1) \( W_{\mu\nu} = \frac{1}{2\pi} \int d^4x \, e^{ixx} \langle p \mid j_\mu(x), j_\nu(0) \rangle \mid p \rangle \)

i.e. we can add \(- j_\nu(0), j_\mu(x)\) to the

original \( j_\mu(x) j_\nu(0) \) without altering the value

of \( W_{\mu\nu} \) or \( \int d^4x \, e^{ixx} \langle p \mid j_\nu(0) j_\mu(x) \rangle \mid p \rangle = 0 \)

Proof: as before put in a complete set of

states:

\[
\sum_x \int d^4x \, e^{ixx} \langle p \mid j_\nu(0) \mid X \rangle \langle X \mid j_\mu(x) \rangle \mid p \rangle = \\
= \sum_x \int d^4x \, e^{ixx} \langle p \mid j_\nu(0) \mid X \rangle \langle X \mid j_\mu(0) \rangle \mid p \rangle e^{-ip\cdot x}
\]

\[
= \sum_x (2\pi)^4 \delta^4(q-p+px) \langle p \mid j_\nu(0) \mid X \rangle \langle X \mid j_\mu(0) \rangle \mid p \rangle
\]

we can show that \( q-p+px \neq 0 \) always!

assume \( q-p+px=0 \) then \((p-q)^2 = px^2 = m^2 \)

which implies \( q_0 = \frac{\sqrt{m^2+q^2-x^2}}{2m} \) in the proton's

rest frame. Since low physical scattering \( W^1 > m^2 \) and \( q^2 < 0 \)

\( \Rightarrow q_0 \leq 0 \) thus contradicts \( q_0 = \gamma - \gamma' > 0 \)

\( q_0 = h, \) loses energy
According to the causality requirement, which states that two observables (operators) lead to independent - interference measurements if they are performed at two space-time points which are not causally connected $\Rightarrow$ spacelike

$\Rightarrow \quad [j_\mu(x), j_\nu(0)] = 0 \quad \text{if} \quad x^2 < 0$

$\Rightarrow$ the integrand in $W_{\mu\nu}$

$$W_{\mu\nu} = \frac{1}{(2\pi)^3} \int d^4x \, e^{i q \cdot x} \langle p | [j_\mu(x), j_\nu(0)] | p \rangle$$

has a support only for $x^2 > 0$

2) Light-cone dominance.

For deep inelastic region: $-q^2 \to \infty, \nu \to \infty$

with $-q^2 / \nu = \text{fixed}$, the dominant contribution to $W_{\mu\nu}$ comes from the region

$0 \leq x^2 \leq \text{const.} \quad \text{i.e. it is all concentrated}$

$\frac{-q^2}{\nu}$ near the light-cone $x^2 = 0$
The proof is very simple. We rely on the theorem that in a Fourier transform
\[ \int dx e^{i k x} f(x) = \hat{f}(k) \]
when \( k \to \infty \) we have to have \( x \sim \frac{1}{k} \to 0 \)
for the integration region. Otherwise \( (x = \text{finite}) \)
\( kx \to \infty \) and \( e^{i k x} \) oscillates rapidly giving a vanishing contribution to the integral.

Now we have:
\[ e^{i q \cdot x} \quad \text{where} \quad q \cdot x = q^0 x^0 - q^2 r 
\]
\[ = q^0 x^0 - |q| r \quad \text{where} \quad r = \frac{x - q}{|q|} \quad \text{is the component of} \ x \ \text{in the direction of} \ q \]
\[ |q| = \sqrt{(q^0)^2 - q^2} 
\]
\[ = q^0 (x^0 - \sqrt{1 - \frac{q^2}{(q \cdot x)} r}) \]

In the target rest frame \( q^0 = \sqrt{-1} \to \infty \) and \( x = \frac{-q^2}{2Mv} = \text{finite} \)
\( q \cdot x = \sqrt{x^0 - \sqrt{1 + \frac{2M}{v^2}} r} \approx \sqrt{x^0 - r} - M^2 r + O(\frac{1}{v}) \)
In order to keep \( q \cdot x \) finite in \( q \) limit, we need:

\[
|x_0 - c| \leq \frac{\text{const.}}{\sqrt{q}} \quad \text{and} \quad q \leq \frac{\text{const.}}{3}
\]

Thus:

\[
x_0 \leq (r + \frac{\text{const.}}{\sqrt{q}})^2 \leq r^2 + \text{const.} \frac{\sqrt{q}}{q} \leq q^2 + \frac{\text{const.}}{3} \leq q^2
\]

which implies

\[
x_0 - x = x^2 \leq \frac{\text{const.}}{-q^2}
\]

Since we have already shown that \( x^2 \geq 0 \), we get

\[
0 \leq x^2 \leq \frac{\text{const.}}{-q^2} \Rightarrow x \approx 0.
\]
Implications of light cone dominance.

In target rest frame: \( p = (p^0, 0, 0) \)

\[ q = (v^0, 0, 0, -\sqrt{v^2 + q^2}) \]

In the Bjorken limit: \( v \to \infty; \quad q^2 \to 0 \quad \frac{q^2}{v^2} \to \text{finite} \)

\[ q \to \left( v^0, 0, v, -v - \frac{1}{2} \left( \frac{q^2}{v^2} \right) \right) = \left( v^0, 0, v, -v - M^2/M_\text{F}^2 \right) \]

\( M_\text{F} \)

Introduce light-cone variables:

\[ \alpha^\pm = \frac{1}{2} \left( \alpha^0 \pm \alpha^2 \right) \]

\[ \alpha^\perp = (\alpha^x, \alpha^y) \]

The scalar product

\[ a_\mu b^\mu = \alpha^+ b^- + \alpha^- b^+ - \alpha^\perp b^\perp \]

Thus:

\[ q^+ = -\frac{M^2}{\sqrt{2}} \quad ; \quad q^- = \frac{2v + M^2}{\sqrt{2}} \to \sqrt{2} v \to \infty \]

\[ \Gamma \to 0. \]

Since

\[ q \cdot x = q^+ x^- + q^- x^+ - q^\perp x^\perp \]

and

\[ W_{\mu\nu} = \frac{1}{2i} \int d^4x^- e^{i(q^- x^-)} \int d^4x^+ d^2x^\perp e^{i(q^\perp x^\perp)} \langle \rho | J_\mu(x) J_\nu(x) | \rho \rangle \]
Thus the inequivalences contribute from \( x^+ \rightarrow 0 \). Note that every component of \( x^+ \) (except \( x^- \)) \( \rightarrow 0 \) in the \( \beta \rightarrow 0 \) when limit.

If we were to think about \( x^+ \) as "time" then since \( x^+ \rightarrow 0 \) on the time scale of photon interaction the target nucleus is essentially "frozen" if typical time scales for strong interactions is

\[
x^+_{\text{phys}} \gg x^+ \rightarrow 0.
\]

Why \( x^+ \) could be regarded as time. (Why not := light cone quantization \( \Rightarrow \) see Riossey + Lepage PRD 22, 2151 (1980).

It is easy to understand \( x^+ \) as time for a system which uses with \( \gamma \rightarrow 1 \) then Lorentz transformation:

\[
t' = \gamma t + p^+ \gamma \Rightarrow \gamma (t + z) \Rightarrow \gamma (t^+) \text{ time in } \gamma \text{ rest frame.}
\]

\[
z' = \gamma z + p^\gamma t
\]

Thus the best way to understand \( \beta \rightarrow 5 \) is to go to \( 0 \) momentum frame.