What if proton has structure?

Then instead of

\[ \langle p \gamma^\nu | j_\mu(0) | p, \gamma \rangle = \bar{u}(p, \gamma) \gamma^\mu \gamma^\nu u(p, \gamma) \]

(we've, structureless quark current)

we could have the most general form which is:

\[ \langle p, \gamma \gamma^n | j_\mu(0) | p, \gamma \rangle = \bar{u}(p, \gamma) \left[ \gamma^\mu F_1(B^2) + \frac{2}{C M} \sigma_1 \cdot F_2(B^2) \right] u(p, \gamma) \]

where \( F_1 \) and \( F_2 \) are some scalar functions that describe proton's structure. The EFT follows from just symmetry arguments based on Lorentz trans formation properties of a vector operator \( j_\mu \), and current conservation \( \partial \mu j^\mu = 0 \).

Repeating the calculation for \( W_1 \) and we in this case you will find that

\[ W_1 = \frac{\alpha^2}{2 \pi s} \left( \frac{\alpha^2}{2 \pi s} - 1 \right) \rightarrow \bar{u} \left[ \frac{F_1(B^2) + F_2(B^2)}{C M} \right] u \]

\[ \bar{u} \left( \frac{\alpha^2}{2 \pi s} - 1 \right) \rightarrow \] similar.
From data on p π 2πB we see that in the deep inelastic limit: \( q^2 \to 0 \), \( x \to 0 \) with \( \omega = \frac{1}{x} = \frac{2m_π}{Q^2} = \frac{2m_π}{Q^2} \) fixed the function \( N(x, \omega) \)

instead of being a function of two independent variables \( x \) and \( Q^2 \), becomes a function of their ratio \( x \) only. If only 'soft' strongly nuclear interactions between the quarks were relevant, we would expect the function \( N(x, \omega) \) to be determined by \( \langle x / m_π \rangle \) and \( F(\omega^2) \) of form factors which would fall off rapidly with \( Q^2 \). Alternatively, we can say that as \( Q^2 \to 00 \) the charge distribution emerges at scales \( \frac{1}{\sqrt{Q^2}} \) and described by the form factors does not go to \( \to 0 \) but is finite resulting in \( \sqrt{N(x, \omega)} \) being finite at large \( \omega^2 \).

Complementing with the elastic \( p \to p \) scattering for which \( \sqrt{N(x, \omega)} = \delta \left( \frac{\omega^2}{2m_π} - 1 \right) = \delta (\omega - 1) \)
is a function of the ratio \( \omega \) we can think that in deep inelastic region the photon scatters free constituents of the proton which have some distribution so that \( \delta (\omega - 1) \to \) regular function of \( x \).
The scattering behaviour i.e. \( N_{\sigma} (\tau, Q^2) \) depending on the ratio \( x = \frac{Q^2}{2m_N} \) was first predicted by Bjorken on a basis of the analysis of various sum rules. Thus the variable \( x \) is now called the Bjorken-\( x \) or \( x_B \), \( x_{ Bj} \).

It was Feynman who came up with the physical interpretation in terms of scattering of pointlike constituents. It was around 1974 before QCD was well accepted as the theory of strong interactions. Now we know that the constituents, that the photon scatters from, called pions by Feynman, are quarks and together with gluons (which are electrically neutral =) photon does not scatter from them directly) make up the proton.
Implications of light cone dominance.

In target rest frame: $p = (m, 0, 0, 0)$

$q = (\sqrt{v^2 + q^2}, 0, 0, -\sqrt{v^2 + q^2})$

In the Bjorken limit $v \to \infty; \frac{q^2}{v} \to 0$:

$q \to (\sqrt{v}; 0, 0; -\sqrt{v} - \sqrt{M^2}) = (\sqrt{v}, 0, 0; -\sqrt{v} - Mx_3)$

Introduce light-cone variables:

$a^\pm = \frac{1}{\sqrt{2}} (a^0 \pm a^3)$

$a_\perp = (a^x, a^y)$

The scalar product $a^\mu b^\mu = a^+ b^- + a^- b^+ - a_\perp \cdot b_\perp$

Thus: $q^+ = -\frac{Mx}{\sqrt{2}}; q^- = \frac{\sqrt{v} + Mx}{\sqrt{2}} \to \sqrt{v} \to \infty$

$D \to \infty$.

