\[ d = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \Psi (\hat{D}_\mu \gamma^\mu - e A^\mu \gamma^\mu - m) \Psi \]

\[ L = \chi (A_\mu, A^\mu; \Psi, \bar{\Psi}) \]

\[ F_{\mu \nu} = \varepsilon^{\mu \nu \rho \sigma} \partial_\rho A_\sigma - \partial_\sigma A_\rho = \partial_\rho F_{\mu \sigma} - \partial_\sigma F_{\mu \rho} = - \partial_{\rho} A_{\sigma}^\mu - \partial_{\sigma} A^\mu_{\rho} \]

\[ B_\mu = -F_{\mu \nu} = -\left( \partial_\mu A_\nu - \partial_\nu A_\mu \right) + \left( \partial_\nu A^\kappa - \partial_\kappa A^\nu \right) = \varepsilon_{ij} \partial_i A^j_k \]

\[ \frac{\delta S}{\delta A^\mu_0} = \frac{\partial}{\partial A^\mu} \left( \frac{1}{2} F_{\mu \nu} F^{\mu \nu} - i \varepsilon^{\mu \nu \rho \sigma} \partial_\rho F_{\nu \sigma} - \bar{\Psi} i \gamma \cdot A \Psi \right) \]

\[ = \frac{1}{2} \delta F_{\mu \nu} F^{\mu \nu} - i \varepsilon^{\mu \nu \rho \sigma} \partial_\rho F_{\nu \sigma} - \bar{\Psi} \gamma \cdot A \Psi \]

\[ = + \bar{\Psi} \gamma^0 D^0 - e \Psi^+ \Psi = 0 \]

\[ \bar{\Psi} \gamma^\mu D^\mu = + e \Psi^+ \Psi \quad (\text{Gauss' Law}) \]

\[ \bar{\Psi} = -\frac{\partial A^\mu}{\partial x^\mu} \]

\[ D^\mu = \bar{\Psi} D^\mu = \bar{\Psi}_{\mu} + \bar{\Psi}_{L} \quad (\bar{\Psi} D^\mu = 0) \]

\[ \Rightarrow \bar{\Psi} \left( -\frac{\partial A^\mu}{\partial x^\mu} - \bar{\Psi} A^0 \right) = + e \Psi^+ \Psi \]

\[ \Rightarrow \frac{\partial}{\partial x^\mu} (\bar{\Psi} A^\mu) + \bar{\Psi} A^0 = - e \Psi^+ \Psi \quad (\Psi) \]

\[ \bar{\Psi} A_\mu Y_\mu \Psi = \bar{\Psi} A_0 \Psi - \Psi^+ \gamma^\mu \bar{\Psi} A^\mu \Psi \]

\[ \bar{\Psi} = \bar{\Psi}_{\mu} + \bar{\Psi}_{L} \]

\[ = \Psi^+ \Psi - \Psi^+ \gamma^\mu \bar{\Psi} A^\mu \Psi \]

\[ \bar{\Psi} A_\mu Y_\mu \Psi = \Psi^+ A_0 \Psi - \Psi^+ \gamma^\mu \bar{\Psi} A^\mu \Psi \]

\[ \bar{\Psi} A_\mu Y_\mu \Psi = \Psi^+ A_0 \Psi - \Psi^+ \gamma^\mu \bar{\Psi} A^\mu \Psi \]

\[ = \Psi^+ A_0 \Psi - \Psi^+ \gamma^\mu \bar{\Psi} A^\mu \Psi \]
\[ A_L^\mu = \frac{\partial A^\mu}{\partial x^\nu} \]

\[ \Sigma A^\mu \mu^4 = \psi^+ \mathcal{D} \psi - \psi^+ \mathcal{A} \mathcal{A} \psi - \psi^+ \mathcal{D} \cdots \mathcal{D} A^\mu \psi \]

\[ e \int d^4x e^{iA^\mu \mu^4} = e \int d^4x e^{iA^\mu \mu^4} \{- e \int d^4x e^{iA^\mu \mu^4} \}
+ \int d^4x \left[ (\psi^+ \psi) \right] \begin{pmatrix} \frac{1}{2} \partial^2 \mathcal{D} \\ \mathcal{D} \mathcal{D} \end{pmatrix} \]

