Application.

Now that we established the similarity of the theories between $T=0$ and $T>0$ transition. Let's see how to consider the effects of interactions in the many-body system at $T=0$. As usual, the main difficulty is to find the reasonable stationary point for an approximate solution so that we reach to the approximation at small order.

Let's consider first the mean field (Harriee-Forch) approximation at $T=0$. This is the simplest self-consistent approximation to the Green's function. Perturbations are treated as if interacting with a constant "average potential" generated.
6. The medium, which by itself depends on
the solution of the S-equation for the
circle, produce a function. Presence of the
Heaviside and pulse reservoir means that
flux is always in energy and pulse number
not allowed. The system is not in the F=0
ground state corresponding to all pulses
lying in bound orbitals up to the Fermi level
(terminus).
As at $T=0$ the HF approximation can be formulated through an approximation to the Dyson's equation for the single particle Green's function:

$$\hat{H} = \hat{H}_0 + \sum_i G_{\sigma}^{\sigma'} \hat{r} \hat{l}_{\sigma} \hat{l}_{\sigma'}$$

On: $G = G^0 + G^0 \Sigma G$

$\Sigma$: proper self-energy

To compare $T=0$ on $T\neq 0$ Feynman rules

recall how this equation looked like at $T=0$
\[ T = 0 \ (\text{remind in p.625}) \ (\text{assume spin-independent potential for fermion}) \]

\[ \text{HF} \Rightarrow \text{1st order with} \ G^0 \rightarrow G \]

\[ 1^{st \ order \ in \ momentum \ space} \]

\[ G(x,y) = G^0(x,y) + i d^4k \delta(x-k) G^0(x,k) \Sigma^*(x,k) G^0(k,y) \]

\[ \Sigma^*(x,y) = \begin{pmatrix} x_{1}^\prime \delta(x_{1} - x_{1}^\prime) \frac{d}{dx_{1}} \mathcal{V}(x_{1} - x_{1}^\prime) G^0(x_{1}^\prime, y) \\
\end{pmatrix} \]

\[ \text{1st order} \quad \text{For HF replace} \ G^0 \rightarrow G \ in \ \Sigma^* \]

\[ G(x,y) = \int \frac{d^4k}{(2\pi)^4} e^{i k \cdot (x-y)} G(k) \]

\[ Z(x,y) = \int \frac{d^4k}{(2\pi)^4} e^{i k \cdot (x-y)} \Sigma(k) \]
\[ S(x_y) = \int \frac{d^4k}{(2\pi)^4} e^{-i\mathbf{k}(\mathbf{x} - \mathbf{y})} \tilde{S}(k) \]

\[ \tilde{S}(x_y) = \int \frac{d^4k}{(2\pi)^4} e^{-i\mathbf{k}(\mathbf{x} - \mathbf{y})} \tilde{S}(k) \]

\[ S(q) = \int d^4(x-y) e^{-iq(x-y)} S(x-y) \]

\[ = S^0(q) + \int d^4(x-y) d^4k \left( \frac{-i}{2\kappa} \right)^{\mu
u} \frac{\partial^4}{(2\pi)^4} \epsilon^{\mu
u} \epsilon_{\lambda\sigma} S^0(k) \]

\[ \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \tilde{S}(k) \epsilon^{\mu\nu} \epsilon_{\lambda\sigma} \tilde{S}(k) \]

\[ S(q) = S^0(q) + \int d^4(x-y) e^{-iq(x-y)} \frac{i(q-k_\lambda)}{2\kappa} \epsilon^{\mu\nu} \epsilon_{\lambda\sigma} S^0(k) \]

\[ \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \tilde{S}(k) \epsilon^{\mu\nu} \epsilon_{\lambda\sigma} \tilde{S}(k) \]

\[ \rightarrow \]

\[ S(q) = S^0(q) + \int d^4(x-y) e^{-iq(x-y)} S^0(q) \tilde{S}(q) \]

