“Mysticism might be characterized as the study of those propositions which are equivalent to their own negations. The Western point of view is that the class of all such propositions is empty. The Eastern point of view is that this class is empty if and only if it isn’t.”

- - - Raymond Smullyan (from IU!)

1. **SHO Me the Functional! (15 pts)**

Consider the 1-D SHO, described by the familiar Hamiltonian:

\[ H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2 \]

We know that the true wave function for the ground state is a Gaussian, with tails extending well into the kinematically forbidden regions. Suppose we didn’t know this, and we guessed that the wave function must “turn off” at some point, just as for the infinite square well. Thus, we might consider a function of the form

\[ \phi_c(x) = \begin{cases} 
(c^2 - x^2)^2 & \text{for } |x| \leq c \\
0 & \text{for } |x| \geq c 
\end{cases} \]

Treating \( c \) as an adjustable parameter, calculate a best-guess for the energy of the SHO ground state, and compare this to the actual value for \( E_0 \).

2. **Linear Thinking**

Consider a potential well that grows linearly with \( x \) when \( x > 0 \), but goes to infinity for all \( x < 0 \), *i.e.*, a potential of the form

\[ V(x) = \begin{cases} 
+\infty & \text{for } x < 0 \\
\kappa x & \text{for } x > 0 
\end{cases} \]

Using variational techniques, estimate the ground state energy using the trial function

\[ \phi_\alpha(x) = x e^{-\alpha x^2} \]
3. I’m Getting Vary Bohr-ed, Adam (15 pts)

(a) Assuming that the orbital angular momentum $\ell$ is zero for the ground state of the hydrogen atom, estimate the energy of this state, using a trial function of the form

$$\phi_c(r) = e^{-cr^2}$$

Compare the answer you get to the actual ground state energy. (b) Without doing any calculations, discuss what your answer would have been if you had instead used a trial function of the form

$$\phi_c(r) = e^{-cr}$$

4. Livin’ on the Ledge

Consider an infinite square well potential in which half of the potential is higher than the other half; more precisely, suppose

$$V(x) = \begin{cases} V_0 & \text{for } 0 < x < a/2 \\ 0 & \text{for } a/2 < x < a \\ +\infty & \text{elsewhere} \end{cases}$$

Check out figure 6.3 in the text if you are having trouble picturing the potential. Then:

(a) Use non-degenerate perturbation theory to estimate $E_n^1$ for each state of the system.

(b) Use the WKB approximation to estimate $E_n$. Express your answer in terms of $V_0$ and $E_n^0$, the energy levels for a system with $V_0 = 0$. (c) Compare your answers to parts a and b, and discuss why they do (or don’t) agree. Put another way: in what regimes of $V_0$ and $n$ would you expect these approximations to yield the same answer? Explain briefly why this seems reasonable.