Applications.

Here we will use the symmetries of strong interactions, discussed above to determine quantum numbers of hadrons. We will ignore electromagnetic and weak interactions.

1) Pion.

Since the virtual transition $\pi^0 \rightarrow N\bar{N}$, $N=p$ or $n$ can occur via strong interaction, the pion isospin must be the same as that of $N\bar{N}$, i.e., 0 or 1. The two lambda's and $\pi^0$ have almost the same masses, resulting a total of $2I+1=3$ degeneracy implies that $I=1$. (The small mass difference comes from electromagnetic interactions and weak $g$-and mass differences.)

Thus, since $\pi^+ \rightarrow p\gamma$ and $(\frac{0}{n})=\left|\begin{array}{l} 1 \end{array}\right|$ while $(\frac{1}{p})=\left|\begin{array}{l} 1 \end{array}\right|$

Thus $\pi^+ \rightarrow (p\gamma) \rightarrow I_2 = +1 \Rightarrow I_2(\pi^+) = +1$

Likewise $I_+(\pi^0)=0$ and $I_-(\pi^0)=-1$
Since $I_N = 1$ and $I_N = \frac{1}{2}$ in (1) the LHS can be

$$I_{N} = \frac{1}{2} \text{ or } I_{N} = \frac{3}{2}, \text{ since } I_{N} = 0 \Rightarrow I_{K} = \frac{1}{2} \text{ or } \frac{3}{2}$$

let $Q_{\pi}, Q_{\pi}, Q_{\pi}$ and $Q_{N}$ be the corresponding electric charges.

Since $Q_{\pi} = I_{3}$ and $Q_{N} = I_{3} + \frac{1}{2}$ (remember $Q = I + \frac{S+B}{2}$)

and since $Q_{N} = I_{N} = 0$ we get

$$\langle \pi \mid p \rangle = (I_{3} + \frac{1}{2}) \langle \pi \mid p \rangle = (I_{3} + I_{3} + \frac{1}{2}) \langle \pi \mid p \rangle$$

$$= (Q_{\pi} + Q_{N}) \langle \pi \mid p \rangle$$

$$\Rightarrow Q_{\pi} \langle \pi \mid O \rangle = (I_{3} + \frac{1}{2}) \langle \pi \mid O \rangle = (I_{3} + I_{3} + \frac{1}{2}) \langle \pi \mid O \rangle$$

$$= (I_{3} + \frac{1}{2}) \langle \pi \mid O \rangle = Q_{\pi} \langle \pi \mid O \rangle$$

Thus $Q_{K} = Q_{\pi} + Q_{N} = I_{3} + \frac{1}{2}$. Consequently if $I_{K} = \frac{3}{2}$

Then we have that $B_{\pi} = 0 \Rightarrow I_{3} = \frac{3}{2}$ must have charge $= \frac{3}{2} + \frac{1}{2} = 2$

Since there are no doubt - charged kaons $\Rightarrow I_{K} = \frac{1}{2}$

Although there are 4 nearly degenerate kaon states: $K^+, K^0, \bar{K}^0, K^-$

Those 4 states have to form isospin doublets

$$(K^+, K^0) \text{ and } (\bar{K}^0, -K^-) \text{ just like } (p, n) \text{ and } (\bar{n}, p)$$
More applications: \( A \) and \( \bar{A} \).

There is only one pahile degeneracy (same mass) with \( A = A^0 \) pahile to its outpahile \( \bar{A}^0 \). Consequently the isospin degeneracy \( 2I_n + 1 \) must be \( \leq 2 \Rightarrow I_n = 0 \) or \( \frac{1}{2} \).

If \( I_n = \frac{1}{2} \) i.e. \((A, \bar{A})\) would be forming isospin multiplet (doublet) then under an isospin rotation, say \( e^{i\theta I_z} \), the state \( |A\rangle \) would be transformed to \( |\bar{A}\rangle \) (wrong numerical sign).

Because of its decay \( A^0 \rightarrow n^+ + \pi^- \), \( A^0 \) has a baryon number \( B = 1 \) (this is a weak decay, but even weak interactions cause the baryon \#)

Likewise \( \bar{A}^0 \rightarrow n^- + \pi^+ \) has a baryon number \( B = -1 \).

Now consider \( \pi^- + p \rightarrow \Lambda^0 + \bar{\pi}^0 \) \( \) \((*)\)

When we apply \( e^{i\theta I_z} \) the left-hand side remains a mixture of \( \pi^0 \) and nucleons: \( e^{i\theta I_z} |\pi^- \rangle \rightarrow |\pi^+ \rangle \) \( e^{i\theta I_z} |p \rangle \rightarrow |n \rangle \)

But this has \( B_{left} = +1 \) but on the right-hand side we would get \( \Lambda^0 + n^+ \) i.e. baryon \# = -1 \( \Rightarrow \) hadronic would imply

Baryon number violation \( \Rightarrow \) \( I_n = 0 \)
Meson and Baryon Octets.

