Reading: Harris, 4.3-4.4, Chapter 5; Eisberg & Resnick, Chapter 5

1. Consider a proton confined to a region of typical nuclear dimensions, about 5 fm. Use the uncertainty principle to estimate its minimum possible kinetic energy in MeV, assuming that it moves in only one dimension.

2. An unusually long-lived unstable atomic state has a lifetime of 1 ms.
   (a) Roughly what is the minimum uncertainty in its energy?
   (b) Assuming that the photon emitted when this state decays is visible (λ ≈ 550 nm), what are the uncertainty and fractional uncertainty in its wavelength?

3. The wave function for a particle in a rigid box of width $a$ is $\psi(x) = A \sin(n \pi x/a)$ for $0 < x < a$ and $\psi(x) = 0$ elsewhere. Show that the requirement that $\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$ leads to $A = \sqrt{2/a}$, independent of $n$. [Hint: Use the identity for $\sin^2 \theta = (1 - \cos (2 \theta)) / 2$.]

4. A particle has a wave function $\psi(x) = A e^{-|x|/b}$, where $A$ and $b$ are positive real constants.
   (a) Find an expression for $A$ such that the wave function is properly normalized.
   (b) Calculate the probability of finding the particle in the region $-2b \leq x \leq 2b$.

5. The $n = 4$ state of a particle in a rigid box of width $a$ is $\psi(x) = \sqrt{2/a} \sin(4 \pi x/a)$ for $-a/2 < x < a/2$ and $\psi(x) = 0$ elsewhere. [Note that the coordinate origin here is shifted relative to our discussion in lecture.]
   (a) Calculate the expectation values $<x>$ and $<x^2>$.
   (b) Calculate the expectation values $<p>$ and $<p^2>$.
   (c) Compute the product $\delta x \cdot \delta p$ where $\delta x$ and $\delta p$ are the uncertainties in position and momentum, and for an observable $q$ the uncertainty is defined as
      $$\delta q \equiv \left( <q - <q>^2> \right)^{1/2} = \left( <q^2> - <q>^2 > \right)^{1/2}.$$

6. Consider a wave function of the form $\psi(x) = F u_1(x) + G u_3(x)$, where $u_1(x)$ and $u_3(x)$ are the wave functions for the $n = 1$ and $n = 3$ states for a particle in a rigid box of width $a$.
   (a) Show that $|F|^2 + |G|^2 = 1$.
   (b) Calculate the expectation value $<K>$ for the kinetic energy for this wave function.
7. Consider the following wave function:

\[
\Psi(x, t) = \frac{1}{\sqrt{2}} \left( u_2(x) e^{-iE_2t/\hbar} + u_3(x) e^{-iE_3t/\hbar} \right),
\]

where \( u_2(x) \) and \( u_3(x) \) are the wave functions for the \( n = 2 \) and \( n = 3 \) states of the infinite square well of width \( a \) where \( U(x) = 0 \) for \( 0 < x < a \). Calculate the probability of finding the particle in the interval \( 0 < x < a/2 \) as a function of time. What is the period of oscillation of the probability?