Since $q \cdot x = q^+ x^- + q^- x^+ - q_\perp \cdot x_\perp$

and

$\omega c^\mu = \frac{1}{(2\pi)^{3/2}} \int d^4x \, e^{i q \cdot x} \int d^4x' \, e^{i q' \cdot x'} <\phi(x) \phi(x')/\mu^2>$. 


Thus the inequivalent receives contribution from

\[ x^+ \rightarrow 0 \] Note that every component of \( x \) (except \( x^-) \rightarrow 0 \) in the \( D_{1} / D_{3} \) when limit.

It we where to think about \( x^+ \) as "time" then

since \( x^+ \rightarrow 0 \) on the time scale of quark interaction the target nucleus is essentially "frozen" i.e.

"typical time scales for strong interactions is

\[ x_{\text{norm}} \gg x^+ \rightarrow 0. \]

Why \( x^+ \) could be regarded as time. (Why not

\[ \Rightarrow \text{ light cone quantization } \Rightarrow \text{ see Brodsky + Lepage} \]

PRD 22, 2152 (1980).

It is easy to understand \( x^+ \) as time for a system

which moves with \( \gamma \rightarrow 1 \) then Lorenz transformation

\[ t' = \gamma (t + \beta x) \Rightarrow t'(t + t) = \gamma t^+ \text{ time in } \]

\[ \text{ rest } \rightarrow 0 \] momentum

Thus the best way to understand (15) is to

go to \( \text{ rest } \) momentum frame.
Figure 39.3: Visual fits to spectra showing the scattering of electrons from hydrogen at $\theta = 10^\circ$ for primary energies 4.88 to 17.65 GeV. The elastic peaks have been subtracted and radiative corrections applied. The cross sections are expressed in nanobarns per GeV per steradian. From Ref.[Q72]
this vanishing term in Eq.(39.13) and write $W_{\mu\nu}$ as the Fourier transform of the commutator of the current density at two displaced space-time points

$$W_{\mu\nu} = \frac{i}{2\pi} (\Omega E) \sum_i \int e^{i\mathbf{q} \cdot \mathbf{z}} d^4z \langle p| [J_\mu(z), J_\nu(0)]|p \rangle$$

(39.16)

Introduce states with covariant norm:

$$|p\rangle \equiv \sqrt{2E \Omega}|p\rangle$$

(39.17)

Equation (39.13) can then be rewritten

$$-\pi W_{\mu\nu} \equiv t_{\mu\nu} = -\frac{1}{4} \sum_i \int e^{i\mathbf{q} \cdot \mathbf{z}} d^4z \langle p|[J_\mu(z), J_\nu(0)]|p\rangle$$

(39.18)

This expression is evidently covariant; it forms the absorptive part of the amplitude for forward, virtual Compton scattering.

A combination of Eqs.(39.7, 39.9, 39.11) yields the general form of the cross section for the scattering of unpolarised (massless) electrons from an arbitrary, unpolarised hadronic target (Prob.39.3)

$$\frac{d^2\sigma}{d\Omega d\epsilon_2} = \sigma_M^\alpha \left[ W_2(\nu, q^2) + 2W_1(\nu, q^2) \tan^2 \theta/2 \right]$$

$$\sigma_M = \frac{\alpha^2 \cos^2 \theta/2}{4e_I^2 \sin^4 \theta/2}$$

(39.19)

Here $\sigma_M$ is the Mott cross section.

Bjorken Scaling. A qualitative overview of the SLAC data on deep inelastic electron scattering from the proton is shown in Fig.39.3 from Ref.[Q72]. On the basis of his analysis of various sum rules, Bjorken predicted, before the experiments, the following behavior of the structure functions in the deep inelastic regime (Ref.[Q73])

$$\frac{\nu}{m} W_2(\nu, q^2) \rightarrow F_2(x) \quad ; q^2 \rightarrow \infty, \; \nu \rightarrow \infty$$

$$2W_1(\nu, q^2) \rightarrow F_1(x)$$

(39.20)

Here the scaling variable is defined by

$$x \equiv q^2/2m\nu \equiv 1/\omega$$

(39.21)

These relations imply that the structure functions do not depend individually on $(\nu, q^2)$ but only on their ratio. The scaling behavior of the SLAC data is shown in Figs.39.4,39.5. The first illustrates the independence from $q^2$ at fixed $\omega = 1/x$; second shows the extracted structure functions $F_{1,2}(x)$.
Quark - Parton Model. The quark-parton model was developed by Feynman and Bjorken and Paschos to provide a framework for understanding the deep-inelastic scattering results (Refs.[Q74, Q75]). The basic idea is as follows:

1) The calculation of the structure functions is Lorentz invariant. Go to the C-M frame of the proton and incident electron with $p = -k_1$. Now let the proton move very fast with $|p| \to \infty$. This forms the infinite-momentum frame; it is illustrated in Fig 39.6.

