Reading: Nelson, Chapter 5

1. Nelson, 5.4

2. Nelson, 5.8

3. (a) *E. coli* swims at about 20 $\mu$m/s by rotating a bundle of helical flagella. If the motors were to turn 10 times faster than normal, what would their swimming speed be? If their fluid environment was made 10 times more viscous, but the motors were to turn at the same rate, what would the swimming speed be? How does the power output of the motor change in these two hypothetical situations?

(b) Two micron-sized spheres, one made of silver and the other gold, sediment (that is, fall under gravity) in a viscous fluid. The silver sphere has twice the radius of the gold one. Which sediments faster?

(c) The left ventricle of the human heart expels about 50 cc of blood per heartbeat. Assuming a pulse rate of 1 heartbeat per second and a diameter of the aorta of about 2 cm, what is the mean velocity of blood in the aorta? What is the Reynolds number?

4. (a) The basis of propulsion by microorganisms with helical filaments is the difference in the viscous drag coefficient for motion along and perpendicular to the axis of a thin rod, given by $\gamma_\parallel$ and $\gamma_\perp$, respectively. Draw a clearly labeled diagram of a helical filament, oriented along the z-axis, that is rotated in the CCW direction as viewed from the positive z-direction. By considering the forces on four consecutive segments the filament (two segments lying into the page and two segments lying out of the page), demonstrate that net propulsion is generated along the z-axis, and determine its direction.

(b) Determine at what speed cilia must move to make stirring the environment more advantageous than waiting for diffusion to bring nutrient molecules to the cell. Take the typical length of a cilium to be $d \sim 1\mu$m, and $D \sim 100 - 1000\mu^2s^{-1}$. What dimensionless constant can you construct that determines the relative strength of mixing versus diffusion in a fluid environment?
5. DNA Twist Experiment: A molecule of DNA (length $L$) is attached at both ends to beads via the two strands of the double-helix, so it is torsionally constrained. Assume that you have twisted one bead relative to the other by a total angle $\phi$, and you keep the molecule under tension to prevent it from supercoiling. Now you attach a small bead of radius $R$ to both the strands and break one of the strands below the tiny bead, so that the molecule can unwind by rotating around the single bond swivel in the unbroken strand as shown in the figure on the left. To understand this experiment, we need to consider the drag forces on the rotating bead.

![DNA twist elasticity experiment](image)

**Left:** DNA twist elasticity experiment. A DNA molecule is constrained rotationally by two beads at both ends. A third bead is used to monitor the unwinding when a nick is introduced. **Right:** Experimental data from DNA twisting experiment. Rotation rate as a function of bead size. (Both figures adapted from Z. Bryant et al., Nature 424:338, 2003, by Phillips, Kondev, Theriot and Orme in Physical Biology of the Cell.

(a) Show that the drag coefficient for a sphere of radius $R$ rotating at angular velocity $\omega$ is given by $K\eta \omega R^3$, where $K$ is a numerical factor. For rotational motion the drag coefficient relates the angular velocity to the frictional torque. Use the approach that led to the Stokes formula in lecture. The idea is to say that the size of the viscous stresses is $\nu/R = \omega$, the area over which they act is $R^2$. Combine these two to get the force, and then the torque.

(b) When DNA is highly twisted, the torque as a function of twist angle $\phi$ is constant at a value of roughly $\tau = 33 \text{ pN} \cdot \text{nm}$. Use the data shown in the figure on the right to determine the numerical factor $K$ in the drag coefficient, in multiples of $\pi$.

(c) Write an expression for the angular velocity of the small bead in terms of the viscosity $\eta$, the radius of the small bead $R$, the constant of torsional stiffness $C$ and the length and number of whole extra turns in the DNA molecule, $N$, where $N = \phi/2\pi$. Estimate the
angular velocity of a $R = 400 \, nm$ bead if the length of DNA is 14.8 $kbp$ and it has been twisted by 50 extra turns. The torsional stiffness of DNA is approximately $400 \, pN \cdot nm^2$. Express your answer in revolutions per second.

(d) Use the results obtained in (a) and (b) to determine the rotational drag coefficient of a sphere rotating around its center of mass. Use the fact that the rotational motion of the bead in the experiment can be decomposed into translational motion of the center of mass and rotation around the center of mass.