I. Introduction

Why nuclear physics:

1) More than 99% of visible mass in the universe is due to nuclei. Out of these 3/4 are protons, and all the rest is in helium with small contributions from deuterium, carbon and other heavy elements.

2) Brief history

It all started after the Big Bang. After a few trillion years there were more quarks than antiquarks, and other stuff (weak bosons, plasmons, gluons, leptons). As the universe cooled and expanded, quarks would bind into hadrons (protons, neutrons, $\Lambda^0$, pions, ...) and soon after only the most stable hadrons $p, n$ would survive. After a couple of minutes, temperatures went down $\sim 10^9 K$ and $K_B T \lesssim 1 MeV$ and bound structures formed.

[*(x) will talk about p, n as if it was one element = nucleon]*

Soon after, deuterons and $\alpha$-particles were formed. The excess of neutrons fused into protons through $\beta$-decay ($n \rightarrow p + e^- + \bar{\nu}_e$) so only primordial deuterium, helium and protons survived (pp do not bind as we shall see). After $10^4$ years temperature went down $10^5 K$, atoms were formed and became transparent to light which stayed with us till now as microwave background radiation. After $10^9$ years, gravity took over, made stars, and reversed the cooling-expansion cycle in which heavier elements were
though stellar nucleosynthesis only light elements to produce heavier ones and energy. After all fuel is used up a star collapses becoming a white dwarf.

[* for small stars where gravity can be balanced by electron's pressure or explodes as a supernova -> planets.]

So much for the history. How are we going to study nuclei:

3) How are we going to study nuclei?

There is a fundamental theory = Quantum Chromodynamics QCD.

But it is, quantum, nonlinear, relativistic, many body field theory. To work, had to get over initial solutions, impossible for nuclei.

But:

We know that there is a choice of effective degrees of freedom in which the many body nuclear problem simplifies to use nucleons

Then nonrelativistic, potential theory seems to be a good starting point.
From a practical point of view

describe N-N interaction
build a systematic approach to finite nuclei and nuclear matter
we do not need to know the microscopic origins of nuclear forces ⇒ we can simply parametrize them

Nowadays, we realize that this is in the same spirit as a general renormalization program.
To study physics at the nuclear scale we do not need to know the structure of the theory at the GUT's scale. Its presence may be parameterized low (nuclear) energies through "irrelevant" interactions which become "more relevant" as we increase energy with which we study the system.
If we knew the underlying theory we could in principle calculate these "irrelevant" interactions, but in practice only few are needed and we may simply parametrize them.
This approach will become less accurate as
the energy increases. At microscopic, pions, quarks
and gluons of about will become relevant.

So initially we will stay in the low, medium energy
domain (E \leq 1 GeV) and use nonrelativistic theory*

* i.e. conserved particle number

to describe N-N interactions.

The holy grail of nuclear physics is ultimately
to understand how this can be derived from QCD*

* even theory in practice you would not use QCD to
solve for $^{10}$O, see above

In the second semester we will therefore study QCD
itself.

This way we have come to realize how important nuclear
physics is from the point of view of building and understanding
effective approaches to a many body system.
Quantum Mechanics - review

Let's start with properties of angular momentum. This is relevant since in absence of a prefered orientation of nuclei in space total angular momentum is conserved. 

1) Operator properties: Start with commutation relations

\[ [J_i, J_j] = i \varepsilon_{ijk} J_k \quad (1) \]

J can be orbital angular momentum

\[ J = L = i \varepsilon_{ijk} \psi_i \phi_j \rho_k \quad \phi_i = -i \frac{\partial}{\partial \psi_i} \]

on spin: \((\frac{1}{2}) J_i \Rightarrow S_i = \frac{1}{2} \psi_i \quad S_x = (0, 1) \quad S_y = (0, -i) \quad S_z = (1, 0) \]

\[(\frac{1}{2}) J_i \Rightarrow S_i = T_i \quad T_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix} \quad T_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad T_z = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ J \Rightarrow L + S \Rightarrow \text{total orbital angular mom.} \]

etc.
Now to work out the properties of representation introduce

\[ J^+ = J_x + i J_y \quad J^2 = J_z J_z - \frac{1}{2} (J^+ J^- + J^- J^+) \]

\[ J_z^+ = J_z \]

From (1) \[ [ J^2, J_z ] = 0 \] \[ J^2 \text{ - Casimir operator} \]

\[ \Rightarrow \text{take } J^2 \text{ and } J_z \text{ and diagonalize simultaneously} \]

\[ J^2 |jm\rangle = \ell (\ell) |jm\rangle \quad J_z |jm\rangle = \ell (\mu) |jm\rangle. \]

in general \[ |\ell\rangle = |\ell \mu \rangle \] where \( \mu \) corresponds to eigenvalues of other operator that commute with \( J^2, J_z \)

Now since \[ [ J_z, J^\pm ] = \pm J^\pm \]

\[ \langle jm' | J^+ J^+ | jm \rangle = \langle jm' | J_z J_z - J^+ J^- + J^- J^+ | jm \rangle = \ell (\ell) \langle jm' | J^+ J^+ | jm \rangle - \ell (\mu) \langle jm' | J^+ J^+ | jm \rangle \]

\[ (\ell - \mu) \langle jm' | J^+ J^+ | jm \rangle = 0 \Rightarrow \mu = \ell - 1 \text{ for } \ell J^+ \neq 0. \]

Thus \[ J^+ |jm\rangle = \ell (\ell, \mu) |jm + 1\rangle \quad (\times) \]

\[ J^- |jm+1\rangle = \ell (\ell, \mu) |jm\rangle \]

\[ A(\ell, \mu) \]