2) The proper motion of the parton constituents of the hadron (proton) is slowed down by time dilation in this frame.

3) The partons are effectively frozen during the scattering process.

4) The interaction between the partons is then not important.

5) The electrons scatter incoherently from the constituents.

6) The electrons scatter from the constituents as if they are pointlike.

7) The parton constituents are quarks (charged) and gluons (neutral).

8) In the limit $q^2 \to \infty$, $\nu \to \infty$, the masses of the constituents can be neglected.\textsuperscript{98}

\textsuperscript{98}The norm of these states is $\langle p | p' \rangle = 2E(2\pi)^3 \delta^{(3)}(p - p')$; this is Lorentz invariant.

\textsuperscript{99}These authors use $W \equiv (1/m)W^{\text{text}}$ where $W^{\text{text}}$ are the structure functions used here.

\textsuperscript{99}From the SLAC data, the ratio of longitudinal to transverse cross section is given by $R \equiv \sigma_L/\sigma_T = 0.13 \pm 0.10$ where $W_L/W_T \equiv (1 + q^2/m^2)\sigma_L/(\sigma_L + \sigma_T)$.

\textsuperscript{99}It is assumed also that the transverse momentum of the parton before the collision can be neglected in comparison with $\sqrt{q^2}$, the transverse momentum imparted as $|p| \to \infty$.
Pauli model:

Let's view the protons in the C-M frame in which both electron and proton were with with large $(p \to \infty)$ momenta.

\[ \vec{P}_{\text{probe}} \]
\[ \vec{P}_{\text{electron}} \]

1) The proper motion of the proton constituents of the hadron is slowed down by time dilation in this frame.

2) The protons are effectively frozen during the scattering process.

- Interaction between protons is then not important.

- The electrons scatter independently from the constituents.

- The electron scatters from the constituents as if they were pointlike, massless objects.
In this frame the $i$-th parton carries a fraction $y_i$ of the incident four-momentum.

$$P_i = y_i P$$

The incident hadron is now just a collection of independent partons.

Let $f^i(y_i)dy_i$ be the number of quarks of type $i$ with four-momentum between $y_iP$ and $(y_i+dy_i)P$.

The total four-momentum is then:

$$P = \sum P_i = \sum_{i} \int_{y_i}^{y_i+dy_i} f^i(y_i)dy_i$$

$$= \frac{2}{\Gamma} \int_{y_i}^{y_i+dy_i} f^i(y_i)dy_i$$

By momentum sum rule:

$$\sum q = \frac{2}{\Gamma} \int_{y_i}^{y_i+dy_i} f^i(y_i)dy_i$$

momentum fraction

run by electrically neutral gluons
Since quark's spin $= \frac{1}{2}$ the reaction $Q_1 \Rightarrow Q_2$ is given by

$$W^i_\nu = \sigma \left( \frac{a^2}{2m_i^2} - 1 \right)$$

where $\nu_i = p_i \cdot q = \nu_i \cdot p \cdot q$.

Thus,

$$W^i_\nu (\nu, q^2) = \sigma \left( \frac{a^2}{2m_i^2} - 1 \right)$$

$$= \sigma \left( \frac{x_B}{m_i} - 1 \right) = \nu_i \sigma \left( \frac{x_i}{x_B} - x_B \right)$$

An incoherent sum over all types of quarks now gives the response tensor $W^i_\nu$ for the composite nuclei:

$$W^i_\nu = \sum_{\nu_i} \sigma \left( \frac{x_B}{m_i} - 1 \right) W^i_{\nu_i}$$

and for the nucleus's $W_2$ we get:

$$W_2 (\nu, q^2) = \sum_{\nu_i} \sigma \left( \frac{x_B}{m_i} - 1 \right) W^i_{\nu_i} (\nu_i) \eta_i \sigma \left( \frac{\eta_i}{x_B} - x_B \right)$$

This demonstrates explicit B-junction scaling, and we define

$$F_2 (x) = \sum_{i=1}^{N} \sigma_i x_i f_i (x_B)$$

$$x_B = \frac{a^2}{2 \rho \cdot q}$$
Sum rules:

Summing over all the contributing quarks must give the quark number of the proton:

Introduce \( f_u(x) \equiv u(x) \)

A quark in the proton

\[ f_u^p(x) = \bar{u}(x) \]

A quark in the proton

\[ f_d^p(x) = d(x) \]

A quark in the proton

\[ F_{\gamma^*p}(x) = \frac{4}{9} \left[ \bar{u}(x) + \bar{d}(x) \right] + \frac{1}{9} \left[ d(x) + \bar{d}(x) \right] + \frac{1}{9} \left[ s(x) + \bar{s}(x) \right] \]

+ \( \bar{u} \) (u, d, s, c, b, t)

\[ q_v(x) = q_u(x) + q_s(x) \]

Valence sea
\[ \int dx W_{1J}(x) = 2 \quad \int dx W_{2J}(x) = 1 \]

Axilov sum rule:
\[ \int_0^1 dx \left[ F_{2J}(x) - F_{1J}(x) \right] = 2 \]
(follows from current conservation). It is exact and receives no QCD corrections.

Bjorken sum rule:
\[ \int_0^1 dx \left( F_{1J}^P(x) + F_{2J}^P(x) \right) = 1 \]

Gottfried sum rule:
\[ \int_0^\infty dx \left( F_{2J}^{ep}(x) - F_{1J}^{nu}(x) \right) = \frac{1}{3} \]
(requires the assumption of axial-isospin-symmetric quarks: \( \bar{c}(x) = \bar{u}(x) \))

Nowak sum rule:
\[ \int_0^1 dx x q_i(x) < 1 \quad \text{(not canceled by gluons)} \]
3.6 Information on quark distributions from experiment

3.6.1 Information from electromagnetic structure functions

The difference of the proton and neutron structure functions given by (3.21) allows a determination of a valence combination \( x(u_n(x) - d_n(x)) \). Fig. 3.6 shows data on this quantity obtained from muons scattered off deuterium and hydrogen. The data certainly are consistent with an \( x^2 \) behaviour as \( x \to 0 \).

Going to large \( x \), the relative magnitudes of the \( u \) and \( d \) distributions can be estimated from the ratio of the neutron and proton structure functions. For, putting \( S(x) = 0 \) gives

\[
\frac{F_2^{\mu n}(x)}{F_2^{\mu p}(x)} = \frac{1 + 4d_n(x)/u_n(x)}{4 + d_n(x)/u_n(x)} \quad (3.59)
\]

and if \( SU(6) \) were exact, \( d_n(x)/u_n(x) = \frac{1}{3} \) and then \( F_2^{\mu n}(x)/F_2^{\mu p}(x) \to \frac{3}{4} \). The data shown in fig. 3.7 however are consistent with \( n/p \to \frac{1}{4} \) as \( x \to 1 \) linearly, i.e. consistent with \( d_n(x)/u_n(x) \sim (1 - x)^1 \) as \( x \to 1 \).

There is a great amount of information on quark functions from muon experiment survey, see the review by Slocum.
3.6 Information on quark distributions from experiment

There is a great amount of data on electromagnetic structure functions from muon experiments at CERN. For a comprehensive survey, see the review by Sloan, Smadja and Voss (1988).

3.6.2 Information from weak structure functions in neutrino scattering

From (3.34) we see that it is possible to extract a pure antiquark distribution from antineutrino cross-sections, e.g. by selecting events where $y$ is close to 1. Then $\sigma^\nu_{NN}$ picks out the combination $\bar{u} + \bar{d} + 2\bar{s}$. If we take a combination of $\sigma^\nu_{NN}$ and $\sigma^{\nu N}$ given by

$$\frac{1}{\sigma_0} \left[ \frac{d^2\sigma^{\nu N}}{dzdy} - (1 - y)^2 \frac{d^2\sigma^\nu_{NN}}{dzdy} \right] = x[\bar{u}(x) + \bar{d}(x) + 2\bar{s}(x)][1 - (1 - y)^4] + 2x[\bar{c}(x) - \bar{s}(x)][(1 - y)^2(1 - (1 - y)^2)]$$

then we get information on the non-charm sea by (a) assuming we are below threshold for exciting charm quarks, $\bar{c}(x) = 0$ or (b) combining information on $x\bar{s}(x)$ from dimuon production. The extra muon is attributed to the decay of a produced charm quark.