As we discussed we are not going to use underlying theory (QED) to derive N-N interactions. Instead we will:

1. Assume that nucleons (protons, neutrons) are the proper degrees of freedom at low energies \( E \ll 1 \text{ MeV} \) (even largely ignore mesons and their interactions can be approximated by instantaneous potentials).

2. Use Schrödinger equation since we have potentials.

3. Consider two-body forces only.

(what are 3 body forces?) Moon - Earth - Satellite

\[ \text{tidal effects through mass distribution} \]
\[ \text{through Earth} \]
\[ \text{through Satellite} \]
\[ \text{force} \]
To describe a system composed of protons and neutrons we will use two different basis:

1. \[ |p\rangle = (\frac{1}{2} | \vec{p}, m_p, e=+1 >) \quad \text{or} \quad | \vec{p}, m_p, e=-1 > \]
\[ |n\rangle = (\frac{1}{2} | \vec{n}, m_n, e=+1 >) \quad \text{or} \quad | \vec{n}, m_n, e=-1 > \]

or

2. Treat \( n \) and \( p \) as two different states of a single particle \( N \) (nucleon)
\[ |N\rangle = (\frac{1}{2} | \vec{p}, m_p, p >) \quad \text{or} \quad | \vec{n}, m_n, n > \]

Such a basis will be useful if there is an underlying symmetry that does not distinguish between \( n \) and \( p \).

In fact, strong interactions are very similar for these two states, and the symmetry is referred to as isospin invariance.

Why is this of relevance? Because there are (strong) interactions that change \( n \rightarrow p \). If this was not the case we would just use two independent \( n \) and \( p \) states.
Second basis:
\[
\begin{align*}
N &= |N\rangle = |p, m, m_s, m_i\rangle, \\
|N\rangle &= |p, m, m_i\rangle, \\
L_i &= \frac{1}{\sqrt{2}} (1^2 - 3^2) \\
M_\pi &= -\frac{1}{\sqrt{2}} \Rightarrow n
\end{align*}
\]

It turns out that isospin symmetry \( (p \leftrightarrow n) \)
can be promoted to a continuous symmetry \( SU(2) \)
acting on the fundamental doublet \((1, 0)\) just like
rotations do on a spin doublet \((1, 0)\).

\[
\begin{align*}
\sigma^2 &= \frac{1}{2} \sigma^2 \\
\hat{\tau} &= \frac{1}{2} \tau
\end{align*}
\]

These are operators acting \( \hat{\tau}(m_t) = \delta_{m_t m_i} \)
acting on the isospin wave function \( |N\rangle \).

Spin wave function:
\[
\begin{align*}
\hat{\tau}(m_t) &= \langle m_t | N \rangle \\
\hat{\tau}(m_t) &= \langle m_t | N \rangle
\end{align*}
\]

Free Nucleon wave function:
\[
\langle x, p, m_t | N \rangle = \Psi_N(x, p, m_t)
\]

Consider neutron with momentum \( \vec{p} \), spin \( \vec{s} \) :
\( N = (\vec{p}, \vec{s}, m_t) \)

\[
\begin{align*}
\Psi_N(x, p, m_t) &= \frac{1}{V_n} e^{i \frac{p \cdot x}{\hbar}} \frac{1}{2} \left( \begin{array}{c}
\vec{p} \\
\vec{s}
\end{array} \right) \\
&= \left( \begin{array}{c}
\frac{1}{\sqrt{2}} \left( \begin{array}{c}
1 \\
0
\end{array} \right) \\
\frac{1}{\sqrt{2}} \left( \begin{array}{c}
0 \\
1
\end{array} \right)
\end{array} \right)
\end{align*}
\]

\[
\begin{align*}
\hat{\sigma} &= \frac{1}{\sqrt{2}} (1 \otimes 1 \otimes \sigma^1) \\
\hat{\tau} &= \frac{1}{\sqrt{2}} (0 \otimes 1 \otimes \tau)
\end{align*}
\]

