The obseved particke density follows then:

\[ n(\mathbf{x}, \mu, \psi, t) \equiv \langle \hat{n}(\mathbf{r}) \rangle = c_{\alpha \alpha}(\mathbf{k} \cdot \mathbf{r}, t^+) \]

\[ = \frac{1}{\beta} \sum_n g_{\alpha \alpha}(\mathbf{k} \cdot \mathbf{r}, \omega_n) e^{i \omega_n t} \]

\[ \lim_{\omega \to \infty} \Theta(\omega^2 - \omega_n^2, \omega^2 - \omega_n^2) \quad \text{(}\omega^2 \text{)} \text{ (}t^+\text{)} \]

\[ = (2s+1) \sum_j \Phi_j(\mathbf{r}) \left( \frac{1}{\beta} \sum_n \frac{e^{i \omega_n t}}{\omega_n - (E_j - \mu)} \right) \]

\[ = f(\beta, E_j, \mu) \quad \text{we will evaluate this soon.} \]

\[ N(\mathbf{r}, \psi, t) = \int d^3 \mathbf{x} n(\mathbf{x}, \mu, \psi, t) = (2s+1) \sum_j f(\beta, E_j, \mu)(\mathbf{x}) \]

Thus we expect \[ f(\beta, E_j, \mu) = \frac{1}{\beta \omega_{j+1} - \omega} \quad \text{femtosecond} \]

\[ \equiv \omega_i \]

In (1+), sum runs over a complete set of \[ \psi_0 \]s.

The particle states \[ v \] are solutions of the HF eq to be determined. From (1+), we can express \[ \mu = \mu(N, \psi, t) \]

and consider \[ N \text{ fixed} \]
With the assumed form of \( G \), the \( \Sigma \) is given by

\[
\Sigma^\pm (x, y; \mu \nu) = (2s + 1) \delta(x-y) \int d^2z \sqrt{\gamma - \gamma} \sum_j \phi_j(x) \phi_j(y) \eta_j^* \eta_j
\]

Now use \( \tilde{\omega}_n - \omega_n(x) \) to act on \( \Phi \).

\[
G(x^r, \omega_n) = G^0(x^r, \omega_n) + \int d^2x \int d^2y \int d^2z \sum_{\sigma} \phi(x, \sigma) \phi(y, \sigma) \eta \phi(z, \sigma) \eta^* \eta
\]

and minimum as in the T=0 case: to get:

\[
\left[ -\frac{\partial^2}{\partial \omega^2} + U(\omega) \right] \phi_j(\omega) + \frac{1}{(2s+1)} \int d^2z \sqrt{\gamma - \gamma} \sum_k \phi_k(\omega)^2 \phi_j(\omega) = \frac{\eta^* \eta}{\omega - \tilde{\omega}_n(x)} \]

Vehicular (non-local)

The eq. 104 as \( T=0 \), except \( \eta \leftarrow \theta(\epsilon_F - \epsilon_k^{\text{HF}}) \).
This set of inhomogeneous differential eg is under to solve

When $a + T = 0 \Rightarrow$ the particle is not fixed

and the sum $j$ was over a complete set of states

(not just up the $T$ level) Furthermore we must adjust $n$

to get desired $N$.

Every of the system:

$$E_q(x, y, z) = (2s + 1) \int d^3 \mathbf{x} \mathbf{u}_{a} \frac{1}{k^2} \mathbf{e}^{i \mathbf{x} \cdot \mathbf{w} - \frac{e^2}{2 \mathbf{w} + u(x) 3\mathbf{u})}}$$

$$= (2s + 1) \sum_{j} \mathbf{E}_j \mathbf{u}_{a} - \frac{1}{2} (2s + 1) \int d^3 \mathbf{x} \mathbf{D}_{j}^{\ast} \mathbf{D}_{j} (x) \sum_{j} (x \cdot \mathbf{y}) \mathbf{D}_{j}^{\ast} (\mathbf{y})$$

= ensemble average of the self-consistent wave function

energies - $\frac{1}{2}$ potential energy to come for the double counting of pairs
For a uniform system: \( \langle \lambda \rangle = 0 \) non-quantum.

In our interest: \( \phi_j(x) = \phi_{E, \lambda}(x) = \frac{1}{\sqrt{V}} e^{iE}\eta_\alpha \)

\[ E_\chi = \frac{\hbar^2}{2m} + \sum^* (E_\mu, \psi T) \]

\[ \sum^* (E_\mu, \psi T) = (2s+1) \sqrt{V} \int_0^{i\hbar} \frac{d^3k'}{(2\pi)^3} \eta \frac{d^3k}{(2\pi)^3} \sqrt{E - E'} n_k \]

This is no longer equal to the first-order perturbation expression, so it was for \( T=0 \)

Sure self-consistency enters through the occupation probabilities

\[ n_k = \frac{1}{e^{E_k} - 1} \]

when \( E_k \) both definite and is determined by \( n_k \)

Finally \[ N(\psi, \eta, T) = (2s+1) \sqrt{V} \int_0^{i\hbar} \frac{d^3k}{(2\pi)^3} 2 n_k. \]
Eulerian of \( f(t, \eta, \beta) \):

\[ \sum_{n} e^{i \omega_n \xi} = \frac{1}{1 + e^{\beta t + 1}} \]

\[ e^{i \omega_n \xi} = \frac{1}{1 + e^{\beta t + 1}} \]

Proof: (ferns, bosco = aurochs)

For \( e^{-\beta} \): His dis pores at \( z = z_n = \frac{(2n+1)\pi}{\beta} \)

Thus: \( e^{-\beta} \)

\[ e^{\beta t + 1} - e^{\beta(2\eta + 1)} - (1 + \beta (2 \eta + \ldots) + 1) \]

\[ = \frac{1}{2 - \xi_n} \Rightarrow \text{ residue } = 1 \text{ at the pole, } \]

\[ z = i \omega_n. \]

From Cauchy Residue:

\[ \sum_{n} e^{i \omega_n \xi} = -\frac{1}{2 \pi i} \int_{\gamma_{\eta}} \frac{e^{\beta t + 1}}{e^{\beta t + 1} \xi - x} \, dt \]

\[ \xi \rightarrow \gamma_{\eta} \rightarrow C \]

\[ z = x \]
define the contour:

\[ \Gamma \]

\[ c \quad c' \quad \Gamma \]

\[ c' \quad c \quad \Gamma \]

\[ \Rightarrow \]

\[ \Rightarrow \quad \mathcal{G} = \mathcal{G} + \mathcal{G} \quad \text{(no poles inside)} \]

\[ \text{But } \quad \frac{1}{2} e^{it} \to 0 \text{ in } \Re t < 0 \quad |z| \to \infty \]

\[ \text{and } \quad \frac{1}{2} \frac{1}{e^{\beta t+1}} \to 0 \text{ as } \Re t > 0 \quad |z| \to \infty \]

\[ \Rightarrow \quad \mathcal{G} = 0 \]

So we are left with \[ \mathcal{G} \] which give

\[ \mathcal{G} \quad \text{at } \quad z = 0 \quad \text{(in the opposite direction)} \]

\[ = \frac{1}{2} \sum \frac{e^{i\omega n t}}{P_n i\omega n - \chi} = \frac{1}{R_n + 1} \quad \text{fermionic} \]