Now consider the general case:

\[ |\Omega \rangle = |q \bar{q} |q \bar{q} \rangle + |q \bar{q} \bar{q} |q \bar{q} \rangle + |q \bar{q} \bar{q} |q \rangle + \ldots \]

Diagonal representation:

\[ \begin{array}{c}
\begin{array}{c}
\text{Diagonal representation.}
\end{array}
\end{array} \]

Sure, \( H_{\text{qg}} \) is a one-body operator acting either on a single quark or antiquark at a time as we have:

\[ \langle H | H_{\text{qg}} | H \rangle = \]

\[ \begin{array}{c}
\begin{array}{c}
\text{Here what we have calculated.}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{This what we have calculated.}
\end{array}
\end{array} \]
You see that except for a single pair of quarks or antiquarks, all other particles are correlated between final and initial states. The SU(3)-flavor indices of this selected pair is correlated with these of $\Delta M_{ij}$ from Higgs and the wave functions, so the SU(3) structure is given by:

\[ Y_a \chi_{a_i a_j} \Delta M_{e j} \chi_{b_i b_j} \]

The product $\chi_{a_i a_j} \Delta M_{e j} \chi_{b_i b_j}$ has to transform as a $(2, 0)$ representation of $SU(3)$.

To describe the product, it uses wave vectors given by the matrices $M_{jb}$ and $M_{kc}$, with $M_{jk} = M_{kc}$.

The only possibility for this to happen is $\Delta M_{ij}$, the SU(3) structure of $TN^x_{ij}$ is

\[ T(N^x Y)_{ij} \sim M_{kc} \delta_{kb} M_{jb} \]
So even from a multiquark(outrigour) wave function we get the same structure for EML/Loxnow.

A similar analysis can be done in a Bayou octet. Just replace $M \rightarrow B$ everywhere, and we get:

\[
3 E_n + E_\Xi = 2(E_N + E_\Xi)
\]

Exp (GeV)
\[
3(1.115) + 1.189 = 4.534 \quad \sqrt{2(1.94 + 1.31)} = 4.5
\]

\% to 1%!
Combining flavor, spin, orbital and color degrees of freedom \( \Rightarrow \) wave functions and simple calculations.

Within a framework of non-Abelian O(3) (eventhough we have not written it explicitly yet) we have agreed that any hadronic state can be written as an expansion in Fock space labeled by degrees of freedom of quarks and anti-quarks.

Quarks/anti-quarks:

\[
|q\rangle = |k, \not{\gamma}, \gamma, \alpha\rangle
\]

Or \(|q\rangle \uparrow \uparrow \uparrow\) 3-momentum (\(\not{E}, \not{P}\))

\[
|\bar{q}\rangle = |\bar{k}, \not{\gamma}, \gamma, \alpha\rangle
\]

Or \(|\bar{q}\rangle \uparrow \uparrow \uparrow\) 3-momentum (\(\not{E}, \not{P}\))

\[
\begin{align*}
\frac{1}{2} & \quad \text{momentum fluence} & \text{velocity} & \text{color} \\
\pm 1 & \quad \text{spin} & \text{velocity} & \text{color} \\
\alpha & \quad \text{quark } & \text{anti-quark} & \text{quark } & \text{anti-quark}
\end{align*}
\]

We have not introduced color with its full glory but have agreed that quarks (anti-quarks) have to have color index \(n=1,2,3\) and that flavor index \(\lambda=1,2,3\) (not the same as SU(3) flavor)

\[
\begin{align*}
\text{SU}(3) & \quad \text{flavor} \\
\text{SU}(3) & \quad \text{color}
\end{align*}
\]
and that only color-singlet states, i.e. belonging to a SU(3) color-singlet representation are physical, are observed.

