Properties of the N-N potential

Since we agree not to try to derive N-N force
from QCD (at least not yet!) we will have to determine
its properties by fitting a given model to N-N data.

\[ \Rightarrow \text{N-N scattering, (including the only NN bound state) } \]

\[ \Rightarrow \text{leptons} \]

Remember, these are QM

equations \( [V, P] \neq 0 \)

Symmetry properties:

The variables are \( \mathbf{r}_1, \mathbf{p}_1, \mathbf{r}_2, \mathbf{p}_2, \mathbf{r}_3, \mathbf{p}_3 \).

1) Hermitian \( \Rightarrow \) elastic scattering only.

2) Invariant under

\[ \text{particle exchange} \Rightarrow V(1, 2) = V(2, 1) \]

3) Invariant under

\[ \text{translations} \Rightarrow V(\mathbf{r}_1, \mathbf{r}_2) = V(\mathbf{r}_1, \mathbf{r}_2) \]

4) Invariant under

\[ \text{Galilean boosts} \Rightarrow \mathbf{U}(\mathbf{p}_1, \mathbf{p}_2) = \mathbf{U}(\mathbf{p}_1, \mathbf{p}_2) \]

5) Invariant under

\[ \text{parity} \Rightarrow \mathbf{r} \rightarrow -\mathbf{r}, \mathbf{p} \rightarrow -\mathbf{p}, \mathbf{q} \rightarrow \mathbf{q} \]

\[ [V, J] = 0 \]

6) Invariant under

\[ \text{time reversal} \Rightarrow \mathbf{r} \rightarrow \mathbf{r}, \mathbf{p} \rightarrow -\mathbf{p}, \mathbf{q} \rightarrow -\mathbf{q} \]

7) Invariant under rotations

\( \text{(generated by } J \equiv \mathbf{L} + S) \Rightarrow [V, J] = 0 \)
Now it follows that the most general isospin dependence is given by

\[ \mathbf{V} = \mathbf{V} + \mathbf{V}_L \mathbf{\tau}_1 \cdot \mathbf{\tau}_2 \]  

(symmetric under \( \mathbf{\tau}_1, \mathbf{\tau}_2 \) exchange)

From 0 \( \Rightarrow \) 7 we get:

\[ \mathbf{\tilde{V}}(1,2) = \mathbf{\tilde{V}}(\tilde{\rho}, \tilde{\rho}, \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_3, \tilde{\tau}_4) \]

\[ \mathbf{\tilde{V}}(\tilde{\rho}, -\mathbf{\tilde{\rho}}, \mathbf{\tau}_2, \mathbf{\tau}_1, \mathbf{\tau}_3, \mathbf{\tau}_4) \]

\[ \mathbf{\tilde{V}}(\tilde{\rho}, -\mathbf{\tilde{\rho}}, \mathbf{\tau}_2, \mathbf{\sigma}_2, \mathbf{\tau}_1, \mathbf{\tau}_4) \]

\[ \mathbf{\tilde{V}}(\tilde{\rho}, -\mathbf{\tilde{\rho}}, -\mathbf{\sigma}_2, -\mathbf{\tau}_2, \mathbf{\tau}_1, \mathbf{\tau}_4) \]

\[ \mathbf{\tilde{V}}(\tilde{\rho}, -\mathbf{\tilde{\rho}}, -\mathbf{\sigma}_2, -\mathbf{\tau}_2, \mathbf{\tau}_1, \mathbf{\tau}_4) \]

The most general form consistent with these assumptions is:

\[ \mathbf{V}(1,2) = \mathbf{U}_d + \mathbf{U}_p \mathbf{\tilde{Z}} \cdot \mathbf{\tilde{S}} + \mathbf{U}_p \mathbf{\tilde{\sigma}}_2 \cdot \mathbf{\tilde{\tau}_2} + \mathbf{U}_o (\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2) (\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2) \]

\[ + \mathbf{U}_e \left[ (\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2)(\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2) + (\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2)(\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2) \right] + \mathbf{U}_e (\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2) (\mathbf{\tilde{\rho}} \mathbf{\tilde{\sigma}}_2) \]

where \( \mathbf{\tilde{Z}} = \mathbf{\tilde{Z}} \cdot \mathbf{\tilde{S}} \) and \( \mathbf{\tilde{S}} = \mathbf{\tilde{S}}_1 + \mathbf{\tilde{S}}_2 = \frac{1}{2} (\mathbf{\tilde{\sigma}}_1 + \mathbf{\tilde{\sigma}}_2) \)

