Chaos Observed with a Buckled Beam

Goal: Study of the onset of chaos in a simple, mechanical system. Acquaintance with strain gauges as mechanical transducers.

Introduction

The study of chaos is a modern development in physics. It deals with the seemingly random behavior of some phenomena in nature. It lead to the realization that some physical systems, although perfectly deterministic in theory, exhibit a very complicated temporal development. This complex behavior is also found in the math underlying these phenomena, thus the development of computers played an important role in the study of chaos. Chaos can be found in many areas of physics, including nuclear physics [BOH88].

The motion of an oscillator (with one degree of freedom, $x$) on which a periodic force acts with frequency $\omega$, is described by a second-order differential equation

$$\ddot{x} + \gamma \dot{x} + g(x) = f \cos \omega t . \hspace{1cm} (1)$$

The second term describes damping. The restoring force $g(x)$ can be taken as a polynomial with odd powers of $x$. For the case where $g(x) = \alpha x$, eq.1 describes a harmonic oscillator. In most cases, eq.1 has a simple, periodic solution, $x(t)$, with a single frequency, that is, with a Fourier spectrum with a single line. For some special choices of the coefficients in eq.1, however, $x(t)$ becomes very complicated. The Fourier spectrum becomes continuous, and the behavior of $x(t)$ is difficult to predict, i.e., a small change of $x$ at $t=0$ grows exponentially in time (expressed by the "Liapunov exponent"). Under these circumstances the system is said to be "chaotic".

The position $x$ and the velocity $v=dx/dt$ fully describe the system at a given instant in time. The two-dimensional space $(x,v)$ is called "phase space". As a function of time the system traces out a curve in phase space. When $(x,v)$ is read only once during every period of the driving force, and the results plotted as points in phase space, one obtains what is called a "Poincaré section".

A mechanical realization of eq.1 is the buckling steel beam clamped at one end. Magnets are used to generate a nonlinear restoring force. A detailed discussion of the forces acting on such a system can be found in Refs.[MOO79,MOO80] (a copy of the second reference is attached).

There are a number of possible observables when investigating chaotic behavior experimentally. An excellent summary of experimental methods can be found in Ref.[MOO87].

Experiment

The beam is a piece of 15 mil steel shim stock (Fig.1). Laminations on both sides of 1 mil steel and double-sided scotch tape provide damping. The position of the buckled beam is translated
into an electrical signal by strain gauges which are attached on both sides of the beam near the clamped end. They are connected in half-bridge configuration to the P-3500 strain gauge readout (Fig.1). We are using CEA-13-062UW-350 gauges from Micro-Measurements Corp.; they have a gauge factor of 2.160±0.5%. In setting up the P-3500, follow step by step instructions. The beam is mounted in a lucite holder. Adjustable magnets give freedom in varying the potential.

The setup is mounted on the stage of a vibrator which provides linear motion. The displacement from equilibrium is given by the current supplied by a distortion-free amplifier. A blower is used to cool the coil inside the vibrator.

The instantaneous velocity of the beam is obtained by processing of the position signal with a low-pass filter and a differentiator (Fig.2). The position and/or velocity of the beam can be displayed by on a scope or can be read by a computer upon command. Familiarity with computer-based data acquisition is assumed for some parts of this experiment.

Measurements

- Check the linearity of the position transducer. Measure the bridge voltage as a function of a static displacement of the lower end of the beam.
- Check the performance of the differentiator. This can be done by using an input from a pulser or by clamping the lower end of the beam and moving the stage of the vibrator with sinusoidal excitation.
- Display \((x(t),v(t))\) on the oscilloscope and study the path of the system in phase space without the magnets. Mount the magnets and search for chaotic behavior.
- Connect to the computer interface and learn to observe phase space paths. Use simple harmonic motion to convince yourself that you get reasonable results.
- Generate Poincaré maps for simple and chaotic motion.
- Possible extensions include the measurement of the Fourier spectrum, or the experimental determination of the potential and subsequent numerical simulation of the system for comparison with the measured Poincaré section.

References