Using current conservation: \[ \partial_\mu j^\mu = 0 \Rightarrow \psi^+ \psi = \frac{1}{it} \psi^+ \psi = -\mathcal{D} \psi \]

\[ \Rightarrow \int d^4x \left[ \begin{pmatrix} \frac{1}{2} \partial^2 \mathcal{D} \\ \mathcal{D} \mathcal{D} \end{pmatrix} \right] \]

\[ = -\int d^4x \left[ \begin{pmatrix} \frac{1}{2} \partial^2 \mathcal{D} \\ \mathcal{D} \mathcal{D} \end{pmatrix} \right] = + \int d^4x (\psi^+ \psi) \frac{1}{2} \frac{1}{\sqrt{4}} \partial^2 \mathcal{D} \mathcal{D} \]

Using (x): \[ \begin{pmatrix} \frac{1}{2} \partial^2 \mathcal{D} \\ \mathcal{D} \mathcal{D} \end{pmatrix} =
- e \int d^4x \psi^+ \psi \frac{1}{2} \partial^2 \mathcal{D} = \int d^4x \psi^+ \psi \frac{1}{2} \partial^2 \mathcal{D} \]

\[ S_{\psi^+} = -e \int d^4x \Sigma A^\mu \mu^4 \psi = -e \int d^4x \psi^+ \mathcal{D} \psi + e \int d^4x \psi^+ \mathcal{A} \mathcal{A} \psi
+ e \int d^4x \psi^+ \mathcal{D} \psi + e \int d^4x \psi^+ \mathcal{A} \mathcal{A} \psi
= e \int d^4x \psi^+ \mathcal{A} \mathcal{A} \psi - e^2 \int d^4x \psi^+ \psi \frac{1}{\sqrt{4}} \frac{1}{\sqrt{4}} \psi^+ \psi \]
\[-\frac{1}{4} F_{\mu \nu} F^{\mu \nu} = -\frac{1}{l} (F_{0i} F^{0i} + F_{ij} F^{ij}) \]
\[= -\frac{1}{l} (-F_{0i} F^{0i} + F_{ij} F^{ij}) = +\frac{1}{2} \bar{E}^2 - \frac{1}{2} \bar{B}^2 \]

with \( \bar{E} = -\bar{E}^\mu x^\mu \).

\[\Rightarrow \int d^4x \frac{1}{l} \bar{E}^2 = \frac{1}{l} \int d^4x \left( \bar{E}_T^2 + \bar{E}_L^2 \right) \]
\[= \frac{1}{l} \int d^4x \bar{E}_T^2 + \frac{1}{l} \int d^4x \bar{E}_L^2 \]
\[= \frac{1}{l} \int d^4x \bar{E}_T^2 + \frac{1}{l} \int d^4x \frac{\bar{D}(\bar{E})}{D^2} \cdot \bar{D}(\bar{E}) \]
\[= \frac{1}{l} \int d^4x \bar{E}_T^2 + \frac{1}{l} \int d^4x \left( \bar{D}(\bar{E}) \frac{\partial}{\partial \bar{E}} (\bar{D}) \right) \]

\[\text{Tr} \quad S = \int d^4x \left( \int d^4x \frac{1}{2} \bar{E}_T^2 - \bar{B}_T^2 \right) - \frac{\gamma}{2} \int d^4x \bar{\Psi}^\dagger \gamma_5 \Psi(x) \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x} \frac{1}{\sqrt{-g}} \bar{\Psi} (x) \Psi(x) \]
\[+ \int d^4x \bar{\Psi}^\dagger \gamma_5 \Psi(x) \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x} \frac{1}{\sqrt{-g}} \bar{\Psi} (x) \Psi(x) \]

\[\Rightarrow \mathcal{L} = \mathcal{L} \left( A_{\mu} ; \bar{E}_T ; \Psi, \bar{\Psi}^\dagger \right) \]

\[H = \int d^4x \left( \frac{1}{2} \bar{E}_T^2 + \bar{B}_T^2 \right) + \frac{\gamma}{2} \int d^4x \bar{\Psi}^\dagger \gamma_5 \Psi(x) \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x} \frac{1}{\sqrt{-g}} \bar{\Psi} (x) \Psi(x) \]
\[- \int d^4x \bar{\Psi}^\dagger \gamma_5 \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x} \frac{1}{\sqrt{-g}} \bar{\Psi} (x) + \int d^4x \bar{\Psi}^\dagger \gamma_5 \Psi(x) \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x} \frac{1}{\sqrt{-g}} \bar{\Psi} (x) \Psi(x) \]
Second order problem \( \psi_1 \).