All \( \mathbf{U} \)'s are real, rotational scalars that you depend on \( \mathbf{\tilde{\rho}}, \mathbf{\tilde{\rho}}, \mathbf{\tilde{\sigma}}_2 \), and \( \mathbf{\tilde{Z}} \), all \( \mathbf{U} \)'s have isospin dependence as in (x)
\[ L^2 = E_j \mu_j \mu_j \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \epsilon_{\mu_1' \mu_2' \mu_3' \mu_4'} \epsilon_{\mu_1'' \mu_2'' \mu_3'' \mu_4''} \epsilon_{\mu_1''' \mu_2''' \mu_3''' \mu_4'''}, \]
\[ = \delta_{\mu_2}^2 \delta_{\mu_3}^2 + \delta_{\mu_1}^2 \delta_{\mu_4}^2 - (\delta_{\mu_1}^2 \delta_{\mu_2}^2 \delta_{\mu_3}^2 \delta_{\mu_4}^2) - (\delta_{\mu_1}^2 \delta_{\mu_2}^2 \delta_{\mu_3}^2 \delta_{\mu_4}^2) = \]
\[ \Rightarrow \text{Thus } V \text{ is also a function of } \delta \text{ required by } T \text{-invariance.} \]
\[ \delta \Rightarrow \text{there are } 6 \times 2 \text{ independent functions in } V \]
\[ \text{for isotropic.} \]

However if we use elastic N-N scattering to determine \( V \) all momenta dependence of \( U \)'s is redundant.*

* This follows because \( p^2 \) dependence of \( V \) in elastic scattering is fixed by \( k^2 = k_f^2 = 2pE \)

Thus one can only write a potential in a form:

\[ V(\gamma^2) = V_\alpha + V_\mu \sigma^2 + V_\sigma \sigma_1 \sigma_2 + V_\delta \left( \frac{2 \sigma_1}{\sigma_2} \right) \]
\[ + V_\epsilon \left[ \left( \frac{C_1}{2} \sigma_1 \right) \left( C_2 \sigma_2 \right) + \left( C_2 \sigma_2 \right) \left( C_1 \sigma_1 \right) \right] \]
\[ V_{\zeta} = V_{\zeta} (\frac{\gamma^2}{2}) \]

Of course in principle all realistic nuclear calculations of your NN system one would need the full potential in terms of \( U \)'s.

The 5 independent \( V \)'s can in principle be determined from 5 independent functions in \( f_\mu(\theta, \rho) \). However, since \( V \)'s depend on \( \gamma^2 \) one would have to know all channels \( 2^{3+1} \) \((J=200)\) which cannot be done,
The most general form that can be determined seems to need still all energies.

Uniquely (in principle) from $N-N$ data contours of $\mathcal{C}$:

$$V(1,2) = V_x + V_p \bar{z} \cdot \bar{z} + V_\perp \bar{e}_1 \cdot \bar{e}_2 + V_\perp (\bar{e}_1 \cdot \bar{e}_2) (\bar{z} \cdot \bar{z})$$

$$V_r = V_r (|r|)$$

(If there was a dependence on $|r|^2$ then again there would be independent $V_r$'s for each channel.)

So let's be optimistic and assume that (1) is (4) is sufficient to describe $(N-N)$ in warhead.

What can we say about $V_x$'s?

1. Attraction:
   - Nuclei are bound $\Rightarrow$ force must generally be attractive
   - Deuteron exists ($J=1$, even parity) has one bound state mostly $3S$, so total attractive
     inside target $S=1$, $T=0$ state.
   - pp scattering has $T=1$, at low energies $L=0$

   $\Rightarrow$ So only Coulomb-nuclear interference it follows that force is attractive.
(3) Spin dependent

For low energy up to $\sim 10\text{MeV}$ in c.o.m. mass system, after $L=1$ scattering (P-wave), contributions:

$$\sqrt{1 - \frac{M}{\sqrt{2\text{GeV}}}} \approx 0.7$$

Thus, the scattering length and effective range

$$\langle r \rangle = \frac{1}{\langle a \rangle} = \frac{1}{2.14} = 0.47\text{ fm}$$

The data is

$$\langle a \rangle = -23.714 \pm 0.015\text{ fm}$$

$$\langle r \rangle = 2.73 \pm 0.05\text{ fm}$$

$$\langle a \rangle = +5.425 \pm 0.004\text{ fm}$$

$$\langle r \rangle = 1.44 \pm 0.08\text{ fm}$$

Scattering length is related to the existence of bound state $a + E = 0$.

$\Rightarrow$ simple almost a bound state

$\Rightarrow$ finite $\Rightarrow$ bound state of nucleons.
Fig. I.16 Neutron-proton scattering cross-section at low energy. (Following Blatt, J. M., and Weisskopf, V. F., Theoretical Nuclear Physics, Wiley, New York, 1952, p 70.)
\[ V(\vec{r}, \vec{v}) = V_0 + \vec{v} \cdot \vec{A} + \frac{1}{2} \vec{v} \cdot \vec{B} \cdot \vec{v} \]

The exact form of \( V \) is shown to be of the form 

\[ V(\vec{r}, \vec{v}) = V_0 + \vec{v} \cdot \vec{A} + \frac{1}{2} \vec{v} \cdot \vec{B} \cdot \vec{v} \]

where \( V_0 \) is a constant, \( \vec{A} \) is a constant vector, and \( \vec{B} \) is a constant matrix. The vector \( \vec{v} \) represents the velocity of the particle, and \( \vec{r} \) represents the position of the particle.

The potential energy function is a scalar function of the position \( \vec{r} \) and the velocity \( \vec{v} \). It is often used in the study of motion and mechanics.