Both \( \hat{\sigma} \) and \( \hat{\tau} \) are given by Pauli matrices.
They are denoted by different letters as they act on different spaces.
Example: Change operator \( P \)

\[
Q |N\rangle = \begin{cases} +1 |N\rangle & \text{if } N = P \\ 0 & \text{if } N = \bar{N} \end{cases}
\]

\[
\Rightarrow Q |N\rangle = \sum N' |N\rangle P_{NN'} \rightarrow Q_{NN} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2} (1 + \tau_3)
\]

The operators can also act on wave functions ("position representation")

Again \( J = \ell + \sum \)

\[
\tilde{S} |m_3\rangle = \sum_{m_3} |m_3\rangle \tilde{S}^z |m_3\rangle = \sum_{m_3} |m_3\rangle \frac{1}{2} \tilde{S}^z |m_3\rangle
\]

\[
\sum_{m_3} |m_3\rangle \tilde{S}^z |m_3\rangle = \frac{1}{2} \sum_{m_3} |m_3\rangle \tilde{S}^z |m_3\rangle = \frac{1}{2} \sum_{m_3} |m_3\rangle \tilde{S}^z |m_3\rangle
\]

\[
\sum_{m_3} |m_3\rangle \tilde{S}^z |m_3\rangle = \frac{1}{2} \sum_{m_3} |m_3\rangle \tilde{S}^z |m_3\rangle
\]

\[
M_{m_3} = \delta_{m_3} \rightarrow \frac{1}{2} \tilde{S}^z
\]

as acting on the wave function.
\[ \mathcal{L} = \bar{\psi} \gamma \cdot p \psi \]

\[ \mathcal{L} = \bar{\psi} \gamma \cdot (i \partial / \partial x) \psi \]

\[ \mathcal{L} = \bar{\psi} \gamma \cdot (i \partial / \partial x) \psi \text{ as acting on the wave function.} \]

\[ \gamma_5, \gamma_3 \text{ - vector operators (ITOS vol. 1)} \]

\[ \gamma_7 \text{ - isovector operator (ITU vol. 2) in isospin space.} \]

Coupling schemes:

Since isospin under isospin transformations behaves just like spin under spin rotations, the (leibin-Gordon) coefficients for coupling at isospin states follow from those at spin.

Example of single particle wave function (different than plane wave):

\[ \Psi_{n \lambda \lambda'} (r, \mathbf{m}; \mathbf{m}') = \int \frac{d^4 \mathbf{p}}{(2\pi)^3} \frac{\psi_{n \lambda \lambda'} \mathbf{p}}{\sqrt{2E_\mathbf{p}}} \langle \mathbf{p} \mathbf{m} | \mathbf{p} \mathbf{m}' \rangle \]

\[ \Psi_{n \lambda \lambda'} (r, \mathbf{m}; \mathbf{m}') = \int \frac{d^4 \mathbf{p}}{(2\pi)^3} \frac{\psi_{n \lambda \lambda'} \mathbf{p}}{\sqrt{2E_\mathbf{p}}} \langle \mathbf{p} \mathbf{m} | \mathbf{p} \mathbf{m}' \rangle \]

\[ \chi = \gamma_z (\mathbf{m}_3) \]

reduced radial wave function.
Two particle wave function:

We have: \[ \vec{L} = \text{relative } \vec{r} \text{ momentum} \]
\[ \vec{S'} = \vec{s}_1 + \vec{s}_2 \]
\[ \vec{T} = \vec{T}_1 + \vec{T}_2 \leq \text{total isospin operator} \]

We will work in the basis of total and relative momentum (position) and drop the free, c.m. motion.

Plane wave: \[ \Psi_\pm (\vec{r}) = \frac{1}{\sqrt{2}} e^{\pm i \vec{p} \cdot \vec{r}} \quad \text{or} \quad (e^{i \vec{p} \cdot \vec{r}} \text{ in other normalization case}) \]

\[ \Psi_N (\vec{r}_{12}, 1/2) = \langle \vec{r}_{12}, 1/2 | NN \rangle \]