Just like we did for the case of a one can show that no baryon can belong to the same multiplet as its antiparticle. From \( \pi^- + p = \Sigma^- + K^+ \) we conclude that \( B_\Sigma = 1 \) and from the known triplet degeneracy \( \Sigma^+, \Sigma^0, \Sigma^- \) we deduce \( I_\Sigma = 1 \). Similarly from \( K^- + p = \Lambda^0 + \pi^0 \) it follows that the baryon number of \( \Lambda^0 \) is \( B_\Lambda = 0 \). Since \( \Lambda^0 \) has no degeneracy we conclude \( I_\Lambda = 0 \). Let's postulate another quantum number \( S \) so that:

\[
Q = I_3 + \frac{S + B}{2}
\]

Our intuition that conserved \( Q, I_3 \) and \( B \) also conserved \( S \) suggests the table illustrates some of the low mass baryons, all with spin-\( \frac{1}{2} \) and even parity and mesons - all pseudoscalars (here \( I_3 = 0 \)).

From the table we see that particles with different \( I_3 \) but within the same isospin multiplet have all the same \( S \) and \( B \) consequently we deduce that these quantum numbers commute with \( I \) i.e.

\[
[Q, I] = [B, I] = 0
\]
On the other hand, since \( Q = I_2 + \frac{Y}{2} \) \( \Rightarrow B + S = \text{hypercharge} \), we have:

\[
[ Q, I_x ] = [ I_2, I_x ] = iI_y
\]

\[
[ Q, I_y ] = [ I_2, I_y ] = -iI_x
\]

\[
[ Q, I_z ] = 0
\]

Strong and electroweak interactions conserve \( S \) but the weak interaction does not. Consequently, a particle with \( S \neq 0 \) can decay to a new state through weak interactions. This means the lifetime of strong particles must be longer.

Then for weak interactions:

\[
\tau \approx \frac{1}{n} \approx \frac{1}{150 \text{MeV} \cdot \text{fm}^2} \cdot \frac{1 \text{fm}}{150 \text{MeV} / c} \cdot \frac{1 \text{MeV}}{c^2} \cdot \frac{1 \text{MeV}}{c^2} \approx 10^{-24} \text{s}
\]

Isospin and weak electroweak and weak interactions.

**Electroweak interactions**

Since protons have different electric charges and do belong to the same isospin multiplet, electroweak interactions cannot commute with \( I_2 \) or \( I_1 \):

\[
\text{New } | lp \rangle \sim | p \rangle
\]

\[
I_2 \text{ New } | lp \rangle \sim I_2 | lp \rangle = (I_2^p + I_2^\pi) | lp \rangle \quad \Rightarrow \quad \Sigma^\pi = 0 \Rightarrow \text{ only one photon}
\]

\[
\text{New } I_2 | p \rangle \sim \text{New } | n \rangle \sim 0 \quad \Rightarrow \quad m_n = 0 \Rightarrow \text{[New] } I_2 \neq 0
\]
However, since under these the $I_3$ component of isospin is unchanged, we will make the assumption that $[Hew, I_3] = 0$.

The electro-weak interactions in the theory in which electro-weak field is introduced through minimal substitution

$$p^\mu \rightarrow p^\mu - eA^\mu$$

are determined by the charge operator $Q$.

Since we have seen that $Q = I_3 + \frac{g}{2}$ with

$$[\gamma, \gamma^2] = 0$$

we can decompose the electric charge operator as well as electric current operators as:

$$J_\mu \rightarrow J_\mu^0 + J_\mu^3$$

$$\text{isospin 0} \quad \text{isospin 1} : \quad [\gamma, J_\mu^0] = 0$$

$$[I^3, J_\mu^3] = \epsilon^{ij} \epsilon^3 j^j J^j$$

where $J_\mu^i$ are the isospin current operators.

So consider a transition: $a \rightarrow b + c$

bounded states

$$\Delta I^\prime_3 = I^\prime_3 - I_3 \quad \Delta Y = \Delta S = 0$$

and $|\Delta I^\prime_3| = 0$ or $\Delta I^\prime_3$ due to $J_\mu^3$

and $\Delta I_3^0$ due to $J_\mu^0$. 

Note that gauge invariance alone does not imply minimal substitution. For example one could have an interaction term in $\text{He}_{\mu} = \frac{i}{\sqrt{2}} \psi_6 \gamma_\mu \psi_6 F^{\nu \mu}$ where $\psi_6, \psi_6$ are particle (fermion) a and b Dirac spinors and $F^{\nu \mu} = \partial^\nu A^\mu - \partial^\mu A^\nu$.

One then imagine using such term to describe \[ \Lambda \geq n + \frac{1}{2} \text{ with } |\Delta I_3| = \frac{1}{2} \]

Such action has never been observed however.

On the other hand $\Sigma^0 = \Lambda + 1$ does occur corresponding to $|\Delta I_3| = 0$ and $|\Delta I| = 1$.

Note that since $[\mathbf{Q}, \hat{J}^3] = 0$

if $\langle a_1 | q_1 b_1 \rangle \neq 0 \Rightarrow I_3^a = I_3^b$.

In the case of $\Sigma^0 = \Lambda + 1$ decay $\langle a_1 q_1 b_1 \rangle = 0$ and this strange rule does not hold. If this decay was mediated by the Q operator then we should get zero, it is however mediated by the magnetic moment i.e. $\mathbf{I}^\mu(x)$ Belshoue part of the electromagnetic current which also implies that in general $[\mathbf{J}^\mu(x), \hat{J}^3] \neq 0$. 