The states \(|q\rangle, |g\rangle\) and \(|\bar{q}\rangle\) are eigenstates of the Hamiltonian and all other operators it commutes with, i.e.:

\[ S^2 = (S_q^+ + S_{\bar{q}}^+ + S_g^+)^2 \rightarrow n = n_{\text{u}, n_{\text{d}}}
\]

\[ S \cdot P \rightarrow \text{helicity}
\]

\[ B \rightarrow \text{baryonic charge}
\]

\[ S^0 \rightarrow \text{baryon}
\]

\[ I_3 \rightarrow \text{third component of isospin}
\]

\[ I \rightarrow \text{isospin}
\]

\[ Q_c \rightarrow \text{color charge}
\]

\[ a = 1, 2, 3 \] for individual quark/anti-quark states,

\[ a = 1, 2, 3 \] for individual quark/anti-quark or glue states.
Thus in QCD we expect:

\[ |\Psi\rangle = \sum_{\mu_1 \mu_2 \mu_3 \mu_4}^{\text{hadron}} \Psi_{\mu_1 \mu_2 \mu_3 \mu_4}^{\text{hadron}} |\mu_1 \mu_2 \mu_3 \mu_4\rangle \] (x)

Now we will make a strong approximation:

We will assume that it is possible to change basis in the Faddeev space such that in the new basis expansion (x) looks like:

\[ |\text{Meson}\rangle \sim \Psi_Q |Q\rangle \]

\[ |\text{Baryon}\rangle \sim \Psi_{\text{aaa}} |\text{aab}\rangle \]

where we use capitalize letters for Quarks to make sure we distinguish

\[ |Q\rangle \text{ from } |\bar{Q}\rangle \]

or \[ |\bar{Q}\rangle \text{ from } |\bar{Q}\rangle \] \{ eigevectors of \(H_0\) \}

Some new basis still describing objects with quantum numbers of quarks but with some different \(H_0\)

where \(H_0\) is the free QCD Hamiltonian
The new quarks have still quantum numbers of 

\[ q \psi, |q\rangle = |\bar{q}\rangle \text{, } \gamma \psi, \bar{q} \text{ are not necessarily } \] 

solitons of massless, free bosons. We do not need to know what they are solutions of. All we need to know is how full \( H \) looks like in a new basis. Since we are assuming that any hadron state is given by a state with a fixed \( (\text{to the order}) \) number of particles \( (\text{Quarks}) \) we could simply forget about deriving \( H \) in this representation and simply try to parameterise it in terms of Quark degrees of freedom.

\[ \Rightarrow \text{ Nucleon physics experience is here very helpful.} \]

This is the Constituent Quark Model (CQM).
We are not going to construct the CQM Hamiltonian here (see for example: S. Capstick and N. Isgur Phys. Rev. D 34, 2809 (1986) and references therein).

We will postulate the wave functions \( \Psi_{ba} \) and \( \Psi_{aabb} \) and study some observables:

**CQM wave functions**

We have already constructed the flavor parts of these wave functions:

**Mesons:** \( Q \otimes \bar{Q} \)

\[ 3 \times 3 = 1 + 8 \]

**Baryons:** \( Q \otimes Q \otimes Q \)

\[ 3 \otimes 3 \otimes 3 = 1 + 8 + 8 + 10 \]

\( \Delta, M_1, M_2, S \)

And in the color to be singlet:

\[ 3 \times 3 = \begin{array}{c} 1 \\ 1(8) \end{array} + \psi(jb) \]

\[ \left| Q_i \bar{Q}_a \right> = \delta_{ia} \left| Q_j \bar{Q}_b \right> \delta_{jb} + \cdots \]

Here I used a now refer to color not \( u_d s \).
\[ 3 \times 3 \times 3 = \begin{bmatrix} 1 & \tau & \tau \\ \tau & \tau & \tau \\ \tau & \tau & \tau \end{bmatrix} \]

\[ |Q; \tau, \tau, \tau\rangle = \left( \varepsilon_{ijk} Q_i Q_j Q_k \right) e_{123} + \ldots \\
\psi_c (L, M, N) \]

Now we can odd spin since quarks and antiquarks belong to the fundamental representation of SU(2) → spin \( \frac{1}{2} \) objects we have:

\[ Q \otimes \bar{Q} \]

\[ 2 \otimes 2 = 1 \oplus 3 \]

