Properties of the N-N potential

Since we agree not to try to derive N-N force from QCD (at least not yet) we will have to determine its properties by fitting a given model to N-N data.

→ N-N scattering, (including the only NN bound state — deuteron)

Symmetry properties:  

Remember, these are Q.M. operations \[ \left[ \mathcal{P}, \mathcal{J} \right] \neq 0 \]

The variables are \( \vec{p}_1, \vec{p}_1, \vec{p}_2, \vec{z}_1, \vec{p}_2, \vec{z}_2, \vec{z}_2 \)

1) Hermitian \( \Rightarrow \) elastic scattering only!

2) Invariant under particle exchange \( \Rightarrow \) \( V(1, 2) = V(2, 1) \)

3) Invariant under translations \( \Rightarrow \) \( V(\vec{p}_1, \vec{p}_2) = V(\vec{p}_1 - \vec{p}_2) \)

4) Invariant under Galilean boosts \( \Rightarrow \) \( V(\vec{p}_1, \vec{p}_2) = V(\vec{p}_1 - \vec{p}_2) \)

5) Invariant under parity

6) Invariant under time reversal \( \Rightarrow \) \( \vec{p} \rightarrow -\vec{p}, \vec{p} \rightarrow -\vec{p}, \vec{z} \rightarrow -\vec{z} \)

7) Invariant under rotations (generated by \( \vec{J} = \vec{P} + \vec{S} \)) \( \Rightarrow \left[ \mathcal{V}, \mathcal{J} \right] = 0 \)
From 8 it follows that the most general isospin dependence is given by:

\[ V = V + V_e \frac{\tilde{r} \cdot \tilde{r}}{2} \]  

(symmetric under \( \tilde{r}, \tilde{r} \))

From 0 \( \Rightarrow 7 \) we get:

\[ V(1,2) = V(\tilde{r}, \tilde{r}, \tilde{r}_1, \tilde{r}_2, \tilde{r}_1, \tilde{r}_2) \]

\[ = V(-\tilde{r}, -\tilde{r}, \tilde{r}_2, \tilde{r}_1, \tilde{r}_2, \tilde{r}_1) \]

\[ = V(-\tilde{r}, -\tilde{r}, \tilde{r}_1, \tilde{r}_2, \tilde{r}_1, \tilde{r}_2) \]

\[ = V(\tilde{r}_1, -\tilde{r}_2, -\tilde{r}_1, -\tilde{r}_2, \tilde{r}_1, \tilde{r}_2) \]

The most general form consistent with these assumptions is:

\[ V(1,2) = U_d + U_\rho \frac{\tilde{r} \cdot \tilde{r}}{2} + U_\sigma \frac{\tilde{r}_1 \cdot \tilde{r}_2}{2} + U_\delta (\tilde{r} \cdot \tilde{r}_1) (\tilde{r} \cdot \tilde{r}_2) \]

\[ + U_e \left[ (\tilde{r} \cdot \tilde{r}_1) (\tilde{r} \cdot \tilde{r}_2) + (\tilde{r} \cdot \tilde{r}_2) (\tilde{r} \cdot \tilde{r}_1) \right] + U_\eta (\tilde{r} \cdot \tilde{r}_1) (\tilde{r} \cdot \tilde{r}_2) \]

where \( \tilde{u} = \tilde{r} \times \tilde{r} \) and \( \tilde{s} = \tilde{r}_1 + \tilde{r}_2 = \frac{1}{2} (\tilde{r} \cdot \tilde{r}_1) \)

All \( U's \) are real, rotational scalars that can depend on \( \tilde{r}, \tilde{r}_1, \tilde{r}_2 \) and \( \tilde{r} \), all \( U's \) have isospin dependence as in (x).
\[ L^2 = E_{ji} y_{ji} P_u + E_{ji} y_{ji} P_u = E_{ji} y_{ji} P_u - y_{ji} P_u \]
\[ = \mathbf{p}^2 \mathbf{p} + R_s [p_u, y_{ij}] P_u - (p_0^2) (\mathbf{p}^2) - R_s [p_u, y_{ij}] P_u \]
\[ = \mathbf{p}^2 \mathbf{p} - 2 \mathbf{p} \mathbf{p}^2 - (p_0^2) (\mathbf{p}^2) \Rightarrow \text{thus } V \text{ is also a function of } \mathbf{p}, \mathbf{p}^2, \mathbf{p}^3 \text{ expressed by T invariant} \]

Thus there are \( 6 \times 2 \) independent functions in \( V \) for isospin.

However, if we use elastic \( N-N \) scattering to determine \( V \), all momentum dependence of \( U \)'s is redundant. *

This follows because \( p^2 \) dependence of \( V \) in elastic scattering is fixed by \( k_i = k_f = 2p \).

Thus one can only write a potential in a form:

\[ V(\ell, 2) = V_d + V_p \mathbf{p} \cdot \mathbf{s} + V_s \mathbf{s} \cdot \mathbf{s} + V_s (\mathbf{p} \cdot \mathbf{s}, \mathbf{p} \cdot \mathbf{s}) \]
\[ + V_s \left[ (\mathbf{p} \cdot \mathbf{s}) (\mathbf{p} \cdot \mathbf{s}) + (\mathbf{p} \cdot \mathbf{s}) \mathbf{p} \cdot \mathbf{s} \right] \]
\[ V_i = V_i (\ell, \mathbf{p}) \]

Of course in principle, for realistic nuclear calculations beyond NN system, one would need the full potential in terms of \( U \)'s.

The 5 independent \( U \)'s can in principle be determined from 5 independent functions in \( f(\ell, \mathbf{p}) \). However, since \( U \)'s depend on \( \mathbf{p}^2 \), one would have to know all channels \( 2s^1\ell_2 \) (\( \ell > \infty \)) which cannot be done,
The most general form that can be determined
-uniquely (in principle) from $N-N$ data contains
at most linear factors of $L^2$:

$$V(1,2) = V_\alpha + V_\beta L^2 \cdot 3 + V_\gamma \sqrt{\sigma_1 \cdot \sigma_2} + V_\delta \left( \sqrt{\sigma_1} / \sqrt{\sigma_2} \right)$$

$$V_i = V_i (1r1)$$

(if there was a dependence on $L^2$ then again there would be
independent $V_i$'s for each channel.)

So let's be optimistic and assume that (t)
is (t) is sufficient to describe ($N-N$) interaction.

What can we say about $V_\alpha$'s?

1. Attraction:
   - Nuclei are bound $\Rightarrow$ force must basically be attractive
   - Deuteron exists ($T=1$, even parity) has one
     bound state mostly $3S_1$, so force is attractive
     in the triples $S=1$, $T=0$ state.
   - pp scattering has $T=1$, At low energies $L=0$
     $\Rightarrow 1S_0$ only. From Coulomb-nuclear interference
     it follows that force is attractive.