\[
H = H_0 + H_{\text{int}} + V_c
\]

\( H = H_e + H_0 \)

\[H_{\text{int}} = O(e)\]

\[V_c = O(e^2)\]

\[
E_{\text{ee}} = E_0 + E_{1e} + E_{2e} + \ldots
\]

\[
H = H_0 + V_1 + V_2
\]

\( H_{\text{int}}' = V_c \)

Basis: \( H_0 |\psi\rangle = E_0 |\psi\rangle, \quad |\psi\rangle = \{ |e\rangle; |1\rangle; |2\rangle; \ldots \} \)

\[
\langle \text{ee}|\text{Holee}\rangle = \langle \text{ee}|\text{Se} |\text{So}\times 4^4(x)\times E \otimes \beta \text{ms} (xy)|\text{ee}\rangle
\]

\[
\approx \sqrt{\omega + \frac{p^2}{2m}} \langle \text{ee}|\text{ee}\rangle \approx \sqrt{\omega + \frac{p^2}{2m}} \langle \text{ee}|\text{ee}\rangle
\]

\[\text{stochastic fields}\]
\[ E_{1}^{\text{ee}} = \langle \text{ee} | V_{1}^{0} | \text{ee} \rangle = 0 \quad \text{since } V_{1} \text{ routes out } A \]

and there is no } A \text{ in the states.

\[ E_{2}^{\text{ee}} = \langle \text{ee} | V_{1}^{1} | \text{ee} \rangle = \sum_{n} \sqrt{\frac{E_{0}^{\text{ee}} - E_{n}^{\text{ee}}}{m}} \langle \text{ee} | V_{1}^{1} | \text{ee} \rangle \]

Second term = 0

Since \[ \langle \text{ee} | V_{1}^{1} | \text{ee} \rangle \rightarrow \langle \text{ee} | V_{1}^{1} | \text{ee} \rangle \]

\[ \rightarrow \frac{e}{m} \quad \text{interaction} \rightarrow \frac{e^{2}}{m} \rightarrow 0 \]

in static limit.

\[ E_{2}^{\text{ee}} = \langle \text{ee} | V_{1}^{0} | \text{ee} \rangle = \sqrt{2} \iint d^{3}x d^{3}y \langle \text{ee} | \delta_{z} \delta(x - y) | \text{ee} \rangle \]

\[ \approx \frac{e^{2}}{4\pi |x - y|} \langle e(x') e(y') | e(x) e(y) \rangle \]

\[ V(\gamma - |x - y|) = \frac{e^{2}}{4\pi |x - y|} \]

\[ \approx \delta(x - x') \delta(y - y') \]

\[ \langle e(x) e(y') | H | e(x) e(y) \rangle \]

\[ = \frac{e^{2}}{4\pi |x - y|} \]

\[ \langle e(x) e(y) | e(x') e(y') \rangle \]
\[ f(x) g(y) \Rightarrow (b^+ b - dd)(b^+ b - d^+ d) \Rightarrow b^+ b b^+ b + d^+ d d^+ d \]

but \( \Theta \) \( b^+ b d^+ d \)

diagonally:

\[
\begin{align*}
\begin{array}{c}
\hline \\
 x \\
\hline \\
 y
\end{array}
\end{align*}
\]

\[ \Rightarrow \frac{1}{4\text{min}(x,y)} \]
(c) Convection (higher order) assume here a light

\[ e^{-\frac{1}{4\pi|x-y|}} \] heavy (static)

\[ \Rightarrow \langle ee | V_c | n \rangle \langle n | V_c | ee \rangle \]

\[ \text{heavy} \quad E_{ee} - E_{ee} \]

\[ V_0 = \frac{e^2}{4\pi|x-y|} \zeta \left( 1 - \frac{e^2}{u^2} \right) \]

\[ V = \frac{\zeta}{|x-y|} \left[ 1 + O(\alpha) + \ldots \right] \]

\[ \approx \frac{\zeta}{|x-y|} \left[ 1 - \frac{\alpha^3}{4\pi} \ln |x-y| + \ldots \right] \]

\[ \approx \frac{\zeta}{|x-y|} \left[ 1 + \frac{\alpha^3}{4\pi} \ln |x-y| \right] \equiv \alpha(r) \]
Country constant is small if we treat at least one class.

\[ x = \frac{1}{13t} = x(\frac{r}{\nu^2 \omega}) \]