For spin - 0, 1

\[ |s = 0\rangle = \chi_s^0 (x, \bar{x}) |Q, Q\rangle \]

\[ |s = 1\rangle \]

\[ |s = 1\rangle = \chi_s^1 (x, \bar{x}) |Q, Q\rangle \]

\[ \chi^0 (x, \bar{x}) = \frac{1}{2} \left( \eta_{\nu \nu} (x) \eta_{-\nu \nu} (\bar{x}) - \eta_{\alpha \nu} (x) \eta_{\nu \alpha} (\bar{x}) \right), \eta (x) = \delta_{\nu \alpha} \]

\[ \chi^1 (x, \bar{x}) = \begin{cases} 
\eta_{\nu \nu} (x) \eta_{\nu \nu} (\bar{x}) \\
\frac{1}{2} \left( \eta_{\nu \nu} (x) \eta_{-\nu \nu} (\bar{x}) + \eta_{-\nu \nu} (x) \eta_{\nu \nu} (\bar{x}) \right) \\
\eta_{-\nu \nu} (x) \eta_{-\nu \nu} (\bar{x}) 
\end{cases} \]
For bosons:

\[ \mathcal{V} \otimes \mathcal{V} \otimes \mathcal{V} \]

\[ 2 \otimes 2 \otimes 2 = (1 \otimes 3) \otimes 2 = 2 \oplus 2 \oplus 4 \]

Mixed symmetric w/ odd antisymmetric

\[ \text{Under-exchange of spins or} \]

\[ \text{quarks} #1 \text{ over} #2 \]

\[ |S = \frac{1}{2}, M_S, M_S\rangle = X_{M_S}^{S = \frac{1}{2}, M_S} |\Omega, \Omega, \Omega, \Omega\rangle \]

\[ X_{\frac{1}{2}}^{\frac{1}{2}} (\gamma, \gamma, \gamma, \gamma) = \frac{1}{\sqrt{6}} \left[ (\eta_{\frac{1}{2}}(\gamma_1) \eta_{\frac{1}{2}}(\gamma_2) + \eta_{-\frac{1}{2}}(\gamma_1) \eta_{-\frac{1}{2}}(\gamma_2)) \eta_{\frac{1}{2}}(\gamma_3) - 2 \eta_{\frac{1}{2}}(\gamma) \eta_{\frac{1}{2}}(\gamma) \eta_{\frac{1}{2}}(\gamma) \right] \]

\[ X_{-\frac{1}{2}}^{-\frac{1}{2}} (\gamma, \gamma, \gamma, \gamma) = -\frac{1}{\sqrt{6}} \left[ (\eta_{\frac{1}{2}}(\gamma_1) \eta_{\frac{1}{2}}(\gamma_2) + \eta_{-\frac{1}{2}}(\gamma_1) \eta_{-\frac{1}{2}}(\gamma_2)) \eta_{\frac{1}{2}}(\gamma_3) - 2 \eta_{\frac{1}{2}}(\gamma) \eta_{\frac{1}{2}}(\gamma) \eta_{\frac{1}{2}}(\gamma) \right] \]

\[ X_{\frac{1}{2}}^{\frac{1}{2}} (\gamma, \gamma, \gamma, \gamma) = \frac{1}{2} \left[ \eta_{\frac{1}{2}}(\gamma_1) \eta_{\frac{1}{2}}(\gamma_2) - \eta_{-\frac{1}{2}}(\gamma_1) \eta_{-\frac{1}{2}}(\gamma_2) \right] \eta_{\frac{1}{2}}(\gamma_3) \]

\[ X_{-\frac{1}{2}}^{-\frac{1}{2}} (\gamma, \gamma, \gamma, \gamma) = \frac{1}{2} \left[ \eta_{\frac{1}{2}}(\gamma_1) \eta_{\frac{1}{2}}(\gamma_2) - \eta_{-\frac{1}{2}}(\gamma_1) \eta_{-\frac{1}{2}}(\gamma_2) \right] \eta_{\frac{1}{2}}(\gamma_3) \]
$\hat{\chi}_{s=3/2, l} (\lambda, \lambda_3) = \psi_{\psi_1} (\lambda) \psi_{\psi_2} (\lambda_3) \psi_{\psi_3} (\lambda_3)$

$\chi_{s=3/2, l} = \frac{1}{\sqrt{3}} (\psi_{\psi_1} (\lambda) \psi_{\psi_2} (\lambda_3) \psi_{\psi_3} (\lambda_3) + \psi_{\psi_2} (\lambda) \psi_{\psi_1} (\lambda_3) \psi_{\psi_3} (\lambda_3) + \psi_{\psi_3} (\lambda) \psi_{\psi_2} (\lambda_3) \psi_{\psi_1} (\lambda_3))$

$\chi_{s=3/2, l} = \chi_{s=3/2, l}$

The flavor wave function: $\phi$

Mesons:

$|\tilde{1}_{13}\rangle = \phi_{\tilde{1}_{13}} (i, a) |Q; \bar{Q}_a\rangle$

Flavor indices

$i = u, d, s$ (no 1, 2, 3)
$a = u, d, s$ (no 1, 2, 3)

$|\tilde{1}_{8}\rangle, a\rangle = \phi_{\tilde{1}_{8}} (i, a) |Q; \bar{Q}_a\rangle$

are given on pages 123 - 124 (Mesons)

and in 115 - 116 (Bayous)
In this notation:

\[ \phi^{[1]}(i,a) = \frac{1}{\sqrt{3}} (\phi_u(i)\phi_u(a) + \phi_d(i)\phi_d(a) + \phi_s(i)\phi_s(a)) \]

with \( \phi_q(i) = \delta_{qi} \).

\[ \phi^{[8]}_g(i,a) = \frac{1}{\sqrt{6}} (\phi_u(i)\phi_u(a) + \phi_d(i)\phi_d(a) - 2\phi_s(i)\phi_s(a)) \]

etc.

For baryons:

\[ R \otimes Q \otimes Q \]

\[ S \otimes S \otimes S = 1 + 8 + \bar{8} + 10 \]

\[ A, M_3, M_A, S \]

\[ |C_{11}\rangle = |\phi^{[1]}_{C_{11}}(i,j,k)\rangle/ \sqrt{8} \otimes \otimes \otimes \]

\[ |C_{81}\rangle, A, M_3 \rangle = |\phi^{[8]}_A, M_3(i,j,k)\rangle/ \sqrt{8} \otimes \otimes \otimes \]

\[ |C_{82}\rangle, A, M_A \rangle = |\phi^{[8]}_A, M_A(i,j,l)\rangle/ \sqrt{8} \otimes \otimes \otimes \]

\[ |C_{10}\rangle, A \rangle = |\phi^{[10]}_A(i,j,k)\rangle/ \sqrt{8} \otimes \otimes \otimes \]
Some examples

\[ \Phi^E_{M^S, M^S}(i; j, k) = \frac{1}{\sqrt{6}} \left[ \left( \phi_u(i) \phi_d(j) + \phi_d(i) \phi_u(j) \right) \phi_u(k) - 2 \phi_u(i) \phi(j) \phi_u(k) \right] \]

\[ \Phi^E_{M^D, M^D}(i; j, k) = -\frac{1}{\sqrt{6}} \left[ \left( \phi_u(i) \phi_d(j) + \phi_d(i) \phi_u(j) \right) \phi_d(k) - 2 \phi_d(i) \phi(j) \phi_d(k) \right] \]

\[ \Phi^E_{M^D, M^U}(i; j, k) = \frac{1}{\sqrt{6}} \left[ \phi_u(i) \phi_d(j) - \phi_d(i) \phi_u(j) \right] \phi_d(k) \]

\[ \Phi^E_{M^U, M^U}(i; j, k) = \frac{1}{\sqrt{6}} \left[ \phi_u(i) \phi_d(j) - \phi_d(i) \phi_u(j) \right] \phi_u(k) \]

etc. following pages 125-126 in the notes.

When constructing boxon wave functions multiplying \( |\psi_{Q^Q}\rangle\) state we have to make sure it is fully antisymmetric. One can show that out of the spin and flavor wave functions listed above the following combinations have well defined symmetry properties under exchange of any pair of quarks, not just #1 and #2:
\[ \sum_{s=\frac{3}{2},[1]}^{S} (\gamma_{i},\gamma_{j},\gamma_{k}) = \chi (\gamma_{i},\gamma_{j},\gamma_{k}) \phi (i,j,k) \Rightarrow \text{fully symmetric} \]

\[ \sum_{s=\frac{1}{2},[1]}^{S} (\gamma_{i},\gamma_{j},\gamma_{k}) = \frac{1}{\sqrt{2}} \left[ \chi (\gamma_{i},\gamma_{j},\gamma_{k}) \phi (i,j,k) + \chi (\gamma_{i},\gamma_{j},\gamma_{k}) \phi (i,j,k) \right] \Rightarrow \text{fully symmetric} \]

\[ \sum_{s=\frac{3}{2},[1]}^{S} (\gamma_{i},\gamma_{j},\gamma_{k}) = \chi (\gamma_{i},\gamma_{j},\gamma_{k}) \phi (i,j,k) \Rightarrow \text{fully antisymmetric} \]

\[ \sum_{s=\frac{1}{2},[1]}^{S} (\gamma_{i},\gamma_{j},\gamma_{k}) = \frac{1}{\sqrt{2}} \left[ \chi (\gamma_{i},\gamma_{j},\gamma_{k}) \phi (i,j,k) - \chi (\gamma_{i},\gamma_{j},\gamma_{k}) \phi (i,j,k) \right] \Rightarrow \text{fully antisymmetric} \]