\[ \tilde{S}(q) = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} S^0(q) \tilde{S}(q) \]
\[
\Sigma^x(q) = \int d^4(x-y) e^{iq(x-y)} \Sigma^x(x-y)
\]
\[
= -i(2s+1) \int d^4(x-y) e^{iq(x-y)} \rho_v(x-y) \int d^3z V(y-z) G_0(z-t) \delta(x-t') \delta(t+t') \delta^{-1}(x-y)
\]
\[
+ i \int d^4(x-y) U(x-y) G_0(y-x) \quad \text{[Hf Hf corr]} \quad G_0 \rightarrow G
\]
\[
\Sigma^x(0) = -i(2s+1) V(0) G(0) \quad \text{[Hf corr]} \quad \text{a} + \text{K} = \text{K}.
\]
\[
\Sigma^x(q) = -i(2s+1) V(0) \int \frac{d^3k}{(2\pi)^3} e^{i(k\cdot x)} \delta(k_y) \delta(k_x) \delta(k_z) \int \frac{d^3\omega}{(2\pi)^3} \frac{d\omega}{\omega} G(k,\omega)
\]
\[
\Sigma^x(q) = -i(2s+1) V(0) \int \frac{d^3\omega}{(2\pi)^3} \frac{d\omega}{\omega} G(k,\omega)
\]
\[
+ e \int \frac{d^3\omega}{(2\pi)^3} \frac{d\omega}{\omega} \delta^3(q-k-x) \int \frac{d\omega}{\omega} G(k+q,\omega) V(E')
\]
\[
\overline{\Sigma^x(q)} = -i(2s+1) V(0) \int \frac{d^3\omega}{(2\pi)^3} \frac{d\omega}{\omega} G(k,\omega)
\]
\[
+ e \int \frac{d^3\omega}{(2\pi)^3} V(q^2-\overline{E}) \frac{d\omega}{\omega} G(k,\omega)
\]
\[
\text{(x)}
\]
Now replace $\int_{(\theta,\omega)} \delta(x, \omega)$ is ill defined

and the correct procedure is to replace it by

$$\int_{(\theta,\omega)} \delta(x, \omega) e^{\text{pick up the order of } \theta, \gamma}$$

So we have "derived" the momentum space result.

From coordinate space, we could simply write it down using momentum space Feynman rules. Also remember (or note from *) that

$$\mathbf{e}(q) = \mathbf{e}(q, q^0) \text{ is q^0 independent}$$

$$\mathbf{e} = \mathbf{e}(q)$$
So we have (with the convention):  

$$
\Sigma_{HF}(x, y, \omega) = \int \delta(x-y) e^{i \omega (x-y)} \Sigma_{HF}(x, y, \omega)
$$

$$
= \Sigma_{HF}(x, y) = \int \frac{d^3 q}{(2\pi)^3} e^{i q (x-y)} \Sigma_{HF}(q)
$$

$$
\Sigma_{HF}(x, y) = -i (2\pi)^3 \delta(x-y) \int \frac{d^3 z}{\lambda} V(\vec{g} - \vec{z}) \int \frac{d\omega}{\omega} e^{i \omega \omega_0} G(x-y, \omega)
$$

Now use the rules for $T \neq 0$ to write $\Sigma_{HF}^{T \neq 0}$:

$$
\begin{align*}
& a + T \neq 0 \\
& (-i) \rightarrow (-1) \\
& \frac{1}{2\pi} \rightarrow \frac{1}{\beta} \\
& \int \frac{d\omega}{\omega} \rightarrow \frac{1}{\beta} \int \frac{d\omega}{\omega} V(q) \\
& \Rightarrow (\frac{1}{\beta} \int \frac{d\omega}{\omega}) \rightarrow \frac{1}{\beta} V(q)
\end{align*}
$$

$$
\Sigma_{HF}(q) = -i \delta(q, q_0) + i \delta(q, q_0) (-i \Sigma_{HF}^{T \neq 0}(q)) i \delta(q, q_0)
$$

