = 2\Phi = 2NAB . \tag{1}
\]

This shows that the voltage induced across the coil, integrated over a time during which its orientation is reversed, equals twice the maximum field flux through the coil. By adding a resistor \( R \) to ground, this voltage can be converted to a current, which is easily integrated as the charge on a capacitor \( \ldots, \) except that all the current now flows to ground! The trick of holding one end of the resistor at ‘ground’, yet not letting any current flow to ground, can be done rather easily using an op amp, which has sufficiently large input impedance (due to a junction field-effect transistor, or JFET, at the front end) that current loss into the op amp is negligible. The capacitor \( C \) provides a path for feedback, so the op amp will do “whatever it takes” to hold the voltage difference between the two inputs at zero. By grounding one input (the non-inverting side), while connecting \( R \) and \( C \) to the other, almost perfect current integration can be achieved. One finds (neglecting overall signs):

\[
V_{out} = V_C = q/C = \frac{1}{C} \int i(t) \, dt = \frac{1}{RC} \int \varepsilon(t) \, dt . \tag{2}
\]

Make sure you thoroughly understand all of the above. It is suggested you open up the ‘black box’ and diagram the circuit for yourself. If you need help understanding the circuit, or are confused about the behavior of an op amp, just ask!