Furthermore, here is still a bunch of more functions of mixed symmetry (i.e., symmetric or antisymmetric with respect to groups #1 and #2 but not all three)

For example:

\[ \sum_{s=\frac{3}{2},[1]}^{S} \chi (\gamma_{i},\gamma_{j},\gamma_{k}) = \chi (\gamma_{i},\gamma_{j},\gamma_{k}) \phi (i,j,k) \Rightarrow \text{symmetric under exchange of } \\
#1 \text{ and } #2 \text{ i.e.} \\
(\gamma_{i},\gamma_{j}) \leftrightarrow (\gamma_{i},\gamma_{j}) \]
It is natural to assume that for low lying states the constituent quarks are in the lowest orbital configuration, which is typically fully spherically symmetric. Thus, for all quarks, \( s^2 \)-waves. For the baryons, the orbital part of the wave function is:

\[
\Psi(E_1, E_2, E_3) |(Q(\bar{q}_1) Q(\bar{q}_2) Q(\bar{q}_3))\rangle
\]

It is thus fully symmetric under permutations of any pair of quarks. Since for 3 quarks color wave function is antisymmetric (\( E_{ij} \)) we need a fully symmetric spin-flavor part to satisfy Pauli principle. Here we assume again (CD was \( SU(3) \)-flavor invariant). If not, we can treat the \( s \)-quark as a different then a and b and we only need to require the 3Q wave function which has one \( s \)-quark to be antisymmetric with respect to the exchange.
of the quark numbers of the two wustrowye quarks. (154)

But let's stick to the SU(3) symmetric case.

Furthermore we can always use our basis we want
out this is a convenient one.

The wave functions of the lowest baryon octet are
thus given by:

\[ |8_{1/2}^{\pm} \rangle = \sum_{ijkl} \chi^{8}_{1/2}(i_1,i_2,i_3,i_4) \mathcal{Y}(k_1,k_2) \mathcal{Y}(a,b,c) \mathcal{Y}(k_3,k_4) \] 

where \( a, b, c \) are color indices.

\[ [\ell N] = \frac{\ell^2 E_1^2 E_2^2}{\ell^2 N_1^2 N_2^2} \]

and from total angular momentum we know

that

\[ \mathcal{Y}(i_1,i_2,i_3) \sim (2i)^3 \delta \left( \ell_1 - \ell_2 - \ell_3 \right) \mathcal{Y}(i_1,i_2,i_3) \]

where the reduced wave function \( \psi_{\text{red}} \) depends only on
relative momenta of the four \( k_i - k_j \).
Similarly the decuplet wave functions \( (\Delta \Sigma^* \Xi^* \Omega^*) \) we obtained by replacing \( X^{s=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}} \) we see that \( SU(3) \) + Pauli principle + quarks having Spin-\( \frac{1}{2} \) leads to a lowest octet of \( S=\frac{1}{2} \) baryons
and decuplet of \( S=\frac{3}{2} \) baryons.

For mesons we also need to insist on symmetric wave function since Pauli blocking is not appreciable \( \xi \neq \bar{\xi} \).
Nevertheless we still anticipate the ground states, lowest lying states have the \( \xi \bar{\xi} \) pair in the fully symmetric \( S=0 \) wave orbital state.

The full \( \psi \) functions for meson octet thus is:

\[ |\Xi_8(\vec{s}=0)\rangle \propto \sum [\Phi_{\psi} X^{\frac{1}{2}}(\vec{s}, \vec{c}) \Phi(\vec{i}, \vec{0}) \Phi_{\Xi^*}(\vec{c})_{\Xi} \Phi_{\Omega}(\vec{c})_{\Omega} \right] \]

\[ [\Phi_{\psi} = \frac{\delta^{ij}_{\vec{s}} \delta_{\vec{i} \vec{c}}}{(16\pi^3)^{1/2}} \quad \text{and} \quad \Phi_{\Xi^*}(\vec{c})_{\Xi} = (16\pi^3)^{1/2} \psi_{\Xi^*}(\vec{c})_{\Xi} \psi_{\Xi}(\vec{c})_{\Xi}] \]
We can also have a $S^1$ octet (replace $\chi_0^0$ by $\chi_\pm^0$).