Intrinsic coordinates

(Operator, isospin is for both nucleons)

\[ \Psi_{(LS)JT} (\vec{r}_{12}, 1/2) = \frac{1}{\sqrt{2}} M \begin{pmatrix} (LS) \otimes \frac{1}{2} \end{pmatrix} \begin{pmatrix} y_{12} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \tilde{\gamma}_L (\bar{\tilde{\gamma}}_L) \otimes (y_{1} \otimes y_{2/2}) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \bar{\tilde{\gamma}}_L (y_{1} \otimes y_{2/2}) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \end{pmatrix} \] 

\[ S = 0, \text{ or } 1 \]

So for no introduction of dynamics, we have just specified a basis. Whether this basis is useful or not depends which \( y \) numbers are "good" i.e. conserved by the interactions.
So let's assume something about doublets and see what we learn:

1. Nucleons are identical fermions \(\Rightarrow\) Hamiltonian symmetric under exchange of particle (0601, 4 = w. function antisymmetric [fermions])

\[
\Psi_{MNP}^{(LS)JT}(\mathbf{r}) = \frac{1}{2} \left[ \Psi_{MNP}^{(LS)JT}(\mathbf{r}_{12}; 1, 2) - \Psi_{MNP}^{(LS)JT}(\overline{\mathbf{r}}_{12}; 1, 2) \right]
\]

Since \(\mathbf{r}_{12} = -\overline{\mathbf{r}}_{12}\), \(Y_{LM}(\mathbf{r}) = (-)^L Y_{LM}(\overline{\mathbf{r}})\)

and from properties of \((-\mathbf{r})\) we get:

\[
\Psi_{MNP}^{(LS)JT}(\mathbf{r}) = \frac{1}{2} \left( 1 - (-)^{L+S+T} \right) \Psi_{MNP}^{(LS)JT}(\mathbf{r}_{12}; 1, 2)
\]

\((*)\)

\(\Rightarrow\) \(L+S+T = \text{odd}\) or wave function vanishes.

2. Hamiltonian parity invariant (true for strong interactions)

\(\Rightarrow\) \(H\) invariant under \(\mathbf{r}_{12} \rightarrow -\mathbf{r}_{12}\) \([H; P] = 0\)

\(\Rightarrow\) wave function eigenvector of parity

\(\Rightarrow\) \(L\) even do not mix with odd
Hamiltonian rotationally invariant \([\mathcal{H}, J^i] = 0\)

\(\Rightarrow J, M\) good quantum numbers

Hamiltonian is charge independent or isospin invariant

\(\Rightarrow [\mathcal{H}, T] = 0\)

\(T, M_T\) good quantum numbers

(In the two nucleon system, the Coulomb interaction which is *not* isospin invariant, does not change \(L\) and \(J\).

Conclusion: Since it acts only in pp state with fixed definite \(T, M_T (1, 1)\))

From (2) \(\Rightarrow S\) is a good quantum number \((S=0, 1)\)

Summary: \(S, J, M, T, M_T\) are "good" (conserved, well defined) quantum numbers

\(L + S + T = \text{odd}\)

\(\Rightarrow S = 0 \Rightarrow L = J \Rightarrow \text{uncoupled typical wave}\)

\(S = 1\) and \(L = J\) \(\Rightarrow \text{uncoupled}\) (since cannot mix with \(J = L \pm 1\) by parity)

\(S = 1\) and \(L = J \pm 1\) \(\Rightarrow \text{coupled} \Rightarrow L\) not a good \(q.\) number.
Catalog of the allowed two-nucleon states using spectroscopic notation:

\[ L = 0, 1, 2, 3, 4, 5, 6 \]

\[ S = 0 \] even parity \( 'S_0', 'D_2', 'G_4 \) \( T = 1 \)

\( S = 0 \) odd parity \( 'P_1', 'F_3', 'H_5 \) \( T = 0 \)

\( S = 1 \) even parity \( (^3S_1, ^3D_1), ^3D_2, (^3D_3, ^3G_3), ^3G_4 \) \( T = 0 \)

\( S = 1 \) odd parity \( ^3P_0, ^3P_1, (^3P_2, ^3F_2), ^3F_3 \) \( T = 1 \)

Deuteron observed in \( ^3S_1, ^3D_1 \) in \( T = 0 \)