$$
\begin{align*}
& (-1)(-i) V \rightarrow \infty \text{ as } q \rightarrow 0 \\
& \frac{1}{\beta} \text{ for loop 1's and 2's} \\
& + (+) (-i) V \text{ as } |q| \rightarrow 0
\end{align*}
$$
\[ G_{\nu \mu} (q, \omega) = G^0(q, \omega) + G^0(q, \omega) \sum_{HF} (q, \omega) G_{\nu \mu}^0(q, \omega) \]

\[ \Sigma^x(q, \omega) = \Theta(2s+1) V(\delta) \int \frac{d^3 \ell}{(2\pi)^3} \sum_{\eta} \frac{1}{p} e^{i \omega \cdot P} \bar{G}(\ell, \omega) e^{i \omega \cdot \eta} \]

\[ \Theta \int \frac{d^3 \ell}{(2\pi)^3} V(\delta, \omega) \sum_{\eta} \frac{1}{p} e^{i \omega \cdot \eta} \bar{G}(\ell, \omega) \]

\[ \Sigma^x(x, \omega) = (2s+1) \delta^3(\mathbf{x} - \mathbf{y}) \int d^3 V(\mathbf{q} - \mathbf{y}) \sum_{\eta} e^{i \omega \cdot \eta} \bar{G}(\mathbf{x} - \mathbf{y}, \omega) \]

\[ - V(\mathbf{x} - \mathbf{y}) \frac{1}{p} \sum_{\eta} e^{i \omega \cdot \eta} \bar{G}(\mathbf{x} - \mathbf{y}, \omega) \]

\[ = \Sigma^x(x, \omega) \text{ independent of } \omega \]

Now let's solve the equation. Recall that

The problem at hand corresponds to

\[ H = T + U + V \text{ where } T = \text{kinetic energy} \]

\[ U = \text{external, out body} \]

\[ V = \text{two body interaction} \]
The free Green's function $G^0$ corresponds to $K_0 = K_0 + \mu I$, adding $U$ does not complicate things too much, as long as we can solve the S-equation (true independently)

\[
(T + U) \hat{\Phi}^0(x) = (-\frac{\partial^2}{\partial x^2} + U(x)) \hat{\Phi}^0(x) = \varepsilon_{j0} \hat{\Phi}^0_j(x)
\]

To express $G^0$ in terms of $\hat{\Phi}^0_j$

\[
G^0(x, y, \omega_n) = \sum_j \frac{\hat{\Phi}^0_j(x) \hat{\Phi}^0_j(y)}{i \omega_n - (\varepsilon_{j0} - \mu)}
\]

where $\hat{\Phi}^0_j(x)$ are simple self-adjoints.

If $U(x) = 0$ then we have the free case discussed earlier with $\hat{\Phi}^0(x) = \frac{1}{\sqrt{2}} e^{-\frac{\mu x}{\sqrt{2}}}$
We may have:

\[
\left[ i \omega_n - V_0(k) \right] G^0(x \bar{y}, \omega_n) = \left[ i \omega_n - \frac{D^2}{\omega_n^2} - U_0(x) + \mu \right] G^0(x \bar{y}, \omega_n)
\]

\[
= \sum_i \frac{\Phi_j^0(x) \Phi_j^0(\bar{y})}{i \omega_n - \epsilon_j + \mu}
\]

\[
= \sum_i \frac{\Phi_j^0(x) \Phi_j^0(\bar{y})}{i \omega_n - \epsilon_j + \mu}
\]

\[
= \delta^3(x - \bar{y})
\]

As \( T = 0 \) we guess that HF approximation leads to an effective one-body problem with a new self-consistent potential. Thus we expect the HF Green's function to have the form:

\[
G(x \bar{y}, \omega_n) = \sum_i \frac{\Phi_j(x) \Phi_j(\bar{y})}{i \omega_n - (\epsilon_j + \mu)}
\]

where \( \Phi_j(x) \) and \( \epsilon_j \) are solutions of the HF equations and depend on \( g \) and \( T \).