Here are:

\[ \pi^- \rightarrow \rho^0 \quad \text{mass} \sim 770 \text{ MeV} \]

\[ K^\pm, \eta' \rightarrow K^{\pm, 0} \quad \text{mass} \sim 890 \text{ MeV} \]

\[ \eta^\pm, \eta' \rightarrow \omega, \phi \quad \text{mass} \sim 782\text{ MeV, and 1020 MeV} \]
<table>
<thead>
<tr>
<th>Particle</th>
<th>I</th>
<th>Y</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^+$</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$n^0$</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\Lambda^0$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>$K^+$</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$K^0$</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\eta^0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$K^0$</td>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$K^-$</td>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 11.1. Quantum numbers of the pseudoscalar meson and the spin-$\frac{1}{2}$ baryon octets.

Any particles in the same isospin multiplet have the same $Y$, $S$, and $N$, as expected.
The fully symmetric 10-dim representation is

\[ \Phi_s \]

\[ \Delta^{++} \]  \[ \frac{1}{6} (dauu + udud + uudd) \]

\[ \Delta^+ \]  \[ \frac{1}{3} (uudd + udud + duud) \]

\[ \Delta^0 \]  \[ ddud \]

\[ \Delta^- \]  \[ ddud \]

\[ \Sigma^{++} \]  \[ \frac{1}{3} (suuu + usuu + ussu) \]

\[ \Sigma^{0*} \]  \[ \frac{1}{6} (sud + sdu + usd + dus + uds + dus) \]

\[ \Sigma^{-*} \]  \[ \frac{1}{3} (sddd + dsd + ddds) \]

\[ \Xi^{0*} \]  \[ \frac{1}{3} (suuu + usuu + ussu) \]

\[ \Xi^{-*} \]  \[ \frac{1}{3} (sddd + dsd + ddds) \]

\[ \Omega^- \]  \[ sss \]
Notice that the antisymmetric state exists because the three objects each have three labels available (uds). In the SU(2) example the restriction to two labels meant that only symmetric and (two) mixed symmetry states could be found.

### Table 3.7

<table>
<thead>
<tr>
<th>( \phi_{M,S} )</th>
<th>( \phi_{M,A} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} [(u + d)u - 2uud] \end{align*} )</td>
</tr>
<tr>
<td>( N )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} [(u + d)d - 2udd] \end{align*} )</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} [(s + u)s - 2uss] \end{align*} )</td>
</tr>
<tr>
<td>( \Sigma^0 )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} \left[ \frac{(u + d)u}{\sqrt{2}} + \frac{(d + u)u}{\sqrt{2}} \right] - 2 \left( \frac{(u + d)u}{\sqrt{2}} \right) \end{align*} )</td>
</tr>
<tr>
<td>( \Lambda^0 )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} \left[ \frac{(s - u)u}{\sqrt{2}} + \frac{(u - d)u}{\sqrt{2}} \right] \end{align*} )</td>
</tr>
<tr>
<td>( \Xi^+ )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} [(s + u)s - 2uss] \end{align*} )</td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} [(u + d)u - 2uud] \end{align*} )</td>
</tr>
<tr>
<td>( \phi_A )</td>
<td>( \begin{align*} \frac{1}{\sqrt{6}} [s(u - d) + (us - dsu) + (du - ud)s] \end{align*} )</td>
</tr>
</tbody>
</table>

*Note:* \( u \rightarrow d \) relates \( P \) and \( N \); \( d \leftrightarrow s \) relates \( P \leftrightarrow \Sigma^+ \); \( u \leftrightarrow d \) and \( d \leftrightarrow s \) relates \( N \leftrightarrow \Xi^- \). Note that in the \( \Sigma^0 \) the ud quarks have \( I = 1 \) while in the \( \Lambda^0 \) they have \( I = 0 \). The locations in the octet hexagon are shown in Fig. 2.2. The antisymmetric singlet state, \( \phi_A \), is also shown.

Identifying uds with the three flavours of quark, the resulting states are identified with the baryons. You can refer back to Table 3.2 showing baryon states and the S, M, A, notation should now be clear.